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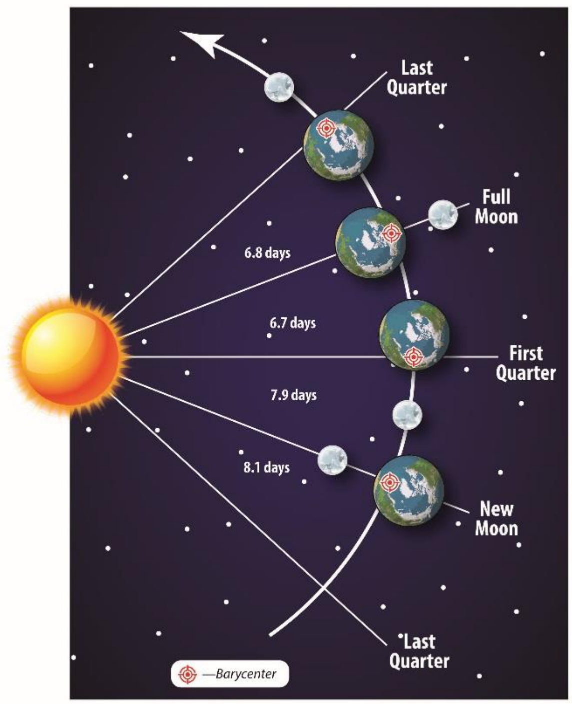
Motion of the Earth-Moon System about their Barycenter: Planetary Free Spheres Motion

Many diagrams of the motion of the Earth-Moon-Sun system show the moon revolving about the center of Earth. At syzygy Earth observes new moon and full moon, and at quadrature it is first quarter and last quarter. Such geocentric diagrams are used to explain the changing lunar phases from new moon to first quarter to full moon to last quarter, *etc*.

In fact though, Earth and its moon revolve about a common center of mass known as the barycenter (*Gr.* “heavy” center). The Earth-Moon system can be thought of as a having a “fulcrum”; the rotation of the two masses is about this point. As with a lever, there is balance when the mass of Earth times its distance to their barycenter is equal to the lunar mass times its distance to said barycenter.

Given that the radius of Earth $a = 6378\ km$, mass of Earth is $M_E = 5.97 \times 10^{24}\ kg$, mass of the moon $M_M = 7.35 \times 10^{22}\ kg$, and the mean distance between centers of mass is $R = 384{,}400\ km$. The barycenter’s location (r) is readily calculated from $M_E\ x\ r = M_M\ x\ (R - r)$; $r = 4675\ km$. Thus their barycenter is $a - r = 6378 - 4675 = 1703\ km$ below Earth’s surface.



It is the motion of the Earth-Moon system barycenter, which forms the ellipse that is the orbit about the sun. This is illustrated in the figure to the right. The orbit of Earth’s center about the sun is sinusoidal about this ellipse.

The revolution about the barycenter is a fundamental frequency (ω) balancing the gravitational attraction of the two bodies with the centrifugal force: $G\frac{M_E M_M}{R^2} = M_M \omega^2 (R - r)$, where G is the universal gravitational constant. Rearranging: $\omega^2 = (\frac{2\pi}{T})^2 = \frac{6.674\times10^{-11}\times5.97\times10^{24}}{(384{,}400\times10^3)^2\times(384{,}400-4675)\times10^3}$; the period of revolution is $T = 27.3\ days$, the sidereal month. As seen in the figure, the Earth-Moon system moves along in its solar orbit and it takes longer than 27.3 days to move from one last quarter to the next last quarter; this is the synodic month of 29.5 days.

Why would this matter?

Appreciating the motion of the Earth-Moon system about their barycenter leads to a clearer understanding of tides – ocean, atmospheric, and solid Earth. It illustrates that the time between lunar quarters is not uniform. It focuses attention on the importance of the barycenter as the point of revolution about the Sun, not Earth’s center.