Initiation of Streamers from Thundercloud Hydrometeors and Implications to Lightning Initiation

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Abstract

TITLE: Initiation of Streamers from Thundercloud Hydrometeors and Implications to Lightning Initiation

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Electric field values measured inside thunderclouds have consistently been reported to be up to an order of magnitude lower than the value required for the conventional electrical breakdown of air. This result has made it difficult to explain how lightning frequently occurs in thunderclouds. One theory that has been offered to explain the lightning initiation process is the theory of lightning initiation from hydrometeors. According to this theory, lightning can be initiated from electrical discharges originating around thundercloud water or ice particles in the measured thundercloud electric field. These particles, called hydrometeors, can cause significant enhancement of the thundercloud electric field in their vicinity, increasing the probability to initiate streamers that are precursor discharges for the hot lightning leader channel.

For this dissertation research, we have focused our efforts on studying streamer initiation and propagation from thundercloud hydrometeors. For the first part of this study, we have further investigated the idea proposed by Liu et al. [2012b] to study streamers from ionization column hydrometeors in thundercloud conditions. We have performed simulations for ambient electric field values as low as $0.3E_k$ at thundercloud altitudes. According to our results, initiation of stable streamers from thundercloud hydrometeors in a $0.3E_k$ electric field is possible,
only if enhanced ambient ionization levels are present ahead of the streamer. We investigate the streamer branching behavior and characteristics, and test a theory that has recently been proposed to explain this phenomenon [Savel’eva et al., 2013].

In order to verify whether an ionization column is a proper representation of a dielectric hydrometeor, for the second part of this dissertation we modify our streamer discharge model to accommodate an isolated dielectric particle representing the hydrometeor inside the computational region. The development of this model has enabled us to accurately simulate the discharges around dielectric hydrometeors with various shapes and physical states. Streamer discharge results obtained from the dielectric hydrometeor have been presented and compared with the results obtained from the first part of this work. We compare our modeling results with laboratory experiments and realistic thundercloud conditions and discuss the implications of this study to lightning initiation and other lightning related phenomena.
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Chapter 1

Introduction

1.1 Lightning Phenomenology

Lightning is a large electrical discharge that occurs in the atmosphere of the Earth as well as other planets and can have a length of tens of kilometers or more. More than 25% of the global lightning activity constitutes of cloud-to-ground discharges. The remaining 75% is called cloud discharges and does not involve the ground. Depending on the polarity of the effective charge being transported downward and the direction of propagation of the developing discharge, four different types of cloud-to-ground lightning have been identified. These include negative downward, positive downward, negative upward, and positive upward discharges. More than 90% of cloud-to-ground discharges are negative downward discharges [Rakov and Uman, 2003]. Figure 1.1 shows each of these discharge types.

The primary source of lightning is the cumulonimbus, more commonly known as the thundercloud. Thunderclouds can be thought of as large atmospheric heat
Figure 1.1: The four types of cloud-to-ground lightning flashes as defined from the direction of leader propagation and the charge on the initiating leader (from Dwyer and Uman [2014]).
engines. Their input energy comes from the sun and water vapor is the primary heat-transfer agent. The principle outputs of a thundercloud include (i) the mechanical work of vertical and horizontal winds, (ii) outflow of condensation in the form of rain and hail from the bottom of the cloud and of small ice crystals from the top of the cloud, and (iii) electrical discharges occurring inside, below and above the cloud [Rakov and Uman, 2003].

1.1.1 Thundercloud Charge Structure

Wilson [1925] suggested that thunderclouds typically have positive charge above negative charge in a positive dipole type of configuration. After Wilson, the generally accepted thundercloud charge structure was considered as three vertically stacked regions: main positive at the top, main negative in the middle, and a lower positive region at the bottom which may not always be present. The dominant positive charge in the upper part of the thundercloud causes a layer of charge of the opposite polarity to form on the upper cloud boundary. This is called the screening charge layer. It is thought that in any region of the thundercloud, charges of both polarity coexist, regardless of the polarity of the net charge in that region [MacGorman and Rust, 1998]. A sketch depicting the charge structure of the thundercloud is shown in Figure 1.2.

The magnitudes of the main positive and main negative charge regions are typically a few tens of coulombs, while the lower positive charge region is about ten coulombs or less. By approximating these charge regions as point charges, the electric field intensity due to the system of three charges can be found by replacing the perfectly conducting ground with three image charges [Rakov and Uman, 2003].
Hydrometeors are defined as liquid or frozen water particles abundant in the atmosphere. There are two main groups of hydrometeors: (i) those of which their motion is mainly influenced by gravity and (ii) all other hydrometeors. The particles of the first group are large enough to have appreciable fall speeds (> 0.3 m/s) and are called precipitation particles. The second group, called cloud particles, are too small to have an appreciable fall speed. Hydrometeors have many different combinations of size, state, and density. Cloud particles consist of cloud water droplets or droplets, and cloud ice particles or ice crystals. Liquid precipitation is sometimes divided by size and terminal fall speed into drizzle (radius ~ 0.1–0.25 mm) and rain (radius > 0.25 mm). Solid precipitation is subdivided by density and fall speed into snow (lowest density, with fall speeds of ~ 0.3–1.5 m/s), graupel (intermediate density, with fall speeds of ~ 1–3
m/s), and hail (greatest density and possibly larger in size, with fall speeds up to 50 m/s) [MacGorman and Rust, 1998].

There is no clear-cut size boundary between cloud and precipitation particles. Smaller particles are loosely called “cloud particles” and larger particles are called “precipitation particles”, without specifying a definite size. Nevertheless, we can still speak of typical sizes for these particles. Usually the following six types of particles are distinguished in clouds [Wang, 2013]. The sizes indicate the radius of the particle, unless otherwise noted:

- Cloud drops - water drops that are suspended in the cloud. Their typical size range is from a few micrometers to \( \sim 500 \mu m \).
- Raindrops - water drops that are falling and may eventually reach the ground. Typical size range is from a few hundred micrometers to \( \sim 3 \) mm. Drizzle drops are a subcategory of raindrops with radius smaller than \( 250 \mu m \).
- Ice crystals - these refer to clean crystalline ice particles that are predominantly of the hexagonal shape. However, needle or long solid column shapes are also frequent. Their typical sizes range from a few tens of micrometers to a few hundred micrometers.
- Snowflakes - crystalline ice particles of relatively large size. Their typical size ranges are from a few hundred micrometers to a few centimeters.
- Graupel - when ice or snow crystals collide with supercooled water drops, eventually a particle is produced called graupel. Graupel is considered the
precursor of hail. By convention, graupel particles must be smaller than 5 mm in diameter. If it grows greater than that it will be called hail.

- Hail - hailstones are the largest particles in a precipitation system. They should be larger than 5 mm in diameter but can reach up to 15 cm.

### 1.1.3 Cloud Electrification

Determining how thunderclouds are electrified has been the goal of many experiments and field observations for decades, during which many different cloud electrification theories have been proposed [e.g., Wilson, 1929, Chiu and Klett, 1976, Takahashi, 1978]. Any cloud electrification mechanism must involve two components: a small-scale process that electrifies individual hydrometeors, and a process that spatially separates these charged hydrometeors, resulting in distances between cloud charge regions on the order of kilometers [Rakov and Uman, 2003].

Here we will briefly overview one of the more popular cloud electrification mechanisms, the *noninductive graupel-ice mechanism*. “Noninductive” refers to any mechanism that does not require polarization of the hydrometeors by an electric field. In this mechanism, electric charges are produced by collisions between graupel particles and ice crystals in the presence of water droplets, and the large-scale separation of the charged particles is due to the action of gravity. The existence of water droplets is necessary for significant charge transfer, as demonstrated by laboratory experiments [Reynolds, 1953]. When heavy graupel particles fall through a suspension of small ice crystals and supercooled water drops, they can acquire negative or positive charges in collision with ice crys-
Figure 1.3: Charge transfer by collision between graupel and ice particles, representing the graupel-ice mechanism of cloud electrification (from Rakov and Uman [2003]).

tals. The resultant charge depends on a critical temperature value called the reversal temperature, $T_R$. When the temperature is below $T_R$, graupel acquires negative charge, and when the temperature is above $T_R$, it acquires positive charge [Rakov and Uman, 2003]. The reversal temperature is assumed to be -15 °C and occurs at 6 km altitude. A sketch of this process is shown in Figure 1.3.

1.1.4 Electric Field Measurements Inside Thunderclouds

The electric field magnitude inside the thundercloud can be measured by many different means which are either remote (outside the cloud) or in situ (inside the cloud). In situ measurements of the cloud electric field have been made
using (i) balloons carrying corona probes, (ii) balloons carrying electric field meters, (iii) aircraft, (iv) rockets, and (v) parachuted electric field mills [Rakov and Uman, 2003].

However, measuring the electric field inside a thundercloud is a very difficult task for several reasons: (1) Thunderstorms are a large and violent environment, which makes it challenging to make in situ measurements. (2) The thunderstorm fields often change rapidly, on the time scale of seconds, and so even a jet aircraft will only sample a small part of the cloud before the field changes. (3) Finally, some lightning initiation models postulate that lightning forms from small water droplets or ice. Placing a large (sometimes wet) object, such as balloon, aircraft or rocket, could artificially discharge the thundercloud before the field has a chance to build up to the point where lightning might be naturally initiated. In other words, the act of observing the system may substantially perturb it [Dwyer and Uman, 2014].

Much research dedicated to measuring thundercloud electric fields can be found in the literature [e.g., Winn and Moore, 1971, Winn et al., 1974, Marshall and Winn, 1982, Marshall and Rust, 1993, Stolzenburg et al., 1998a, Stolzenburg et al., 2007]. Many electric field measurements from inside active storms show electric field magnitudes that are typically less than 150 kV/m [e.g., Marshall and Rust, 1991], with a few extreme values of up to 1000 kV/m [e.g., Winn et al., 1981]. The maximum electric field magnitudes measured inside the cloud have significant importance in understanding the processes that govern lightning initiation. Table 1.1 summarizes the reported maximum electric field magnitudes measured inside thunderclouds.
Table 1.1: Maximum electric field magnitudes measured in thunderclouds.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Type of measurement</th>
<th>Maximum field (kV/m)</th>
<th>Altitude of max. field (km)</th>
</tr>
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<tbody>
<tr>
<td>Gunn (1948)</td>
<td>Aircraft</td>
<td>340</td>
<td>4</td>
</tr>
<tr>
<td>Winn et al. (1971)</td>
<td>Rocket</td>
<td>60</td>
<td>5.5</td>
</tr>
<tr>
<td>Winn et al. (1974)</td>
<td>Rocket</td>
<td>400</td>
<td>6</td>
</tr>
<tr>
<td>Winn et al. (1981)</td>
<td>Balloon</td>
<td>1000</td>
<td>6</td>
</tr>
<tr>
<td>Weber et al. (1982)</td>
<td>Balloon</td>
<td>110</td>
<td>8</td>
</tr>
<tr>
<td>Byrne et al. (1983)</td>
<td>Balloon</td>
<td>130</td>
<td>8</td>
</tr>
<tr>
<td>Fitzgerald (1984)</td>
<td>Aircraft</td>
<td>120</td>
<td>8.8</td>
</tr>
<tr>
<td>Marshall and Rust (1991)</td>
<td>Balloon</td>
<td>146</td>
<td>–</td>
</tr>
<tr>
<td>Stolzenburg et al. (1998)</td>
<td>Balloon</td>
<td>120</td>
<td>5</td>
</tr>
<tr>
<td>Stolzenburg et al. (2007)</td>
<td>Balloon</td>
<td>127</td>
<td>13.4</td>
</tr>
<tr>
<td>Stolzenburg et al. (2007)</td>
<td>Balloon</td>
<td>200</td>
<td>12.2</td>
</tr>
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1.1.5 Electric Fields Observed Near Lightning Initiations

Obtaining data on the electric field magnitude close to the lightning initiation region is even more difficult. Statistics show that the majority of sounding instruments are not near a localized region of lightning initiation, either intentionally or non-intentionally [Rakov and Uman, 2003]. The instruments that are exposed to such extreme conditions are unlikely to acquire useful data due to the damage to the instruments. A thundercloud is a very large region, and the volume of large electric field values near lightning initiation may be very small. Hence the observations of electric field magnitudes right before a lightning flash occurs are rare.

Stolzenburg et al. [2007] examined more than 250 balloon soundings of the cloud electric field magnitudes. Among these, only 9 samples were close enough to the lightning initiation locations that the balloon or instruments were severely affected. The largest electric field magnitude observed from these cases was 626 kV/m scaled to ground pressure (about 127 kV/m at 13.4 km altitude), and the largest estimated magnitude was 929 kV/m scaled to ground pressure (about 200 kV/m at an estimated altitude of 12.2 km).
1.1.6 Lightning Initiation Altitudes

The altitude of lightning initiation is one of the parameters that can be inferred from electric field measurements. Using a vertical profile of the thundercloud electric field, a one-dimensional approximation to Gauss’s law is sufficient to carry out this analysis [Stolzenburg et al., 1998a]. This can be done because the largest component of the electric field in a sounding is usually in the vertical direction. Furthermore, electric potential as a function of altitude can be estimated from electric field soundings by integrating upward from the ground [Marshall et al., 2001].

Figure 1.4 shows observed electric field values from 50 different balloon measurements, as a function of altitude. The figure shows the theoretical threshold electric field for conventional and runaway breakdown processes (see two theories for lightning initiation, discussed in Section 1.3). The measurements in the figure are for three different storm types, each corresponding to different color marks. The circled values are extreme values associated with close lightning. The two values enclosed by red boxes are at known distances of 1100–2100 m from a subsequent lightning initiation location.

Figure 1.4 gives an indication of typical altitudes for lightning initiation. The most likely electric field values to initiate lightning are the points closest to the threshold curves. For the positive values, this altitude ranges between 4–6 km. This altitude range typically corresponds to the region between the lower positive and main negative charge regions in the storm, making negative cloud-to-ground lightning the most probable type of lightning to occur [e.g., Stolzenburg et al., 2003]. On the negative side of measured electric fields, the peak values are observed between about 6–11 km, corresponding to the region
Figure 1.4: Observed electric fields from 50 balloon soundings, as a function of altitude. The filled marks show typical values within updrafts, and open marks show values for outside of updrafts for three different storm types. The circled values are extreme values associated with close lightning. The two values enclosed by red boxes are at known distances of 1100–2100 m from a subsequent lightning initiation location (from Stolzenburg and Marshall [2009]).
above the main negative charge layer [e.g., Marshall and Rust, 1991]. This is the region that intra-cloud lightning flashes are more likely to occur [e.g., Stolzenburg et al., 1998b]. Generally, the altitude of lightning initiation greatly depends on the type of the thunderstorm, the charge structure of the cloud, and the location of measurements [Rakov and Uman, 2003]. Different initiation altitudes give rise to different types of lightning discharges [e.g., Krehbiel et al., 2008].

1.2 Electrical Discharge Processes in Air

Air is generally a good insulator and can maintain it’s insulating properties until the applied electric field exceeds about $3.2 \times 10^6$ V/m at standard atmospheric temperature and pressure. If the background electric field exceeds this critical value, free electrons in air will gain enough energy to ionize other atoms and molecules through collisions. This ionization leads to an increase in the number of electrons, which initiates the electrical breakdown of air. This critical threshold value is called the conventional breakdown threshold field of air ($E_k$) and is defined as the equality of the ionization and dissociative attachment coefficients in air [Raizer, 1991, p.135]. Its value scales with atmospheric density, $N$, as follows:

$$E = E_0 \frac{N}{N_0}, \quad (1.1)$$

where $E_0$ and $N_0$ are the critical electric field and the density of air at standard atmospheric conditions, respectively. The air density in the Earth’s atmosphere decreases exponentially with height as $N = N_0 e^{-z/H}$, with $H \approx 7$ km being the
atmosphere scale height. So the critical electric field required to cause electrical breakdown in the atmosphere will also decreases with height:

\[ E = E_0 e^{-z/H}. \]  

(1.2)

Different types of electrical discharges that occur in the atmosphere consist of four basic processes, electron avalanches, streamer discharges, corona discharges, and leaders. A brief description of each of these discharge types will be provided below.

1.2.1 Electron Avalanche

An electron avalanche is the primary element of any breakdown mechanism. Consider a free electron moving under the influence of a background electric field. If the background field is larger than \( E_k \), the electron may produce another electron through ionization collisions. These two electrons will give rise to two more electrons and this process continues [Cooray, 2012, p.68]. Let \( \alpha \) be the number of ionizing collisions per unit length, and \( \eta \) the number of electron attachments per unit length. In traveling across a length of \( dx \), \( n_{e0} \) number of electrons will give rise to \( dn \) additional electrons:

\[ dn = n_{e0}(\alpha - \eta)dx. \]  

(1.3)

This equation shows that the number of electrons will increase exponentially:

\[ n_e = n_{e0}e^{(\alpha-\eta)x}. \]  

(1.4)
The exponential growth of the number of electrons with distance is called an electron avalanche. The critical electric field beyond which air will electrically breakdown is achieved when $\alpha = \eta$. For electric fields below the breakdown threshold, $(\alpha - \eta) < 0$, and for fields higher than the threshold, $(\alpha - \eta) > 0$ [Cooray, 2012, p.68].

1.2.2 Streamer Discharges

Streamer discharges are narrow filamentary plasma discharge channels, which are the precursors of the lightning leader [Rakov and Uman, 2003, p.137]. A streamer is a weakly ionized thin channel formed from an avalanche in a sufficiently strong electric field [Raizer, 1991, p. 334]. As an electron avalanche is moving forward, the number of charged particles in the avalanche head is increasing. It has been estimated that an avalanche will transition into a streamer if the number of positive ions in the avalanche head reaches a value of about $10^8$ [Loeb and Meek, 1940] at ground pressure. Thus the condition for the transformation of an avalanche to a streamer can be written as below [Cooray, 2012]:

$$\exp\left(\int_0^{x_c} [\alpha(x) - \eta(x)] dx\right) = 10^8 - 10^9,$$

(1.5)

where $x$ is the distance from the origin of the avalanche, and $x_c$ is the distance from the origin of the avalanche to where the background field falls below $E_k$.

A streamer discharge is either positive or negative depending on the polarity of the charge in its head. Each of these types has a different propagation mechanism. The mechanism of propagation of a positive streamer is depicted in Figure 1.5a. The propagation direction of the positive streamer is the same...
Figure 1.5: (a) A positive streamer at two consecutive moments of time. The wavy arrows represent photons that generate seed electrons for avalanches. Secondary electron avalanches move toward the positive head of the streamer. (b) A negative streamer at two consecutive moments of time. Secondary avalanches move in the same direction as the streamer channel (from Raizer [1991]).

as the direction of the ambient electric field. Multiple electron avalanches move toward the streamer head and increase the electron density rapidly, while leaving positive ions behind. As the electron avalanches propagate towards the streamer head, not only do they ionize neutral molecules, but they also excite them. The excited atoms emit energetic photons which produce photoionization. The photoelectrons will be attracted toward the positive space charge and neutralize them, while exciting new molecules and leaving new positive ions in their trails. The positive ions form the new streamer head and the process starts over again.

For a negative streamer, the propagation mechanism is somewhat different due to the fact that electrons drift in the same direction as the streamer propagation. As a result of photoionization, secondary avalanches are produced in front of the negatively charged streamer head. The plasma channel evolves in
turb the electric field and are no longer independent, which leads to the streamer mechanism of discharges. Streamer discharge theory was put forward in the 1930's to explain spark discharge [Loeb and Meek, 1940]. The theory is based on the concept of a streamer. Streamers are narrow filamentary plasmas, which are driven by highly nonlinear space charge waves [e.g., Raizer, 1991, p. 327]. The dynamics of a streamer is mostly controlled by a highly enhanced field region, known as a streamer head, and the streamer polarity is defined by the sign of the charge in its head. A schematic illustration of a positive streamer propagating in an ambient electric field is given in the left panel of Figure 2.2. The right panel of the figure gives the distribution of electron density, electric field and space charge along the central axis of the streamer that is usually considered having cylindrical symmetry. The propagation direction of the positive streamer is the same as the direction of the ambient electric field. A large amount of space charges exists in the streamer head (the shaded region in the figure), which strongly enhances the electric field in the region just ahead of the streamer, while screening the ambient field out of the streamer channel (the region behind of the streamer head). The peak space charge field can reach a value about 4-7 times breakdown field $E_k$ [e.g., Dhali and 28]

Figure 1.6: Schematic of a positive streamer propagating in an ambient electric field (from Bazelyan and Raizer [1998]).

the same direction as the electron avalanches close to its head [Raizer, 1991, p. 337]. Figure 1.5b shows the propagation of a negative streamer.

For both positive and negative streamers, the dynamics of the streamer is greatly dependent on the very high field region of the streamer head. Figure 1.6 shows a positive streamer propagating in an ambient electric field. The streamer head hosts a large amount of space charge which greatly enhances the electric field ahead of the streamer, and screens out the ambient electric field behind the streamer head (inside streamer channel). The right panel of Figure 1.6 shows profiles of electric field, electron density, and space charge along the streamers central axis (along the streamer body).

1.2.3 Corona Discharges

In many cases, the high electric field around a sharp object is strongly localized. In this case, the discharge activity will be concentrated in a very small volume around the object. This type of discharge activity is called a corona discharge [Raizer, 1991]. During corona discharges ionic space charge of both polarities accumulate near the highly stressed electrode, which modifies the electric field

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distribution. Similar to streamer discharges, corona discharges have positive and negative polarity depending on the polarity of the electrode.

Corona discharges can occur from conductors in a variety of different locations and situations in the atmosphere. For example, corona can occur from pointed objects connected to the ground, from cables and wires above the ground, and in regions of high electric field magnitudes in thunderclouds from rain and wet ice particles [e.g., Loeb, 1966]. The electrons emitted by corona can form regions of space charge in the thundercloud and some of these sources make a significant contribution to other forms of electrical discharges in the atmosphere (see section 3.9 for more details).

1.2.4 Leader Discharges

A leader is a highly ionized, highly conductive channel that grows along the path prepared by preceding streamers [Raizer, 1991]. Compared to a streamer, a leader discharge is a hot discharge ($\sim 5000–6000$ K) and the conductivity of its channel is very high. The leader head has strong electric field magnitudes. Therefore, many streamers are sent out from the leader tip, and they draw a large total current from it. The current heats the gas and creates a new head, so the strongly ionized leader channel can advance [Raizer, 1991].

The most important condition necessary for leader formation is an increase in gas temperature, leading to a sustained high channel conductivity. An individual streamer is a cold discharge (about room temperature) and the current associated with it cannot heat the air sufficiently to sustain its high conductivity in the channel. However, if the current from a group of streamers with a common origin (called the streamer stem) is combined, the common channel
will heat up and the conductivity of the channel can be maintained [Cooray, 2012]. Thus, the current flow will be concentrated to a thin channel (i.e. the streamer stem) and produce more heat and more ionization [Raizer, 1991]. With increasing ionization, the process of transformation of the streamer stem into a hot and conducting leader channel will take place.

1.3 Lightning Initiation

One of the important unanswered questions in atmospheric electricity is how lightning is initiated inside thunderclouds. Despite the progress in the field of atmospheric electricity in the recent decades, the specific mechanism and the exact ambient electric field magnitude available for lightning initiation remain unknown. Two theories have been proposed as the underlying mechanism of lightning initiation; conventional and runaway breakdown. We will discuss them in the following sections.

1.3.1 Conventional Breakdown Theory

One theory of air electrical breakdown that has been applied to explaining the initiation of lightning discharges is the conventional breakdown theory [e.g., MacGorman and Rust, 1998, p.86, Rakov and Uman, 2003, p.121]. According to this theory, in order for a lightning leader to form, the electric field value inside some part of the cloud must exceed the conventional breakdown threshold field of air. Despite the constant lightning activity observed in the atmosphere, years of in situ measurements of the thundercloud electric fields have consistently shown electric field values of well below the breakdown field. The maximum
values recorded in Table 1.1 are insufficient to initiate electron avalanches and then the electrical breakdown process.

To overcome this obstacle, the theory of lightning initiation from thundercloud hydrometeors (water drops, ice crystals, etc.) was brought forward [e.g., Dawson, 1969, Griffiths and Latham, 1974, Crabb and Latham, 1974]. Hydrometeors are abundant in the thundercloud environment (see section 1.1.2) and when they are subject to an external electric field, they can cause significant field enhancement in their vicinity. A critical component of the conventional breakdown theory is to demonstrate that streamers are able to form and propagate in an electric field magnitude similar to the observed thundercloud electric fields. Investigation of this topic is the focus of this dissertation research.

1.3.2 Runaway Breakdown Theory

Another theory for lightning initiation is the runaway breakdown theory. Wilson [1925] discovered the runaway electron mechanism in which fast electrons may obtain large energies from static electric fields in air. When the rate of energy gain from an electric field exceeds the rate of energy loss from interactions with air then the energy of an electron will increase and it will “run away” [e.g., Dwyer, 2003, Dwyer et al., 2012]. The threshold electric field at which this happens is about $2.8 \times 10^5$ V/m and is scaled with air density. Gurevich et al. [1992] showed that runaway electrons can undergo avalanche multiplication, resulting in a large number of high energy runaway electrons. This avalanche mechanism is commonly referred to as the Relativistic Runaway Electron Avalanche (RREA) mechanism [Babich et al., 1998, Babich et al., 2001]. Gurevich and Zybin [2001] hypothesized that the RREA mechanism can lead to the electrical breakdown
of air, which they called “runaway breakdown”.

The threshold electric field for runaway breakdown is an order of magnitude less than the conventional breakdown threshold field. This fact has intrigued many researchers to investigate the implications of this mechanism on lightning initiation and lightning related phenomena [e.g., Dwyer, 2005, Dwyer, 2007, Dwyer, 2010, Dwyer et al., 2012]. Although this theory offers an advantage over the conventional breakdown theory, a lower threshold field, it has difficulty in producing the spatially compact lightning channel and its high electron density. Also, several authors have noted that runaway breakdown process itself may discharge the large-scale electric fields inside thunderstorms, shutting down the processes leading to lightning initiation [e.g., Gurevich et al., 1992, Marshall et al., 1995, Solomon et al., 2001]. The RREA processes can at most provide ambient conditions (e.g., producing a plasma domain) for the compact lightning channels to develop [Babich et al., 2012].

1.4 Scientific Contributions

The occurrence of a lightning flash entails highly complicated processes. A comprehensive understanding of lightning requires understanding all the microphysical processes involved in electrical breakdown, and the macroscale processes in flash propagation that typically covers tens of kilometers of path length in a single flash. For this dissertation, we investigate the initial stage of the lightning initiation process, by means of streamer emission from thundercloud hydrometeors, through theoretical and numerical modeling. The major scientific contributions resulting from this dissertation work are summarized as follows:
1. We have developed a model capable of simulating streamers initiated from isolated dielectric hydrometeors. This model is a modification of the sprite streamer model by Liu and Pasko [2004]. It can be used to simulate any dielectric hydrometeor shape with irregular curved boundaries. This model allows studies of positive and negative streamers forming from a hydrometeor, and analysis of the streamer and hydrometeor interactions.

2. We have established that streamers can be initiated from model hydrometeors in maximum observed thundercloud electric fields. The lowest ambient electric field capable of initiating a streamer from realistic hydrometeors at 7 km altitude is found to be $0.3E_k$, where $E_k$ is the conventional breakdown threshold value.

3. Background ambient electron density existent inside the thundercloud has been identified to promote the formation and propagation of stable streamers. Depending on the magnitude and distribution, this density affects the streamer formation stage, and can be a determining factor on whether the streamer branches, recovers after the pre-branching stage, or continues propagating stably.

4. Geometry of the streamer head has been determined to play an important role in the streamer branching phenomenon. The fast radial movement of the maximum streamer head curvature, combined with the slow reduction of the maximum curvature value causes the streamer head to eventually pull the maximum electric field off of the symmetry axis. The redistribution of the electric field profile with the maximum moving away from the axis is a critical turning point for the streamer head. As the location of
the maximum electric field is also the fastest growth point of the streamer head surface, this phenomenon results in the streamer propagating off-axis and hence branching.

5. The dimension of the model hydrometeor has also been identified as a critical parameter for streamer formation. The minimum length required for the hydrometeor in order to initiate streamers in a $0.3E_k$ field has been obtained to be between 5 and 8 mm, when a background ambient density on the order of $10^{15}$ m$^{-3}$ is included. The minimum length decreases slightly with increasing background density. This range overlaps with the range of lengths obtained from experimental studies on streamers from hydrometeors.
Chapter 2

Overview of Previous Experimental and Modeling Studies

As described in Chapter 1, the conventional breakdown theory for lightning initiation suggests that hydrometeors in the thundercloud locally enhance the electric field to a value greater than the conventional breakdown threshold so that the discharge process can be initiated. In this chapter, we will give a brief overview of the laboratory experiments and theoretical studies that have been dedicated to test this theory. Later in the chapter we will describe the numerical model that we have used for the present work.
2.1 Laboratory Experiments on Streamers from Hydrometeors

The majority of the experiments designed to investigate the conditions under which streamer corona discharges may be produced have been conducted on two main hydrometeor groups: water drops and ice crystals. Richards and Dawson [1971] suggested that the collision of a pair of raindrops temporarily produces an elongated filament of water, which provides a favorable site for the initiation of streamers. The second group, ice crystals, have been used to examine the conditions under which streamers can be initiated from their extremities. Blyth et al. [1998] concluded that these two are the only two microphysical situations that appear to be capable of initiating a discharge in the low thundercloud electric fields. A summary of these experimental studies is presented below.

2.1.1 Collision Between Water Drops

Crabb and Latham [1974] measured the electric field values required to produce streamer corona discharges from a pair of colliding water drops. According to their experiments, a pair of raindrops colliding within a thundercloud produces a deformed elongated object whose shape is particularly favorable for corona emission in weak electric fields. Figure 2.1 shows a schematic of the experimental setup with two drops colliding, and a photograph of the drop formed after collision. Their experiments primarily focused on positive corona emission, as it may play an important role for lightning initiation. From their measurements in the laboratory, maintained at ground pressure, electric field values for corona onset ranged from about 250–500 kV/m for the combined length of the filament.
between 5–25 mm. They also confirmed the pattern of decreasing onset field with the increasing combined length of the liquid filament suggested by Griffiths and Latham [1974] for ice specimen. They concluded that the emission of positive coronas from colliding raindrops can readily occur in thunderclouds and therefore be a possible source for triggering lightning.

### 2.1.2 Ice Particles

Experimental studies conducted by Griffiths and Latham [1974] have shown that positive streamers and various forms of corona discharges (e.g., steady glows, Trichel pulses, breakdown streamers, and sparks) can be produced from ice hydrometeors a few millimeters in length suspended in a uniform electric field, for temperatures above −18°C. They studied many different types of ice specimen, all showing that the currents obtained upon onset are sufficient to produce all
the mentioned types of discharges, including the continuous initiation of positive streamers. They suggested that hydrometeors with longer lengths can produce streamer discharges easier than shorter lengths, due to higher polarization charge and field enhancement at their tips. They concluded that the role of streamer corona discharges from ice hydrometeors in a thunderstorm is of great importance to lightning initiation, and the ambient fields necessary for corona emission from ice in the central regions of thunderclouds are probably in the range of 400−500 kV/m at ground pressure.

In further work, Griffiths [1975] examined the dependence of the electric field required for corona onset on the electric charge carried by ice crystals. It was found that the charges on the order of 10^{−10} C carried by the specimens reduced the required electric field value by as much as 20%. Additionally, Griffiths and Latham [1974] established that the temperature below which substantial corona currents were inhibited due to surface conductivity reduction depended on the purity of the ice sample. This value is reduced from −18°C to −25°C when the surface of a pure sample was contaminated with a solution containing 4.5 mg of ammonia per kilogram of water.

More recently, Petersen et al. [2006] conducted a series of laboratory experiments on positive streamer emission from ice hydrometeors. The vertical dimensions of the ice crystals ranged from 0.5−3 mm. Their results show that positive streamer discharges are able to occur at temperatures as low as −38°C when subject to electric fields between 500−850 kV/m at ground pressure. This result is in contrast with the finding by Griffiths and Latham [1974], which indicate that positive streamers can only occur at temperatures greater than −18°C for pure ice crystals. Figure 2.2 shows a sample photograph from their experiments;
an ice crystal cluster and a positive streamer forming from it. They concluded that the results from their experiments suggest that positive streamers from frozen precipitation particles in the high altitude regions of the thundercloud may occur and may in fact be a necessary element for the lightning initiation process.

Petersen et al. [2014] extended their previous work by reporting a new laboratory investigation on the initiation of streamer discharges from small ice crystals. They observed different types of electrical discharges (glow corona, pulse corona, and positive and negative streamers) from a variety of vapor-grown ice crystal shapes. Figure 2.3 shows a grown ice crystal habit and examples of such discharges. The primary parameters that effect the threshold electric field for positive streamer initiation were determined to be the length of the ice crystal and air density. Observed threshold electric fields for positive streamers at $-14^\circ$C and 570 mb pressure (about 5 km altitude) were between 700–2000 kV/m, for ice crystal lengths decreasing from 5 mm to 1 mm, larger than values observed previously [e.g., Griffiths and Latham, 1974, Petersen et al., 2006]. The
sharpness of the tip of the ice hydrometeors also had an effect on the threshold electric field, with the sharpest tips inhibiting positive streamer formation while promoting glow coronae. A general equation to express the threshold electric field required for positive streamer generation in terms of vertical ice crystal length was determined. The authors concluded that single streamers can form from individual ice hydrometeors at very cold temperatures. Further, they concluded that this fact along with the large number of individual ice hydrometeors in a meter-scale volume suggests that the positive streamer system mechanism of lightning initiation, described by Griffiths and Phelps [1976], may be possible at the very cold temperatures characteristic of high altitudes.
2.2 Theoretical Studies on Streamers from Hydrometeors

Phelps [1974] hypothesized that a single positive streamer, subjected to a strong electric field, would rapidly intensify and branch. While this intensifying streamer system propagates, it carries an increasing amount of positive charge in its tip, and deposits negative charge in its trail. This charge distribution results in a local dipole that enhances the electric field at the positive streamer origin. If a series of these streamer systems were to propagate sequentially, each one advancing into the charge debris of its predecessors, the local electric field can intensify up to the conventional breakdown threshold, implying that the resulting strong electric field could support the emergence of a lightning leader.

Griffiths and Phelps [1976] used a numerical model to calculate the electric field enhancement in a thundercloud due to the propagation of a growing system of positive streamers, similar to what was described in Phelps [1974]. Their model assumes that the streamer system propagates through a conical volume in a series of steps. The positive charge in the head of the system is assumed to be uniformly distributed on a circular disc, constituting the base of the cone. The deposited negative charge is assumed to be located on a series of discs, also uniformly charged, located halfway between each step. Supposing the streamer system starts with an initial charge \( Q_0 \), advancement of each positive disc is assumed to add \( \Delta Q \) charge onto it. An amount of charge \(-\Delta Q\) will be deposited uniformly on a disc halfway between the step, forming the next negative disc. For the passage of the first streamer, the electric field is equal to the chosen constant ambient field. However, in order to model subsequent pas-
sages of streamers into the deposited charge debris of the preceding streamers, the field computed at each step must be a sum of the ambient field along with the field due to the charge deposited by all previous streamer systems. The authors conclude that the described model is capable of producing electric field magnitudes as strong as the conventional breakdown, by the multiple passages of streamer systems.

Schroeder et al. [1999] conducted a modeling study of positive streamer emission around a simulated coalesced water drop. They modeled positive discharges using a one-dimensional drift-diffusion equation, paired with an analytical solution to the electric field. They studied the positive streamer mechanism and obtained results appearing to be consistent with the laboratory observations of Crabb and Latham [1974]. Their results show that the minimum electric field required for a positive streamer to form from a pair of colliding water drops (drop sizes similar to Crabb and Latham [1974]) is 500 kV/m at ground pressure. They also concluded that the hydrometeor shape is an important factor in determining the minimum electric field for positive streamer formation. Solomon et al. [2001] used the same model to evaluate the two mechanisms for lightning initiation. They found the required electric field for positive streamers to form from ice particles must be about 600 kV/m at 500 mb pressure. For positive streamers from colliding raindrops, this field was determined to be about 200 kV/m at 500 mb.

Although there are important experimental studies on streamer corona discharges from hydrometeors, the theoretical studies that have been carried out to investigate the formation of a streamer near a hydrometeor all use primitive plasma discharge models. These models have several disadvantages that
result in an inaccurate and incomplete description of the streamer initiation process. For one, they are one-dimensional models, despite the streamer being a two-dimensional object at the least, under the consideration of cylindrical symmetry. There is strong nonlinear coupling between the space charge field generated by the streamer and the particle transport, which is not included in any of these models. Also, these models lack the ability to simulate the interactions between the streamer and the hydrometeor. Liu and Pasko [2004] developed a complete, very accurate and robust streamer model to study the dynamics of air discharges, called "sprites", in the upper atmosphere (see Section 2.3 for details). The successful application of this model to studying sprite discharges have been reported numerously [e.g., Liu and Pasko, 2004, Liu et al., 2006]. Further development of this model provides the ideal tool to study lightning initiation by streamer emission from thundercloud hydrometeors.

Liu et al. [2012a] investigated the inception conditions of positive corona discharges around thundercloud hydrometeors that are simulated as a spherical point electrode. The inception of positive corona discharges occurs when the electrical discharge around a hydrometeor becomes self-sustaining, i.e., when the discharge can produce enough numbers of seed electrons to sustain itself around the hydrometeor. For a 1 mm radius hydrometeor at thundercloud altitudes, it takes about 590 pC to charge the hydrometeor to trigger the corona, and the onset surface field is $5.3 \times 10^6 \text{ V/m}$. The required amount of the charge increases as the size of the hydrometeor increases. It was also found that the corona discharge current is about $0.06 \mu\text{A}$, which agrees with the corona onset current of $0.1 \mu\text{A}$ measured by Griffiths and Latham [1974]. This current could remove all the charge on the hydrometeor if the self-sustaining discharge lasted for several
milliseconds. The limit of hydrometeor charge set by corona discharges was found to agree well with measurements of precipitation charge [e.g., Weinheimer et al., 1991, Marshall and Rust, 1993, Bateman et al., 1999, Mo et al., 2007].

In a different study, Liu et al. [2012b] presented a modeling study on streamer initiation from thundercloud hydrometeors in subbreakdown (below the conventional breakdown threshold electric field) conditions. They conducted a simulation for an electric field value of $0.5E_k$ at ground pressure. Their model hydrometeor had a length of 5 mm and radius of 0.1 mm. They observed successful streamer emission from this hydrometeor, as shown in Figure 2.4. The figure shows cross-sections of electron density and electric field. Theirs was the first theoretical study to show streamers are able to form from isolated model hydrometeors in electric fields close to the measured thundercloud field.

### 2.3 Streamer Discharge Model

For the present study, we further investigate the idea proposed by Liu et al. [2012b] to study streamers from hydrometeors in thundercloud conditions. To describe the dynamics of a single streamer, we use the cylindrically symmetric streamer discharge model developed by Liu and Pasko [2004]. In this model, the streamer dynamics are described by the electron and ion drift-diffusion equations coupled with Poisson’s equation:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot n_e \mathbf{v}_e - D_e \nabla^2 n_e = (\nu_i - \nu_{a2} - \nu_{a3})n_e - \beta_{ep} n_e n_p + S_{ph}, \quad (2.1)$$
Figure 2.4: Cross-sectional views of distributions of electron density and electric field of a streamer at ground pressure (Liu et al. [2012b]).

\[
\frac{\partial n_p}{\partial t} = \nu_i n_e - \beta_{ep} n_e n_p - \beta_{np} n_p n_p + S_{ph},
\]

\[
\frac{\partial n_n}{\partial t} = (\nu_{a2} + \nu_{a3}) n_e - \beta_{np} n_p n_p,
\]

\[
\nabla^2 \phi = -\frac{e}{\varepsilon_0} (n_p - n_e - n_n),
\]

where \(n_e, n_p, \) and \(n_n\) are the electron, positive ion, and negative ion number densities, respectively; \(\vec{v}_e = -\mu_e \vec{E}\) is the drift velocity of electrons, with \(\mu_e\) being the absolute value of electron mobility and \(\vec{E}\) the electric field; \(\nu_i\) is
the ionization frequency; $\nu_{a2}$ and $\nu_{a3}$ are the two-body and three-body electron attachment frequencies, respectively; $\beta_{ep}$ and $\beta_{np}$ are the electron-positive ion and negative-positive ion recombination coefficients, respectively; $D_e$ is the electron diffusion coefficient; $S_{ph}$ is the electron-ion pair production rate due to photoionization; $\phi$ is the electric potential; $e$ is the absolute value of electron charge; and $\varepsilon_0$ is the permittivity of free space. Ions have much smaller mobilities compared to electrons and for the timescales involved in this study (a few tens of nanoseconds) they can be considered immobile. All the coefficients in the model are functions of the reduced local electric field $E/N$, where $E$ is the magnitude of the electric field and $N$ is the neutral air density. The coefficients are obtained from the solution to the Boltzmann equation [Moss et al., 2006], with a few exceptions discussed by Liu and Pasko [2004]. The photoionization rate $S_{ph}$ is calculated using the methods developed in Bourdon et al. [2007] and Liu et al. [2007]. The boundary conditions on the sides of the simulation region are Dirichlet boundary conditions. The direct integral solution to the electric potential is used to calculate the potential at the boundary [Liu and Pasko, 2006].
Chapter 3

Streamers from Ionization

Column Hydrometeors

3.1 Model Description

For this part of our study we model the hydrometeors using an isolated column consisting of free electrons and positive ions. The column consists of two hemispherical caps attached onto a cylindrical body. The initial distributions of electrons and positive ions in the column are described by the following equations:

for $z > z_t$:

$$n_{e0}(r, z) = n_{p0}(r, z) = \frac{n_0}{2} \left[ 1 + \tanh \left( \frac{a - \sqrt{r^2 + (z - z_t)^2}}{\sigma} \right) \right]$$

(3.1)

for $z < z_b$:

$$n_{e0}(r, z) = n_{p0}(r, z) = \frac{n_0}{2} \left[ 1 + \tanh \left( \frac{a - \sqrt{r^2 + (z - z_b)^2}}{\sigma} \right) \right]$$

(3.2)
and for $z_b < z < z_t$:

$$n_{e0}(r, z) = n_{p0}(r, z) = \frac{n_0}{2} \left[ 1 + \tanh\left( \frac{a - r}{\sigma} \right) \right]$$

(3.3)

where $z_t$ and $z_b$ are the altitudes of the center of the top and bottom hemispherical caps, respectively; $n_0$ is the peak density; $a$ represents the radius of the column; and $\sigma$ controls the sharpness of the distribution. Equations 3.1 and 3.2 represent the distribution in the two hemispherical caps, while equation 3.3 describes the cylindrical body. This column shape for the model hydrometeor best models an elongated water drop or ice needle that is aligned with the direction of the thundercloud electric field. In Liu et al. [2012b], by approximating the column as a perfect conductor, the required dimension of the column in order for a streamer to be initiated was provided as a simple relation. We have used their relation to estimate the required length for our hydrometeors. It is noteworthy to mention that although Liu et al. [2012b] have used a Gaussian profile for the initial distribution of electrons and positive ions in the column, we have mostly used a hyperbolic tangent (tanh) profile for this study. This is because the radius and sharpness of the column or the hemispherical cap are separately controlled by $\sigma$ and $a$, allowing flexibility in tuning the parameters for reducing the simulation run time.

In addition, if we assume our hydrometeors to be liquid water filaments, we can calculate an initial density for the model hydrometeor by allowing the Maxwellian relaxation time of the column hydrometeor to be equal to the dielectric relaxation time of liquid water. Liquid water has a dielectric relaxation time ($\tau_D$) on the order of tens of picoseconds [Raju, 2003]. Following the re-
lation $\tau_M = \varepsilon_0/\sigma$, where $\tau_M$ is the Maxwellian relaxation time of the column and $\sigma$ the conductivity, the column conductivity should be about 0.885 S/m, if $\tau_M = \tau_D$ and $\tau_D = 10^{-11}$ s. This conductivity yields an initial electron density of $1 \times 10^{20}$ m$^{-3}$ if the column is placed inside a uniform electric field $E_0 = 0.3E_k$ at ground pressure, where $E_0$ determines the mobility of electrons.

Using an ionization column in place of a liquid water or ice dielectric filament is justified by the following reasons. First, both water and ice have a very high dielectric constant. When they are polarized, the same field enhancement factor can be obtained as a polarized dense plasma column. This is the most important factor determining the streamer initiation. Second, the seed electrons initiating the electron avalanches that propagate toward the positive tip are entirely created by photoionization of air molecules. The discharge activity around the positive tip of a dielectric filament is therefore accurately modeled. In addition, it can be expected that corona discharges surrounding both of the positive and negative tips prior to streamer initiation create an ionization cover around each tip, similar in nature to the ionization column. Finally, the timescale of ambient field variation is much longer than the dielectric relaxation time of water and ice. Steady-state polarization of the water and ice hydrometeors can therefore be assumed, i.e., their dielectric constant takes its dc field value. However, the space charge field varies on a much shorter timescale than the dielectric relaxation time of ice, so inaccuracies might be introduced when modeling ice particles by using an ionization column. Given that the space charge field is highly localized and it quickly moves away from the ice particle, we assume the inaccuracies are negligible but future studies are required to verify it.
3.2 Modeling Results for $0.5E_k$

In this section, we present the results of our streamer simulations. We have performed simulations for a $0.5E_k$ ambient electric field at thundercloud altitudes and obtained successful streamer formation, thus confirming the work of Liu et al. [2012b]. We place the model hydrometeor inside the simulation domain and apply a uniform ambient electric field $E_0 = 0.5E_k$ in the downward direction to start the simulation, which simulates the electric field inside the thundercloud. The simulation domain is placed at 7 km altitude in all of our simulations, which fits within the range of lightning initiation altitudes (see Section 1.1.6). However, the principal conclusions of this study stay the same for other thundercloud altitudes. Note that $E_k$ is about $1.4 \times 10^6$ V/m at 7 km altitude.

Figure 3.1 shows a model hydrometeor 5.8 mm long and 0.6 mm wide, with an initial column density of $n_0 = 2 \times 10^{19}$ m$^{-3}$ (initial column density as calculated in the previous section, scaled to 7 km altitude pressure, i.e., $1 \times 10^{20}$ m$^{-3}$ $(N/N_0)^2$, where $N$ and $N_0$ are air densities at 7 km altitude and ground, respectively). The figure shows cross-sections of electron density and electric field distributions at a few different moments of time. At about 30 ns, the hydrometeor is nearly fully polarized and an enhanced electric field is present at the positive tip. At 32.1 ns, the streamer is born from the positive tip. Each panel shows a snapshot of the streamer, while propagating through the domain. The last panel corresponding to 47.6 ns shows a fully formed streamer that has stably propagated to the end of the simulation region.
Figure 3.1: Cross-sectional views of distributions of (a) electron density and (b) electric field of a streamer from a hydrometeor in $E_0 = 0.5E_k$ at 7 km altitude. The length and radius of the model hydrometeor are 5.8 and 0.6 mm, respectively.

3.3 Modeling Results for $0.3E_k$

When simply reducing the ambient electric field to $0.3E_k$, the maximum observed thundercloud electric field, initiation of stable streamers from isolated thundercloud hydrometeors becomes impossible. In this case, the streamer either does not form or branches during initiation. The branching of the streamer interrupts the stable streamer formation and changes the dynamics of the streamer.

In Figure 3.2, the model hydrometeor is 6.6 mm long and 0.75 mm wide, and has $n_0 = 2 \times 10^{19} \text{ m}^{-3}$. An ambient electron density distribution was included in the computational domain before the simulation started. This addition increases the preionization level ahead of the electrically stressed hydrometeor to control the formation and propagation of the discharge. This background density is uniform along the axial direction and follows a Gaussian distribution along
Figure 3.2: Cross-sectional views of distributions of (a) electron density and (b) electric field of a streamer from a hydrometeor in $E_0 = 0.3E_k$ at 7 km altitude. The length and radius of the model hydrometeor are 6.6 and 0.75 mm, respectively.

the radial direction. The Gaussian distribution has a radius of 0.88 mm, on the order of spatial corona region extents, and a peak plasma density of $n_{e0} = 2 \times 10^{15}$ m$^{-3}$, consistent with typical corona densities (see Section 3.9 for further discussion).

Figure 3.2 shows the hydrometeor is nearly fully polarized at 27.7 ns and an enhanced electric field is present at the positive tip. At 31.0 ns, the positive streamer starts forming and the last panel corresponding to 47.6 ns shows a fully formed streamer that has stably propagated to the end of the simulation region.

If we compare this result with Figure 3.1, the change in the density of the column during and after streamer formation is interesting. For the streamer in $0.5E_k$, the column density does not change much. On the other hand, Figure
3.2 shows an obvious drop in the column density. The processes that might be responsible for changes in electron density of the column include two-body electron attachment, three-body electron attachment, and recombination. A simple estimation shows that three-body attachment is the dominant process responsible for the density drop in the column, as shown in Figure 3.2. Assuming the electric field in the column to be about $0.0025E_k$, the three-body attachment frequency at 7 km altitude is equal to $5.5 \times 10^7$ s$^{-1}$. Consequently, the three-body attachment timescale is equal to about 18 ns, explaining the reduction in density during the simulation time of 47 ns.

Figure 3.3 shows the time evolution of electron density and electric field profiles of the streamer from Figure 3.2, along the symmetry axis, at a few representative moments of time. The direction of propagation of the positive streamer is to the left. As the simulation starts, the ionization column starts to polarize. Electrons in the column start moving in the opposite direction of the electric field, towards the top of the column. As electrons move upward, positive ions are left behind. Due to their low mobilities, positive ions are assumed to be stationary in our model, being more and more exposed as electrons move upward. This results in a compact positive tip at the bottom of the column. On the other hand, electrons are constantly moving upward and out of the column, resulting in a diffuse region on the upper tip of the column. As the column is polarizing, a space charge electric field starts to build up due to the resultant charge separation. This space charge field reduces the electric field inside the column, while enhancing the electric field at the tip. Polarization of the column continues and the electron density distribution at the positive tip looks sharper and sharper, while the negative tip looks more and more diffused.
Figure 3.3: Time evolution of axial profiles of (a) electron density and (b) electric field corresponding to the streamer in Figure 3.2, for several moments of time. Note that $E_k = 1.4 \times 10^6$ V/m at 7 km altitude.
due to electrons spreading outward. During this time, the electric field value continues increasing at the positive tip. The bump in electron density around \( z = 4.5 \) mm at 27.7 ns results from the ionization due to the highly enhanced field at the positive tip. After 31 ns, the electric field at the hydrometeor tip has increased to about \( 6.4E_k \) and starts to quickly move forward. At this moment, the positive streamer is born from the positive tip. New electron avalanches continuously move toward the streamer head, leaving a new positive charge region behind and advancing the streamer head forward. The streamer continues to propagate stably through the low field region until it reaches the bottom of the domain. The streamer head field is about \( 4.25E_k \) during this propagation stage, while the channel density is about \( 2 \times 10^{19} \text{ m}^{-3} \).

### 3.4 Effects of Hydrometeor Dimension

According to the results obtained from our simulations, the dimension of the model hydrometeor plays a significant role in the streamer initiation. As previously mentioned, in Liu et al. [2012b] the dimension required for streamer initiation from an ionization column was estimated by treating the initial column as a perfect conductor. The required enhanced electric field \( E_m \) at the tip of a cylindrical conductor in a uniform electric field was assumed to be the value of typical streamer head fields \( 3-5E_k \), if the radius of the conductor takes a value of typical streamer radii. This would result in the following relation:

\[
l = a\left(\frac{1}{0.56}(E_m/E_0 - 3)\right)^{1/0.92},
\]

where \( l \) and \( a \) are the length and radius of the conductor, respectively.

Using this relation, with a radius equal to 0.75 mm, the length of the model
A Gaussian background electron density distribution with a peak density of \(2 \times 10^{19} \text{ m}^{-3}\) and 0.88 mm radius is included. The length of the model hydrometeor in Figure 3.2, 6.6 mm, is consistent with that relation, even though our case differs slightly from Liu et al. [2012b] due to the inclusion of ambient density. We have performed simulations for a wide variety of dimensions for the model hydrometeor, using this relation as a guideline. The dependence of streamer initiation on the length of the hydrometeor is illustrated in Figure 3.4. The figure shows that at \(t = 0\) s two model hydrometeors are placed inside a uniform ambient electric field value of \(E_0 = 0.3E_k\), with a background electron density included, which has a Gaussian distribution with
a peak density of $2 \times 10^{15}$ m$^{-3}$ and 0.88 mm radius. Both model hydrometeors have the same radius and initial column density of 0.75 mm and $2 \times 10^{19}$ m$^{-3}$, respectively, as the simulation case for Figures 3.2 and 3.3. The hydrometeor in the left panel of Figure 3.4 has a length of 5.5 mm, while the one in the right panel has a length of 4.4 mm. After about 99.6 ns, from the two hydrometeors that are identical otherwise, only the one with the longer length is able to initiate a streamer. The length of the hydrometeor in the left panel is in agreement with the estimate obtained by the relation.

However, additional simulation results show that streamers can be initiated from the shorter hydrometeors when $E_0$ is increased to $0.5E_k$. The dependence of the discharge initiation on the length of the sample hydrometeor has also been shown by laboratory experiments. In the study by Griffiths and Latham [1974], although they did not present a clear-cut relationship between $E_c$ (critical applied electric field necessary to initiate a streamer corona, as defined in that study) and the length of the sample through their experiments, they concluded that longer crystals have lower values of the critical applied field. In other words, the critical applied field value required for streamer corona initiation is lower when the length of the sample hydrometeor is longer. This is also suggested by the relation between the required dimension and the applied field discussed above.

### 3.5 Effects of Ambient Density

Introducing ambient electron density into our simulations promotes the formation of stable streamers in the ambient electric field of $0.3E_k$. Recently, Qin and
Pasko [2014] show that the stable propagation of streamers originating from a highly stressed electrode connected to an external circuit is not solely controlled by the external field but also by the physical dimensions of streamers. Depending on different initial conditions (i.e., the dimension of the model hydrometeor, and the magnitude and distribution of background density), the streamer initiation from our model can result in a few different scenarios. One case is that the streamer is able to form and propagate stably through the simulation domain (Figure 3.2). For some cases, the forming streamer goes through pre-branching and recovery stages many times. This results in an interesting form of the streamer channel with fluctuating radius demonstrating the same variation of the streamer head radius. We have named it a “wavy” streamer. The streamer from Figure 3.4a is an example of such a case. Figure 3.5 shows the time evolution of electric field profiles along the symmetry axis of this streamer. The “wavy”-like structure of this streamer is more apparent from the profiles. For this streamer, at some moments of time, for instance 76.3 ns, the transverse radius of the streamer increases, and the streamer head flattens. At this point, the streamer is in a pre-branching state, and the electric field in the streamer head drops. After this widening, some streamers continue widening until the streamer head splits. For this particular simulation case, after the widening of the streamer head, the streamer recovers from the pre-branching stage, reducing its transverse radial size, and hence continuing propagation with one single head. This process corresponds to the dips and rises in the streamer head electric field as shown in the time evolution of the axial electric field profiles, and is repeated a few times throughout the streamer propagation.

The distribution of the background density is a key parameter for the stable
Figure 3.5: Time evolution of axial profiles of electric field corresponding to the streamer in Figure 3.4.
streamer formation. According to our simulation results, a nonuniform ambient
density distribution is required for a stable streamer to form. In some cases, even
with the inclusion of a significantly large uniform ambient electron density, the
forming streamer still branches. Figure 3.6 demonstrates the effects of different
background density distributions. Three different simulation cases are shown.
Each case includes a model hydrometeor 7.7 mm long and 0.75 mm wide. For
panel a, the background electron density included is uniform and has a value of
\( \times 10^{15} \text{ m}^{-3} \). For panel b, the background density is uniform along the axial
direction and Gaussian along the radial direction. The radius of the Gaussian
distribution is 0.88 mm, and the peak plasma density is \( 2 \times 10^{15} \text{ m}^{-3} \). For panel
c, all the parameters are the same as in panel b, except that the radius of the
ambient density distribution is 0.44 mm.

The three simulation cases show quite different results. The simulation case
with uniform background density has resulted in a branched streamer (Figure
3.7), while the other two cases have formed stable streamers. The two cases with
stable streamers are also different, demonstrating different streamer character-
istics. In order to better illustrate the effects of the different ambient density
distributions on the streamers, the axial electric field and electron density pro-
files at \( t = 46.5 \text{ ns} \) corresponding to each case are depicted in Figure 3.8. As
can be seen from the figure, the streamer with the Gaussian distribution and
\( \sigma = 0.44 \text{ mm} \) has the highest peak electric field and channel density. Table
4.1 summarizes the streamer characteristics associated with the streamers from
Figure 3.8, where all the parameters have been calculated at roughly the same
streamer length. In Table 4.1, \( l_s \) is the streamer length; \( E_h \) is the peak electric
field in the streamer head; \( E_b \) is the electric field in the streamer body; \( N_s \) is
Figure 3.6: Cross-sectional views of electron density distributions of (a) a streamer that is about to branch, and (b, c) stable streamers, from model hydrometeors at 7 km altitude. The background electric field is $0.3E_k$, and a background ambient electron density is included in each case. For all three cases, the length and radius of the hydrometeor are 7.7 and 0.75 mm, respectively. For (a), the ambient density is uniform with a value of $2 \times 10^{15}$ m$^{-3}$. For (b), the ambient density has a uniform distribution along the axial direction and Gaussian distribution along the radial direction. The radius of the Gaussian distribution is 0.88 mm and the peak density is $2 \times 10^{15}$ m$^{-3}$. For (c), all parameters are the same as (b) except that the radius of the ambient density distribution is 0.44 mm.
Figure 3.7: Time evolution of the splitting of the streamer from Figure 3.6a. The figure shows cross-sectional views of electron density.

the streamer electron density; \( r_s \) is the streamer radius; and \( v_s \) is the streamer speed.

As can be seen from Table 4.1, the streamer radius decreases as the background ambient density becomes nonuniform. The streamer with \( \sigma = 0.44 \) mm has the smallest radius, while propagating with the highest speed. This streamer also has the maximum streamer head electric field-to-breakdown field ratio \( (E_h/E_b) \). The calculated streamer speeds show that the branched streamer was propagating slower than the other two cases, which lead to stable streamers. The streamer that eventually branched has the smallest streamer head peak electric field and displays the largest radius. In brief, the inclusion of a background density decreasing radially causes the streamer to propagate faster with a smaller radius.

The slower the streamer propagates, the more time the dynamically extending streamer channel will have to be polarized and the closer the electric field inside the channel will approach zero. A decrease in the channel electric field increases the value of the \( E_h/E_b \) ratio. The larger the \( E_h/E_b \) value for
Figure 3.8: Axial profiles of (a) electric field and (b) electron density corresponding to the streamers presented in Figure 3.6, at \( t = 46.5 \) ns. This time corresponds to a moment of time before the streamer in Figure 3.6a branched.
Table 3.1: Streamer characteristics associated with the streamers from Figure 3.8.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>$l_s$ (mm)</th>
<th>$E_h/E_k$</th>
<th>$E_b/E_k$</th>
<th>$E_b/E_h$</th>
<th>$N_s$ (m$^{-3}$)</th>
<th>$r_s$ (mm)</th>
<th>$v_s$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>1.80</td>
<td>3.9</td>
<td>0.07</td>
<td>51.6</td>
<td>$1.5 \times 10^{19}$</td>
<td>0.33</td>
<td>$1.93 \times 10^{5}$</td>
</tr>
<tr>
<td>Gaussian ($\sigma = 0.88$ mm)</td>
<td>1.77</td>
<td>4.3</td>
<td>0.09</td>
<td>48.7</td>
<td>$2.5 \times 10^{19}$</td>
<td>0.28</td>
<td>$2.08 \times 10^{5}$</td>
</tr>
<tr>
<td>Gaussian ($\sigma = 0.44$ mm)</td>
<td>1.82</td>
<td>4.9</td>
<td>0.10</td>
<td>47.3</td>
<td>$3.2 \times 10^{19}$</td>
<td>0.25</td>
<td>$2.40 \times 10^{5}$</td>
</tr>
</tbody>
</table>

the branched streamer indicates the streamer channel approaches the “ideal conductivity” condition, similar to the streamer branching study reported by Arrayás et al. [2002]. However, in that study, the splitting of the streamer head in the limit of ideal conductivity was analytically demonstrated when perturbations were present in the streamer head front. For the case of our study, even without the presence of perturbations, the streamer head is unstable when the ideal conductivity limit is reached, and it tends to split.

3.6 Streamer Branching and Head Curvature Analysis

Branching of streamer discharges is a complex phenomena that is not clearly understood. Many studies have tied streamer branching to the photoionization process, indicating it to be a critical factor defining the branching dynamics of streamers [e.g., Liu and Pasko, 2004, Pancheshnyi, 2005, Luque and Ebert, 2011, and references cited therein]. The photoionization process is responsible for generating seed electrons to increase preionization ahead of the streamer head, and the occurrence of branching is quite sensitive to the preionization level. A higher preionization level overall acts to suppress the occurrence of branching, while a lower preionization level typically leads to the streamer branching phenomena.
According to our simulation results for the lower electric field value of $0.3E_k$, the photoionization process being the sole source for increasing the preionization level has proven inadequate to initiate a stable streamer from isolated thundercloud hydrometeors.

Pancheshnyi [2005] has investigated the effects of other sources of preionization production like “natural” ionization sources (cosmic radiation, terrestrial radiation, etc.) or accumulation of residual charged particles in repetitive discharges, on streamer branching. His results imply that preionization ahead of the front of a positive streamer discharge affects the streamer head structure and properties, and may be the parameter determining streamer branching. A recent experimental study by Takahashi et al. [2011] has explored the situation in which streamer branching can be controlled in atmospheric argon, by increasing the amount of background ionization before the streamer discharge. In their study, branching structures were suppressed as the streamer propagated through a region with enhanced ionization density produced by laser irradiation. This phenomena is depicted in Figure 3.9.
Inside thunderclouds, background ionization could be present due to several mechanisms such as cosmic radiation, corona discharges, or electrons remaining from a previous discharge. As a result, streamers may not be developing and propagating through virgin air. In particular, other forms of electrical discharges may have preceded the streamer initiation stage, causing enhancement in the background ionization levels.

A more recent study by Savel’eva et al. [2013] has investigated the reasons for branching of a positive streamer in a nonuniform electric field in a point-to-plane electrode setup. They have found that flattening (or as they call it, “blunting”) of the streamer head is a premonitory sign of branching, consistent with earlier modeling studies [Arrayás et al., 2002, Liu and Pasko, 2004]. In a blunting streamer, the maximum curvature of the streamer head is moving away from the “pole” (the tip of the streamer head on the symmetry axis). As a result, the maximum electric field and the maximum ionization rate shift away from the symmetry axis as well. This causes the streamer to start branching at an angle to the symmetry axis. In that study, a standard two-dimensional drift-diffusion model is used to describe the streamer dynamics. They analyze the results of their model in the context of a simplified analytical model for the streamer head. In this analytical model, the streamer is approximated as an ideally conductive semielliptical head with a cylindrical channel. It is assumed that the streamer head grows due to ionization on its surface. The growth rate of a point on the streamer head surface is determined mainly by the ionization frequency of that surface point. The ionization frequency of each point depends on the magnitude of the local electric field. For an ideal conductor, the distribution of the electric field depends on its shape. So with a known shape for the streamer
Figure 3.10: (a) Electric field distribution on the surface of an ideal conductor in the shape of a cylinder with a semielliptic head. The semiaxes of the ellipsoid are $a$ and $b$. $U$ is the conductor potential with respect to ground, and $z$ is the symmetry axis (reproduced from Savel’eva et al. [2013]). (b) Determination of points $A$, $B$, and $C$ on the streamer surface (modified from Figure 6 in Savel’eva et al. [2013]). Point $A$ is located on the pole of the streamer head. $B$ is the point on the streamer surface with a distance $b/2$ away from the symmetry axis, and $C$ is on the surface with a distance $b$ away ($b$ being the maximum transverse distance of the streamer surface from the symmetry axis).

head, the electric field distribution can be determined, and hence the growth rates of the streamer head. Considering the semielliptical surface of the streamer head, the half-axis along the symmetry axis is $a$, and the half-axis perpendicular to the symmetry axis is $b$. If charged to a constant potential, the electric field distribution of an ideal conductor in the shape of a cylinder with a semielliptical head can be determined by solving Laplace’s equation numerically. Figure 3.10 illustrates the electric field distribution on the surface of the streamer if the results are reduced to a dimensionless form.
For a semi-spherical head \((a = b)\), the maximum electric field is on the symmetry axis, at the streamer head pole. From Figure 3.10a, for values of \(a\) and \(b\) with \(a/b \geq 0.95\), the maximum electric field continues to remain at the pole. However, for \(a/b \leq 0.90\), the maximum of the electric field shifts off of the symmetry axis. In other words, for a sharp streamer head, the maximum electric field remains on the pole, while for a blunt streamer head, the maximum electric field shifts off-axis. This result also implies that the maximum electric field does not necessarily move off-axis as soon as the streamer head deviates from a semi-spherical form. It is only when \(b\) exceeds \(a\) by about 10\% that the maximum field starts to shift. With an initial semi-spherical streamer head, the maximum electric field moves away from the pole in a rapidly widening streamer provided that \(b\) grows faster than \(a\), i.e., \(db/dt > da/dt\). This conclusion has been verified by their simulation results from the two-dimensional drift-diffusion model.

According to the simulation results from the present study, flattening of the streamer head is indeed a premonitory sign of streamer branching. In order to analyze the surface dynamics of a streamer, we need to define the streamer surface. One way to define the streamer surface is the points along the streamer’s transverse direction with the maximum electric field value. Another can be the points along the streamer’s transverse direction with the maximum value of charge density. For the analysis presented here, we define the streamer surface as the points along the streamer’s transverse direction, beyond the maximum charge density, that are \(1/e\) of this peak value. This definition results in a smoother streamer surface in comparison with the others.

Using a similar convention as that used by Savel’eva et al. [2013], we focus
on three points $A$, $B$, and $C$ on the streamer surface as shown in Figure 3.10b. Point $A$ is located on the pole of the streamer head. $B$ is the point on the streamer surface with a distance $b/2$ away from the symmetry axis, and $C$ is on the surface with a distance $b$ away ($b$ being the maximum transverse distance of the streamer surface from the symmetry axis).

Considering the three streamer simulation cases presented in Figure 3.6, Figure 3.11 shows the streamer surfaces for each of these cases, at different moments of time. The three points $A$, $B$, and $C$ are marked on each profile. Figure 3.12 shows electric field values on the streamer surfaces corresponding to each of the cases. For cases (b) and (c), all the profiles show the electric field on the streamer surface is maximized at point $A$. Also, the maximum electric field value stays about constant throughout the streamer propagation. On the other hand, for case (a), initially at $t = 44.3$ ns, the surface electric field values drop in the direction away from the streamer’s symmetry axis, with the maximum field value being at point $A$. As time passes, the electric field value of point $A$ decreases, and the difference between $A$ and $B$ is reduced. At $t = 48.7$ ns, the electric field at point $B$ exceeds the value of point $A$ and the maximum field has shifted off of the symmetry axis. This trend continues as the streamer propagates, moving the maximum electric field further and further away from the axis. If we mark the time when the maximum electric field has moved off-axis to be the branching initiation time, this streamer has started branching at $t = 48.7$ ns.

In the study by Savel’eva et al. [2013], the maximum electric field moves off of the symmetry axis after the maximum curvature of the streamer surface has moved off-axis. We have performed a similar curvature analysis for our streamer
Figure 3.11: The streamer surface profiles corresponding to the cases in Figure 3.6 (a), (b), and (c), shown at a few different moments of time. The diamonds correspond to point A, circles correspond to point B, and squares correspond to point C on the streamer surface.
simulation results. Figure 3.13 presents streamer surface curvature profiles corresponding to the same moments of time presented in Figures 3.11 and 3.12. To calculate the curvature value for each point on the streamer surface, the surface of the three-dimensional streamer body is obtained by rotating the two-dimensional streamer surface profile around the symmetry axis. Each point on the streamer surface may have different curvatures for different planes containing that point. Each curvature value measures how much the surface bends along a particular tangential direction at that point. The principal curvatures at each point on the surface, denoted by $\kappa_1$ and $\kappa_2$, are the maximum and minimum values of all the different curvature values at that point. In other words, as defined in differential geometry, the principal curvatures are the eigenvalues of the shape operator at that point. The average of the two principal curvatures at every point is the mean curvature, denoted $H$, where $H = \frac{1}{2}(\kappa_1 + \kappa_2)$. We use this definition for $H$ to calculate the mean curvature (henceforth denoted as “curvature”) of every point on the streamer surface, corresponding to the moments of time presented in Figure 3.12.

Looking at Figure 3.13, the main feature that distinguishes our results from Savel’eva et al. [2013] is that neither of the curvature profiles gives a maximum value on the symmetry axis. This means that even at the beginning of the streamer propagation, the streamer tip is not the sharpest point of the streamer head. This is true even for the two streamer cases that do not branch at all. For these two cases, our results show that the maximum electric field is on the symmetry axis and until the end of the simulation, it does not shift off-axis with the shifting of the maximum curvature. A common misconception is that the electric field at the surface of an isolated conductor is greatest where the
Figure 3.12: The electric field values on the streamer surfaces corresponding to the same moments of time illustrated in Figure 3.11. The diamonds correspond to point A, and circles correspond to point B on the streamer surface. The direction of propagation of the streamer in each case is from right to left.

Figure 3.13: The curvature of the surface of the streamer, corresponding to the same moments of time illustrated in Figure 3.11. The diamonds correspond to point A, circles correspond to point B, and squares correspond to point C on the streamer surface.
curvature is greatest. In fact, it has been shown by Price and Crowley [1985] that in general, these two maxima are located at different points on the surface. This is based on the fact that these two parameters depend on the shape of the surface in entirely different ways. Curvature depends only on the local shape of the surface, while the electric field is determined by the charge density distribution on the entire surface. Changing the shape of the surface away from a particular point does not affect the curvature at that point. However, changing the shape of the surface in one region influences the charge density distribution over the entire surface. This suggests that the non-collocation of the maximum curvature and maximum electric field cannot be the sole parameter responsible for streamer branching, since the two streamer cases that have not exhibited branching have not shown collocation of these two parameters either.

From Figure 3.13, a main feature of the branching streamer case (a) is that the maximum curvature of the streamer surface moves away from the symmetry axis with a faster rate than the other two cases. Focusing on the moments of time before the streamer has branched (until $t = 48.7$ ns), the location of the maximum curvature at the time of branching is considerably farther than the first profile’s. The curvature profiles for the other two cases show much less shifting of the maximum values, with some profiles hardly shifting at all. The farther the maximum curvature value is from the symmetry axis, the flatter the streamer head becomes. On the other hand, the two non-branching cases show a fast drop in the maximum curvature value as time progresses. This is consistent with acceleration and radial expansion of the streamer channel as it propagates forward [Liu and Pasko, 2004]. The curvature profiles for the branching case, however, do not show much reduction in the maximum values and the rate of
reduction is definitely less than the other two cases.

According to the discussion above, our simulation results suggest that the geometry of the streamer head plays an important role in the streamer branching phenomena. The fast radial movement of the maximum streamer head curvature, combined with the slow reduction of the maximum curvature values, causes the streamer head to eventually pull the maximum electric field off of the symmetry axis. The redistribution of the electric field profile with the maximum moving away from the axis is a critical turning point for the streamer head. As the location of the maximum electric field is also the fastest growth point of the streamer head surface, this phenomenon results in the streamer propagating off-axis and hence branching.

3.7 Comparison with Realistic Hydrometeors

We have demonstrated that streamers can be initiated from model hydrometeors in subbreakdown electric fields as low as $0.3E_k$, which is a measured maximum electric field value inside the thundercloud. It is also important that these hydrometeors possess similar physical properties as the hydrometeors present in the thundercloud. A summary of different types of hydrometers and their corresponding dimensions can be found in Table 3.2. As discussed in Section 2.1, the most favorable configurations for streamers to originate from hydrometeors have been concluded to be glancing collisions of two water drops [Crabb and Latham, 1974] and individual ice crystals [Griffiths and Latham, 1974]. The simple columniform shape that we have chosen for our model hydrometeor characterizes the geometries of both cases.
Table 3.2: List of different hydrometeor types and typical sizes (summarized from Section 1.1.2).

<table>
<thead>
<tr>
<th>Hydrometeor Type</th>
<th>Dimension (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cloud drops</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Raindrops</td>
<td>&lt; 3</td>
</tr>
<tr>
<td>Ice crystals</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Snowflakes, Ice Pellets</td>
<td>0.5 - 6</td>
</tr>
<tr>
<td>Graupel</td>
<td>&lt; 5</td>
</tr>
<tr>
<td>Hail</td>
<td>5 - 150</td>
</tr>
</tbody>
</table>

As summarized in Table 3.2, except for hail, most types of hydrometeors are in the sub-centimeter range. The minimum length necessary for the model column hydrometeor to initiate a streamer depends on its radius and the ambient electric field. As discussed in the previous section, the value for the minimum length increases with decreasing ambient electric field value, and the presence of background ambient electron density shortens the required length to some extent. For example, in an ambient electric field of $0.5E_k$, if there is no ambient density present, the length of the hydrometeor must be $\sim 3$ mm in order to initiate a streamer. If an ambient density with a peak of $2 \times 10^{12}$ m$^{-3}$ is present, the streamer is able to form from a hydrometeor as short as $\sim 2$ mm. For an ambient electric field value of $0.3E_k$, a stable streamer has always failed to form in the absence of background density. The minimum length required to initiate a stable streamer for this field depends on the magnitude of the background ambient density, increasing with decreasing background density. As shown in Figure 3.4, the minimum length of a model column hydrometeor with a radius 0.75 mm to initiate a streamer in $0.3E_k$ is about 5 mm, when a nonuniform background density with a magnitude comparable to corona discharges (see Section 3.9) is included. We can conclude from these results that realistic hydromete-
ors can initiate streamers in the measured thundercloud electric fields, with the presence of ambient ionization density.

3.8 Comparison with Laboratory Experiments

In this section, we compare the results of the present study with experiments conducted on streamer emission from hydrometeors, discussed in Chapter 2. In the work of Griffiths and Latham [1974], corona streamers initiated from ice hydrometeors were studied. They experimentally determined the relation between air pressure ($P$) and the critical applied electric field ($E_c$) required to produce corona streamers, for various ice specimen at $-12^\circ$C. They considered ice needles, prismatic ice crystals, flat plates of ice, and hailstones as different ice hydrometeor groups. The ice needles considered are the most similar case to the model hydrometeors in the present study. For their experiments, they used chambers that were carefully maintained at a constant temperature and the desired pressure. The ice sample was placed inside the chamber with its long axis parallel to the field lines. After the chamber was sealed, no other source could contaminate the experiment. Then the electric field was turned on and slowly increased until a corona streamer discharge was initiated from the sample. The onset of the discharge was marked by the appearance of glow around the ends of the sample. This was viewed from the window in the chamber lid, with the room previously darkened to allow the eye to adapt. They reported the range of lengths that can produce streamer activity around ice hydrometeors to be between 4–17 mm, for a $0.3E_k$ ambient field at 7 km pressure.

From the results of our current study, the range of minimum hydrometeor
lengths necessary for streamer initiation has been obtained to be approximately 5–8 mm (for an ambient electric field value of 0.3$E_k$ at 7 km with a background density range between $10^{15} - 10^{17}$ m$^{-3}$), which overlaps with the range reported by Griffiths and Latham [1974]. This range fits into the upper boundary of typical hydrometeor sizes listed in Table 3.2. In these experiments, the slowly varying electric field inside the chamber results in corona discharges preceding the streamer formation. This can be similar in nature to the ambient density that we have included in our simulations.

Petersen et al. [2006] have also observed positive streamer discharges from ice crystals grown under controlled temperature and pressure. The dimension of the longer axis of the ice crystals in their study ranged from 0.5–2 mm for thicker crystals, and from 2–3 mm for thinner crystals. In their experiments the electric field values necessary to start positive streamer discharges were recorded to be between 500 and 850 kV/m at ground pressure. This range is a bit smaller than the value used in our study (960 kV/m at ground pressure). Similar to the experiments conducted by Griffiths and Latham [1974], the potential difference applied to the two parallel plate electrodes used to establish a uniform electric field was increased manually. The threshold electric field was recorded upon observation of luminous filamentary traces around the ice crystal, via the image intensifier. The corona discharges occurring before streamers in these experiments could be the cause of the difference between the threshold electric fields reported in the literature (as low as 500 kV/m for a $\sim$ 2 mm long hydrometeor) and our study. However, at present it is unclear how a stable streamer can be formed at such a low magnitude electric field.

When streamers are fully formed, they are known to be able to propagate
Figure 3.14: Sample photographs of discharges from ice hydrometeors. The different discharge types have been marked on each photograph. The positive streamers appear as luminous filaments extending from the anodic tip (the left of each photograph) of the ice crystal to the cathode (the right of each photograph) (from Petersen et al. [2014]).
far even after branching, as shown by laboratory experiments on the discharges originating from highly-stressed electrodes connected to a power supply [e.g., Nijdam et al., 2010]. However, experiments on streamers from isolated hydrometeors in an external electric field close to the measured thundercloud field display a single channel that propagates without branching, much similar to what we have observed from our simulation results. In particular, Petersen et al. [2014] and Petersen et al. [2006], have observed positive streamer discharges from simulated ice hydrometeors in the laboratory. The positive streamers from their study display a single illuminated channel moving from the ice hydrometeors anodic tip into the electrode gap, as depicted in Figure 3.14.

Petersen et al. [2014] have observed different discharge types from glow corona to positive and negative streamers, initiated from ice crystals. Figure 12 in that paper provides a plot of positive streamer threshold electric fields as a function of ice crystal length for 570 mb pressure (about 5 km). The threshold ambient field for positive streamer initiation from a 6–7 mm long ice crystal is about 700 kV/m, which is in good agreement with our value of 0.3E\textsubscript{k} = 550 kV/m.

Overall, the results presented here complement the experimental studies mentioned above. The general trend of the decreasing electric field necessary for streamer initiation with increasing hydrometeor length (Figure 3.4) obtained from our study is consistent with the results of these studies, as well as with other previous experimental and theoretical work [e.g., Crabb and Latham, 1974, Schroeder et al., 1999].
3.9 Role of Corona Discharges

An important conclusion from this study is that streamer initiation from realistic thundercloud hydrometeors in a $0.3E_k$ electric field is impossible without the presence of a nonuniform background electron density distribution. Corona discharges inside the thundercloud can be a possible source for this ambient density. Positive and negative corona discharges normally occur in the vicinity of an electrically stressed discharge electrode. Chen and Davidson [2002] have numerically determined the electron density distribution for a positive dc corona discharge along a wire. The electron density distribution is obtained from the 1-D charge carrier continuity equations and Maxwell’s equation. In order to study corona discharge in a wire-cylinder electrode geometry, the computational domain is a thin cylindrical annulus, which only includes the plasma region. For the wire-cylinder electrode setup, the corona discharge consists of a corona plasma region directly around the high voltage electrode and a unipolar ion region that is spread outward to the grounded electrode. The two regions are separated by the ionization boundary, defined as the radial position where the ionization coefficient and the attachment coefficient are equal. Electron number density in the positive corona plasma was obtained for six different wire radii and six linear current densities. For a 100 $\mu$m wire radius and a 100 $\mu$A/cm current density, the electron density drops from $10^{15}$ m$^{-3}$ at the wire surface to $10^{10}$ m$^{-3}$ as the distance from the wire surface increases, up to 300 $\mu$m away. The electron density increases with increasing current, and smaller electrodes produce thinner plasmas, compared to those produced by larger wires. According to this study, the provided range is sufficient to represent the most common uses of dc coronas. Another study by Chen and Davidson [2003] has been conducted on negative
dc coronas. This study suggests that in negative corona, except very near the surface of the wire, the density of electrons is nearly four orders of magnitude greater than that in the positive corona. For a wire radius of 100 $\mu$m and a current density of 100 $\mu$A/cm, the electron density increases from $10^{10}$ m$^{-3}$ at the wire surface to $10^{14}$ m$^{-3}$ at 500 $\mu$m away. The electron density increases with increasing gas temperature and current density.

Recently, Sin’kevich and Dovgalyuk [2014] have presented a review of theoretical and laboratory studies of corona discharge initiation in clouds. Theoretical estimation of the rate of corona discharge appearance in clouds was reviewed. This rate is necessary to estimate the role of corona discharges in the formation of ambient density in the cloud. A study by Shishkin [1968] used a simple numerical model of the cloud and assumed that each collision of large hydrometeors leads to a discharge. He showed that if the intensity of precipitation in a cloud amounts to 10 mm/hr, the rate of collisions of large hydrometeors and, correspondingly, the discharge rate per unit volume is equal to $10^{-3}$ m$^{-3}$ s$^{-1}$.

Later studies [e.g., Stalevich and Uchevatkina, 1979a, 1979b, Shishkin, 1983] considered that corona discharges will occur not only during collisions, but also when hydrometeors approach each other. They estimated the approach number for different distances between falling hydrometeors. This number is from $10^{18}$ up to $10^{30}$ times greater for small hydrometeors (radius = 0.2 mm) compared to large ones (radius = 4–5 mm) when the precipitation intensity varies from 100 mm/h to 0.5 mm/h. This means that the approach rate per unit volume for smaller hydrometeors is $10^{19} – 10^{31}$ m$^{-3}$ s$^{-1}$. Since the lifetime of the electrons at thundercloud altitudes is about 100 ns, the number of encounters will be $10^{12} – 10^{24}$ m$^{-3}$. If we consider the volume of a corona discharge around a small...
A hydrometeor is about $10^{-9}$ m$^3$, the number of encounters is $10^3 \sim 10^{15}$. Even if a small fraction of these small hydrometeor encounters lead to corona discharges, a persistent ionization may be expected in the thundercloud high field region. It is noteworthy to mention that to the best of our knowledge, no recent papers have been published on this subject.

The total number density of cloud particles is different for various types of clouds as well as various types and sizes of hydrometeors. For water drops, the total number of drops with diameters from 0.1–0.4 mm can be as large as $10^8$ m$^{-3}$, which means that the average distance between two particles is about 2 mm. From our simulation results, the cross-sectional area of the initial streamer channel is about 0.3 mm$^2$. Therefore, it is very likely that a streamer will encounter a small hydrometeor after propagating through a short distance on the order of a few mm. The total number density of large water drops with diameters between 2–7 mm is about $10^4$ m$^{-3}$ with an average distance between them about 4 cm. Hence, there will be a few small hydrometeors in the vicinity of a large hydrometeor. This presents a great chance that corona discharges will appear near a large hydrometeor, before streamer formation, and then in the path of the streamer after it has formed.

According to the results of the studies discussed above, a nonuniform ambient density with a maximum value of $10^{15}$ m$^{-3}$ may exist in a localized region inside thunderclouds. The spatial extent of the corona region depends on the discharge geometry and corona polarity, but the spatial scale in the transverse direction of the background density distribution used in this study ($\sim 400–800$ µm) fits within the range of common coronas.
3.10 Role of Electron Detachment

The role of electron detachment from $O^-$ in increasing ionization in the upper atmosphere has recently been studied [e.g., Luque and Gordillo-Vázquez, 2012, Liu, 2012, and Neubert and Chanrion, 2013]. Liu [2012] investigated the role of electron detachment from $O^-$ ions in sprite halo dynamics. This study indicates that the detachment process affects the dynamics of the halo by allowing the growth of electron density in the upper atmosphere under subbreakdown conditions. Below, we show that this detachment process can be ignored for streamer formation in subbreakdown fields at the thundercloud altitudes considered here.

$O^-$ ions are created by two-body dissociative electron attachment to molecular oxygen. However, for an ambient electric field of $0.3E_k$ at 7 km altitude, the three-body electron attachment has a frequency of about $1.7 \times 10^6 \text{s}^{-1}$, while the two-body electron attachment frequency is $2.2 \times 10^5 \text{s}^{-1}$, an order of magnitude smaller than the three-body reaction. Consequently, the three-body attachment reaction is faster, converting electrons to $O_2^-$ ions. In addition, even when the two-body attachment is faster than the three-body attachment, converting most of the electrons to $O^-$ ions, at stronger electric fields, $O^-$ ions are mainly lost due to a three-body attachment process transforming them to much more stable ozone ions [Luque and Gordillo-Vázquez, 2012], instead of releasing electrons through the detachment process. Therefore, the role of electron detachment from $O^-$ ions can be ignored at thundercloud altitudes.
3.11 Negative Streamers

In this study, we have conducted simulations mainly for two electric field values of $0.5E_k$ and $0.3E_k$. The lengths of the ionization column hydrometeors used vary between 4 and 10 mm, which is consistent with the dimension of ice crystals and snowflakes presented in Table 3.2. For these simulations, the negative streamer has always failed to form. This is also true for the case reported by Liu et al. [2012b], and the sprite streamer initiation from ionization patches in subbreakdown fields reported by Kosar et al. [2012] and Kosar et al. [2013]. For sprite streamers, it has been discussed that positive streamers developing in an electric field greater than their critical propagation field draw exponentially increasing currents, which deposit a large amount of negative charge in the trail of the streamer faster than the charge removed by the conducting trail [Liu, 2010]. Luque and Ebert [2010] have also suggested negative charging of the positive sprite streamer channel, which may eventually lead to the emergence of negative streamers. The positive streamers formed from our ionization column simulations propagate very short distances, and the negative charging of the positive streamer channel is not yet able to initiate negative streamers. This is due to the high computational time that restricts the use of a longer computational domain. It is very likely that continuous propagation of the positive streamer from the ionization column may lead to initiation of negative streamers.

Experimental studies of Dawson [1969] (individual water drops) and Crabb and Latham [1974] (colliding water drop filaments) also had difficulty producing the negative streamer. However, it may as well be possible that the discharge around the negative tip of a physical hydrometeor can not be resolved with our current model. Replacing the ionization column by a dielectric hydrometeor
may offer a more accurate understanding of processes around the negative tip to fully address the problem of the negative streamer initiation.
Chapter 4

Streamers from Dielectric Hydrometeors

4.1 Dielectric Hydrometeor

For the first part of this dissertation study we used an ionization column in place of a liquid water or ice dielectric filament. As discussed in Section 3.1, this is justified in part by the high dielectric constant of water and ice. In this chapter, we modify our streamer simulation model to accommodate an actual dielectric water or ice particle. The inclusion of an actual dielectric material inside the simulation domain results in a very complex dynamical model, because the dielectric constant is not uniform anymore and a curved boundary is contained in the simulation domain. We will describe the numerical models and methods used to implement a dielectric hydrometeor into our model, and present the results we have obtained.
4.2 Model Description

To simulate streamer initiation from a dielectric hydrometeor, we accommodate a single isolated dielectric particle inside the simulation domain. This geometry is different from previous streamer simulation studies where the electrodes lie outside of the computational domain and/or are connected to an external circuit [e.g., Liu and Pasko, 2004]. The model equations need to be solved throughout the simulation domain in two different regions: outside the dielectric hydrometeor, and inside. Outside of the dielectric hydrometeor, Poisson’s equation is solved in order to find the electric field, and particle transport equations are solved to obtain the distribution of space charges in the simulation domain (as described in Section 2.3). Inside the dielectric hydrometeor, there is no free charge density, so there is no need to solve particle transport equations, and Poisson’s equation will reduce to Laplace’s equation.

The streamer model in our study has cylindrical symmetry, and a uniform rectangular grid system is used for discretization of the model equations. When including a dielectric particle inside the computational domain, the shape (e.g., sphere, ellipsoid, column, etc.) and physical state (e.g., liquid, solid) of the hydrometeor play a defining role in choosing the numerical methods to solve the model equations. Let’s assume that our shape of interest is a spherical dielectric. Due to the inclusion of a spherical dielectric inside the computational domain, uniform rectangular simulation cells will be cut by the curved boundary of the hydrometeor. For these cells, part of the cell is inside the dielectric material, while the other part is in air. Two key aspects that need to be considered in this situation are: (1) accurate discretization of the governing equations in the cells that are cut by the boundary, and (2) the imposition of proper
boundary conditions on the curved boundary. In the current study, we will concentrate on two particular shapes for our dielectric hydrometeors: spherical, and columniform particles. The columniform shape consists of a cylinder body, with a hemispherical cap attached to each end. Figure 4.1 shows this geometry.

Appropriate numerical techniques have been chosen to accommodate the new geometries in the computational domain. These techniques and their implementation methods are described below.
4.3 Cut-Cell Method for Resolving the Curved Boundary

The “cut-cell” method is a technique proposed by Ye et al. [1999] as a method to simulate viscous incompressible flows with complex immersed boundaries. When dealing with a problem that has immersed boundaries with an arbitrary shape, the obvious complication is that the boundary may cut through the underlying domain grids in an arbitrary manner. The main challenge is to construct a boundary treatment which does not seriously impact the accuracy and conservation property of the solution. For our case, the shape of the immersed boundary is usually known. By knowing the shape of the boundary, the methodology to apply the cut-cell method for proper handling of the cells that have been cut by the boundary is described as follows. For the sake of simplicity, we will describe this method for the case of a spherical dielectric hydrometeor. However, the same algorithm to implement this method can be applied to any other curved boundary structure.

For the first step, all the cells adjacent to the hydrometeor that are cut by the curved boundary are identified. For each of these cells, the intersection points of the boundary with the sides of the cell are determined (Figure 4.2). It must be pointed out that the intersection of the boundary is found with the half-grid lines (dash-dotted lines in a maroon color), which make up the boundaries of each individual cell.

For each cut-cell, the location of its center is first determined, along with each of its corners (Figure 4.3). If the center of the cell is located outside the sphere boundary, the cell is reshaped by discarding the part that lies inside of
Figure 4.2: Cells that are cut by the boundary are identified and the intersection of the boundary with the sides of these cut cells are determined (black dots). The solid black lines are the simulation domain grid lines, and the dash-dotted lines in maroon are the half-grid lines which make up the boundaries of the individual cells.
the sphere. For a cell whose center lies inside the boundary, the piece that is left outside is absorbed by neighboring cells (Figure 4.4).

In order to establish a uniform merging system, the normal to the curved boundary is obtained at the location of the cell that needs merging, which is also the cell with its center lying inside the boundary. If the angle between the normal to the boundary and the vertical direction is less than 45° or greater than 135°, that cell will be merged with its neighboring vertical cell. If this angle is greater than 45° and less than 135°, the cell will be merged with its neighboring horizontal cell. After a boundary-cut cell is merged with its neighboring cell, a new cell with a new shape is produced. Depending on the location of the cell and the local orientation of the boundary, cells of a variety of shapes and dimensions are produced.

4.4 Surface Area and Volume of the Cut Cells

In order to properly extend the Finite Volume Method (FVM) used to solve the continuity equations to the cut cells, we must identify the particle fluxes moving into and out of the boundary-cut cells. The fluxes are dependent on the new surface areas and volumes of these cells. The regular domain cells are uniformly shaped, leading to a straightforward calculation of the side surface areas, and the ring shaped volume that results from the cylindrical symmetry of the system. However, for the new boundary cut cells, a variety of different shaped cells may be produced. So a new method must be developed to systematically deal with each individual cell, and obtain the required quantities.

All the different shapes of the cut cells can be summarized into a few different
Figure 4.3: The centers of the cells cut by the curved boundary (light blue dots) are identified along with their corners (dark blue dots).
Figure 4.4: Examples of reshaping of cut cells. The cells cut by the boundary whose cell centers lie outside the dielectric are reshaped by discarding the part of these cells that lie inside. Pieces of the cut cells whose centers lie in the dielectric are absorbed by neighboring cells.
scenarios: triangles, quadrilaterals, and irregular pentagons (Figure 4.5a). The black points in the figure are the cell centers, the dashed lines are the new cell boundaries, and the solid lines are the simulation domain grid lines. Figure 4.5b illustrates a cut cell, and a method to calculate the surface area produced due to the slanted side of the cell. This surface is the difference in surface area between the large and small cones illustrated in the figure: \( \pi r_2 s_2 - \pi r_1 s_1 \), where \( s_1 = \sqrt{r_1^2 + z_1^2} \), and \( s_2 = \sqrt{r_2^2 + z_2^2} \); \( r_1 \) and \( r_2 \) are the perimeter radii of the small and large cones, respectively; \( z_1 \) and \( z_2 \) are the respective heights; and \( s_1 \) and \( s_2 \) are the respective lateral heights of the cones.

The total volume of the cut cell is the sum of the two volumes \( V_1 \) and \( V_2 \). In order to calculate \( V_1 \), the surface area of side A is multiplied by the length of B. For calculating \( V_2 \), the difference in surface areas of sides C and A, da, is multiplied by \( h \). \( h \) is an average height for the cell, calculated by averaging the
lengths of B and D. Based on the method of section 4.3, all the required cell information has been recorded. This information includes the position of the cell center and each of the cell corners, as well as the cell location with respect to the spherical boundary. Equipped with this information, values of parameters like $r_1$, $z_1$, and the length of each of the sides of the cell can easily be obtained. Following a similar approach for each individual cut cell, the appropriate surface areas and volumes of these cells can be collected.

4.5 Continuity Equation Solution Around the Curved Boundary

The continuity equations of the streamer model have been solved using a finite volume method [Liu and Pasko, 2004]. These methods are commonly used in solving problems of fluid dynamics and are especially designed to solve hyperbolic partial differential equation systems where discontinuities of physical quantities or extremely high gradients of those quantities are present. They are based on the integral form of the hyperbolic system. Instead of pointwise approximations at grid points, the study domain is broken into grid cells and the particle density in each cell is an average value, approximated by dividing the total number of particles in the cell by the volume of the cell [LeVeque, 2002].

The streamer model equations have been discretized on a uniform, cylindrically symmetric grid system. The solutions of the continuity equations are based on keeping track of the total number of particles in each of the grid cells. At each time step, these values are updated by using the value of the particle flux,
through the boundaries of each grid cell. A numerical scheme called the modified Scharfetter-Gummel method is used to calculate the particle flux density [Liu and Pasko, 2004]. This method is a discretization scheme for variables in convection-dominated or particle drifting-dominated problems, which employs an exponential fitting technique to characterize the rapid density variation in neighboring cells [Scharfetter and Gummel, 1969, and Kulikovsky, 1995].

4.5.1 Particle Flux Calculation and Boundary Conditions

After properly resolving the local geometrical parameters for the boundary-cut cells, the next key issue is to accurately evaluate the particle fluxes moving into and out of these cells. Figure 4.6 shows examples of reshaped boundary-cut cells, along with arrows indicating the fluxes on each side of the cell. The dark shaded arrows correspond to the fluxes that need special treatment.

Considering the reshaped boundary-cut cell in Figure 4.6, we describe the method used to calculate the flux component due to electron drift, \( \vec{J} = -\mu_e \vec{E} n_e \), below. The figure shows four primary components for flux on the sides of the boundary cell, \( \vec{F}_P, \vec{F}_S, G_P, \) and \( G_M \), each component pointing to the direction of positive flux. From here forth we will drop the vector sign, referring only to the absolute values of the particle flux. \( G_P \) can be broken down into two separate components, \( G_{P1} \) and \( G_{P2} \). For each side, the related flux component is calculated at the center of the side. The “donor-cell” or “upwind” method is used to calculate these irregular flux components, by estimating the electric field, mobility, and electron density from one side of the cell surface. Focusing on \( G_{P1} \), the electric field and electron mobility values at the center of AB can be calculated by linear extrapolation, provided that the values for correspond-
Figure 4.6: Schematic of reshaped boundary-cut cells. The arrows indicate the fluxes on each side of the cell. The dark shaded arrows correspond to the fluxes that need special treatment.

In order for neighboring nodes to be known, for the donor-cell method, depending on the direction of the drift velocity, the appropriate electron density value either from the node above or the node below AB will be used. Knowing these three parameters will obtain the flux component for AB ($G_{P1}$).

To simulate the boundary of the dielectric sphere, we use an insulated boundary condition. This condition is translated into the dielectric serving as an impermeable medium with charged particles not allowed to enter the dielectric:

$$\hat{n} \cdot (\mu_e \vec{E} n_e - D_e \vec{\nabla} n_e) = 0$$  \hspace{1cm} (4.1)

This boundary condition dictates the value of $F_S$ to be zero. $F_S$ is the normal component of the flux vector on the boundary of the hydrometeor. On the other hand, the electrons are free to move on the dielectric’s surface.

An important issue that must be considered for choosing a method to cal-
culate the irregular flux components around the cut cells is assuring particle flux balance between adjacent cells. This can be a bit tricky, due to the use of different methods of flux calculation for different sides of a single cut cell, and different methods may have different order of accuracies. For example, in Figure 4.6, $F_P$ is calculated using the modified Scharfetter- Gummel method, while $G_P$ and $G_M$ are calculated using a combination of the donor-cell method with linear extrapolation, and $F_S$ is set to zero due to the boundary condition. The results from the combination of methods that we have chosen here have been carefully monitored and tested, and have proven accurate.

4.6 Electric Field Calculation Around the Curved Boundary

In the streamer model, the electric field of the system is calculated by numerically solving Poisson’s equation, transformed into a set of linear differential equations upon discretization. The successive over-relaxation method (SOR) is used to find the solution of this linear differential equation system, which gives the electric potential. The electric field calculation is dependent on the physical state of the dielectric particle. We have modified the electric field calculation to accommodate two states of a hydrometeor: solid ice crystals and liquid water filaments. The method and implementation details for each state is discussed below.
4.6.1 Contant Polarization: Solid Ice Particles

To simulate solid ice hydrometeors, we have assumed the dielectric achieves steady-state polarization at the beginning of the simulation, and the polarization state is unvarying over time. This assumption is justified by the relatively large dielectric relaxation time of ice ($\sim 10^{-4}$ s) [Raju, 2003] compared to the simulation time ($\sim 10^{-9}$ s). Assuming the solid ice hydrometeor is in the shape of a sphere, the electric field produced by a uniformly polarized dielectric sphere with dielectric constant $\epsilon$ in a uniform electric field $E_0$ is as follows:

$$\vec{E}_{\text{in}} = \frac{3}{\epsilon/\epsilon_0 + 2} E_0 \hat{z} \quad r < R, \quad (4.2)$$

$$\vec{E}_{\text{out}} = \left[ 1 - \left( \frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2} \right) \frac{R^3}{\epsilon} + 3 \left( \frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2} \right) \frac{R^3}{\epsilon} z^2 \right] E_0 \hat{z}$$

$$+ \left[ 3 \left( \frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2} \right) \frac{R^3}{\epsilon} z r \right] E_0 \hat{r} \quad r > R \quad (4.3)$$

where $R$ is the radius of the sphere; $r$ and $z$ are the horizontal and vertical coordinates, respectively; $\hat{z}$ is the distance from the point of interest to the center of the sphere and can be written as $\sqrt{r^2 + z^2}$; and $\vec{E}_{\text{in}}$ and $\vec{E}_{\text{out}}$ are the electric fields inside and outside of the dielectric sphere, respectively.

Figure 4.7 shows a schematic illustration of the simulation domain, including the spherical dielectric hydrometeor, located inside a uniform electric field distribution. At every point inside the simulation region, the electric field has three components; the uniform ambient Laplacian field, the electric field due to the polarized dielectric sphere, and the electric field generated by space charges. The first two components are calculated using Equations 4.2 and 4.3. These val-
Figure 4.7: Schematic illustration of the computational domain used for the streamer simulation from spherical dielectric hydrometeors in a uniform electric field.

Values are set upon initialization of the simulation, and don’t change over time. The solution of the space charge potential at every node is obtained by numerically solving Poisson’s equation, according to the method described in Liu and Pasko [2004]. The sum of these three potential components at each node is used to find the electric field value of that node.

It is important to note that for this method, it is necessary to be able to obtain the analytical solution of the electric field inside and outside of the hydrometeor. This is the main drawback of this method since there are a limited number of dielectric shapes with known electric field distributions. A more generalized method is described in the next section.
4.6.2 Variable Polarization: Liquid Water Filaments

The case of a dielectric hydrometeor with variable polarization is more complex than the constant polarization. In this situation, the electric polarization inside the dielectric hydrometeor changes as a response to the varying space charge density in the system. In order to solve for the electric potential inside the dielectric, proper boundary conditions must be implemented at the dielectric-air interface. The two general electrostatic boundary conditions that must be imposed on this boundary are as follows:

\[
D_{n1} - D_{n2} = \sigma_f \quad (4.4)
\]
\[
E_{t1} - E_{t2} = 0 \quad (4.5)
\]

where the medium inside the dielectric is denoted as 1, and the air outside is denoted as medium 2; \( n \) refers to the normal to the plane, and \( t \) refers to tangential; \( D_n \) is the normal component of the displacement vector \( \vec{D} \), and \( E_t \) is the tangential component of \( \vec{E} \). These two conditions imply that at the interface, the normal component of the displacement vector is discontinuous if free charge is present on the surface, and the tangential component of the electric field vector is continuous. For our simulations, there are no free surface charges deliberately placed on the surface of the dielectric. In this case, the right hand side of Equation 4.4 will also be equal to zero, meaning that both components are continuous.

In order to properly enforce the boundary conditions on the curved boundary, we utilize a boundary condition capturing method proposed by Liu et al.
[2000] for solving Poisson’s equation on irregular boundaries. This method has been presented specifically for solving Poisson’s equation where the solution may be discontinuous across an immersed boundary. A major advantage of this method is that it has been developed so that the coefficient matrix associated with the linear system produced by discretization of Poisson’s equation will keep its standard format used in the absence of immersed boundaries. In other words, any numerical method that has been utilized to solve Poisson’s equation on a standard domain without an immersed boundary, can still be used. This greatly simplifies the numerical implementation of this technique.

This method is based on a similar approach as the Ghost Fluid Method (GFM) [Fedkiw et al., 1999]. The generalized GFM captures the appropriate boundary conditions at an interface, without explicitly enforcing them. Instead, the method creates an artificial fluid which implicitly induces the proper conditions at the interface. Simply put, the boundary conditions will be satisfied without actually enforcing them on the physical curved/irregular shaped boundary. A detailed description of the implementation of this method in our streamer model will be given below.

Assume a simulation setup similar to that shown in Figure 4.8. We want to solve for the electric potential $V$, from Poisson’s equation:

$$\nabla \cdot (\varepsilon \nabla V) = -\rho,$$  \hspace{1cm} (4.6)

where $\varepsilon$ is the permittivity; and $\rho$ is the free charge density. In cylindrical coordinates, the two dimensional Poisson’s equation can be written as follows:
Figure 4.8: Schematic illustration of the computational domain used for the streamer simulation from columniform liquid dielectric hydrometeors in a uniform electric field. The hydrometeor in this case is a long cylindrical column, most similar to ice needles in the thundercloud.

\[ \nabla \cdot (\varepsilon \nabla V) = \frac{1}{r} \left( \varepsilon \frac{\partial V}{\partial r} \right) + \frac{\partial}{\partial r} \left( \varepsilon \frac{\partial V}{\partial r} \right) + \frac{\partial}{\partial z} \left( \varepsilon \frac{\partial V}{\partial z} \right), \]

where \( r \) and \( z \) are the horizontal and vertical coordinates, respectively. Along the boundary of the dielectric, \( \varepsilon \) is discontinuous. For grid cells located outside the dielectric hydrometeor (the dark grey shaded area in Figure 4.8), permittivity equals to its value in free space, \( \varepsilon_0 \). Inside the dielectric, \( \varepsilon = 80\varepsilon_0 \).

In order to properly capture the discontinuities in \( D \) and \( \varepsilon \) along the bound-
ary, the first step is to represent the boundary location by a signed distance function, \( \phi \). This function will be equal to zero at the location of the boundary, and will have positive values outside of the dielectric, and negative values inside. The value of \( \phi \) is equal to the distance of each grid node to the dielectric surface.

\( \varepsilon \) is calculated on the cell sides (domain half-grid lines), in the same location where the flux components are calculated. If grid points \( (i, j) \) and \( (i+1, j) \) are in the same medium, \( \varepsilon_{i+1/2,j} \) is evaluated between \( r_i \) and \( r_{i+1} \), in accordance with the side of the boundary that the cell side is located in, as determined by \( \phi \). The \( \varepsilon \) values on all sides of the cell \( (\varepsilon_{i\pm1/2,j\pm1/2}) \) are evaluated similarly. Since \( \phi \) is known at the grid nodes, a linear average of the nodal values can be used to find the value of \( \phi \) on the half-grid lines.

For added accuracy, we take the subcell location of the boundary into account. Assume that the boundary lies between \( r_i \) and \( r_{i+1} \), and splits the grid line into two pieces \( \theta \Delta r \) and \((1-\theta)\Delta r\). The value of \( \theta \) can be calculated as below:

\[
\theta = \frac{|\phi_{i,j}|}{|\phi_{i,j}| + |\phi_{i+1,j}|}.
\] (4.8)

Consequently, to evaluate \( \varepsilon \) if \( r_i \) and \( r_{i+1} \) are located on the opposite sides of the dielectric boundary, we define an “effective” \( \varepsilon \), say \( \hat{\varepsilon} \), which is calculated as below:
\[ \dot{\varepsilon} = \frac{\varepsilon_{i+1,j} \varepsilon_{i,j}}{\varepsilon_{i+1,j} \theta + \varepsilon_{i+1,j}(1 - \theta)}. \] (4.9)

With these definitions, we can discretize the right hand side of Equation 4.7 as follows:

\[
RHS = \frac{1}{\Delta r} [\varepsilon_{i+1/2,j}(V_{i+1,j} - V_{i,j}) - \varepsilon_{i-1/2,j}(V_{i,j} - V_{i-1,j})] + \\
\frac{1}{\Delta z} [\varepsilon_{i,j+1/2}(V_{i,j+1} - V_{i,j}) - \varepsilon_{i,j-1/2}(V_{i,j} - V_{i,j-1})] + \\
\frac{1}{2r_{i,j}} [\varepsilon_{i+1/2,j}(\frac{V_{i+1,j} - V_{i,j}}{\Delta r}) + \varepsilon_{i-1/2,j}(\frac{V_{i,j} - V_{i,j-1}}{\Delta r})].
\] (4.10)

Substituting Equations 4.8, and 4.9 into Equation 4.10, finalizes the implementation of the boundary condition capturing method proposed by Liu et al. [2000]. It is important to note that the last term in Equation 4.10 has an \( r_{i,j} \) term in the denominator, which will be problematic for the grid points along the symmetry axis of the domain, where \( r_{i,j} = 0 \). To overcome this problem, we use L'Hôpital's rule to evaluate this indeterminate term. As a result, Equation 4.10 will be written as below, for the grid points along the symmetry axis only:

\[
RHS = \frac{2}{\Delta r} [\varepsilon_{i+1/2,j}(\frac{V_{i+1,j} - V_{i,j}}{\Delta r}) - \varepsilon_{i-1/2,j}(\frac{V_{i,j} - V_{i-1,j}}{\Delta r})] + \\
\frac{1}{\Delta z} [\varepsilon_{i,j+1/2}(\frac{V_{i,j+1} - V_{i,j}}{\Delta z}) - \varepsilon_{i,j-1/2}(\frac{V_{i,j} - V_{i,j-1}}{\Delta z})].
\] (4.11)

The resulting linear equations in 4.10 and 4.11 can be transformed into a
system of linear equations, and solved using the SOR method. The boundary capturing method used here has the great advantage of simplicity of implementation. The algorithm described above, can be applied to any dielectric hydrometeor shape, whether the analytical solution of the electric field is available or not.

4.7 Modeling Results from the Dielectric Hydrometeor

In this section, we present streamer simulation results from each of the two states of the dielectric hydrometeor presented above.

4.7.1 Solid Ice Particles

Figure 4.9 shows cross-sectional views of distributions of electron density and electric field of a streamer that has successfully formed from an isolated spherical dielectric hydrometeor. The hydrometeor is placed in an ambient electric field value of $E_0 = 0.9E_k$, at 7 km altitude. The radius of the hydrometeor is equal to 3.1 mm, consistent with typical hydrometeor dimensions, and an initial plasma cloud is present to provide seed electrons. The plasma cloud has a Gaussian distribution with $\sigma = 0.22$ mm and peak density $= 1 \times 10^{17}$ m$^{-3}$.

Figure 4.10 shows profiles of electron density and electric field along the symmetry axis of the streamer, at different moments of time. To avoid confusion, the profiles show the positive tip of the hydrometeor only. The direction of propagation of the streamer in each profile is from right to left. From these figures, the streamer channel density value is about $5 \times 10^{19}$ m$^{-3}$, and the
Figure 4.9: Cross-sectional views of distributions of electron density (top) and electric field (bottom) of a streamer formed from a spherical dielectric hydrometeor (solid ice particle) at a few consecutive moments of time. The ambient electric field is $E_0 = 0.9E_k$, in the downward direction. The sphere radius is 3.1 mm, located at 7 km altitude.
streamer head electric field is about $8.5 \times 10^6$ V/m which is about $6E_k$ at 7 km altitude.

Figure 4.11 demonstrates the streamer profiles along the symmetry axis differently. The profiles of electron density, electric field, and space charge density along the symmetry axis are shown at two different moments of time. The solid lines correspond to a moment of time when the streamer is forming, and the dashed lines correspond to a fully formed streamer. The maximum space charge density is slightly behind the peak electric field in the streamer head, while colocated with the peak in electron density.

This is the first study to successfully model streamer discharges from an isolated dielectric hydrometeor in an external electric field below $E_k$. We compare the results from this streamer to previous studies dedicated to streamer formation and dynamics; streamers formed from a conducting sphere connected to an external circuit [Liu and Pasko, 2006], and an ionized spherical patch [Liu et al., 2012b, Sadighi et al., 2015]. The simulation setups for all three cases are as similar to each other as possible. For these cases, $E_0$ is $0.9E_k$, and the domain is placed at 7 km altitude. For the conducting sphere, the radius of the electrode is 3.1 mm, and the initial seed plasma cloud parameters are $\sigma = 0.22$ mm and peak density $= 1 \times 10^{17}$ m$^{-3}$. The ionized spherical patch also has a radius of 3.1 mm, and the peak density of the patch is $2 \times 10^{19}$ m$^{-3}$. Figure 4.12 illustrates cross-sectional views of electron density and electric field for these two simulation cases at the top, and profiles of electron density, electric field, and charge density along the symmetry axis for each streamer on the bottom.

To be able to effectively compare the three presented simulation cases, we have summarized the important streamer characteristics in Table 4.1. The ef-
Figure 4.10: Profiles of electron density (top) and electric field (bottom) along the symmetry axis of the streamer in Figure 4.9. The profiles only show the positive end of the dielectric hydrometeor, and in each plot the direction of propagation of the streamer is from right to left.
Figure 4.11: Profiles of electron density, electric field, and space charge density along the symmetry axis of the streamer in Figure 4.9. Solid lines correspond to a moment of time when the streamer is forming. Dashed lines correspond to a fully formed streamer.

Effects of the different electrodes on streamer parameters are demonstrated in the table, calculated when the streamers are about the same length. The exponential growth rate associated with the streamer radius and speed is found for each case. The growth rates are on the same order for all three cases. The dielectric streamer has the highest streamer head electric field, and all three streamers have similar channel densities.

4.7.2 Liquid Water Filaments

Figure 4.13 presents modeling results for the case of a hydrometeor with variable polarization. The figure shows cross-sectional views of distributions of electron
Figure 4.12: (top) Cross-sectional views of distributions of electron density and electric field of streamers formed from two other setups with the same dimension as the hydrometeor in Figure 4.9. For these cases, $E_0$ is also $0.9E_k$, and the domain is placed at 7 km altitude. (bottom) Profiles of electron density, electric field, and charge density along the symmetry axis for each streamer. Solid lines correspond to a moment of time when the streamer is forming. Dashed lines correspond to a fully formed streamer. (a) Conducting sphere with $r = 3.1$ mm, (b) Ionized sphere with $r = 3.1$ mm, peak density $= 2 \times 10^{19}$ m$^{-3}$. 
Table 4.1: Streamer characteristics measured for the three streamers in Figures 4.9 and 4.12, when the streamers are approximately the same length, demonstrating the effects of different electrodes on the streamer.

<table>
<thead>
<tr>
<th>Type</th>
<th>$E_h/E_k$</th>
<th>$N_s$ (m$^{-3}$)</th>
<th>Growth rate for $r_s$</th>
<th>Growth rate for $v_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conducting Sphere</td>
<td>5.8</td>
<td>$4.78 \times 10^{19}$</td>
<td>$2.1 \times 10^8$</td>
<td>$1.4 \times 10^8$</td>
</tr>
<tr>
<td>Ionized Sphere</td>
<td>5.4</td>
<td>$3.28 \times 10^{19}$</td>
<td>$2.3 \times 10^8$</td>
<td>$3.3 \times 10^8$</td>
</tr>
<tr>
<td>Dielectric Sphere</td>
<td>6.5</td>
<td>$3.89 \times 10^{19}$</td>
<td>$3.6 \times 10^8$</td>
<td>$3 \times 10^8$</td>
</tr>
</tbody>
</table>

density and electric field of a streamer that has formed from a dielectric hydrometeor filament. The hydrometeor is placed in an ambient electric field value of $E_0 = 0.8E_k$, at 7 km altitude. The length and radius of the hydrometeor are 2.9 mm and 0.44 mm, respectively, consistent with typical hydrometeor dimensions.

An initial plasma cloud with a Gaussian distribution is present to provide seed electrons, with $\sigma = 0.22$ mm and peak density $= 1 \times 10^{17}$ m$^{-3}$.

Figure 4.14 shows profiles of electron density and electric field along the symmetry axis of the streamer from Figure 4.13, at different moments of time. In these profiles, only the positive tip of the hydrometeor is demonstrated. The direction of propagation of the streamer in each profile is from right to left. From these figures, the streamer channel density value is about $1 \times 10^{20}$ m$^{-3}$, and the streamer head electric field is between $9 \times 10^6$ and $1 \times 10^7$ V/m which is about $6.5-7E_k$ at 7 km altitude.

Figure 4.15 provides a comparison between two streamers, one formed from a dielectric hydrometeor, and the other from an ionization column hydrometeor. The physical conditions of the two simulations are identical. The hydrometeor lengths and radii are 2.9 mm and 0.44 mm, respectively. The simulation domains are located in a uniform ambient electric field value of $0.8E_k$, pointing in the downward direction, at 7 km altitude. The ionization column hydrometeor has
Figure 4.13: Cross-sectional views of distributions of electron density (top) and electric field (bottom) of a streamer formed from a columniform dielectric hydrometeor (liquid water filament) at a few consecutive moments of time. The ambient electric field is $E_0 = 0.8E_k$, in the downward direction. The length and radius of the hydrometeor are 2.9 mm and 0.44 mm, respectively. The simulation domain is located at 7 km altitude.
Figure 4.14: Profiles of electron density (top) and electric field (bottom) along the symmetry axis of the streamer in Figure 4.13. The profiles only show the positive end of the dielectric hydrometeor, and in each plot the direction of propagation of the streamer is from right to left.
a peak density value of $9 \times 10^{19}$ m$^{-3}$.

The two streamers have different visual characteristics. We will refer to the simulation with the dielectric hydrometeor as case (1), and the ionization column as case (2). The streamer formed from case (1) seems to have a much smaller radius compared to case (2). The streamer channel density for case (1) is about $1 \times 10^{20}$ m$^{-3}$, while case (2) has a channel density of about $1 \times 10^{18}$ m$^{-3}$. The maximum streamer head electric field for case (1) is about $9.5 \times 10^6$ V/m, while case (2) has a slightly smaller streamer head field, equal to $7 \times 10^6$ V/m. As expected, the streamer from case (1) has started from an ionized region ahead of the hydrometeor tip. This seems to also be the case for the streamer in case (2).

Case (1) has a slightly longer time scale, compared to case (2). This could be a consequence of the dielectric hydrometeor having less initial seed electrons present around its tip. This will delay the initiation of electron avalanches and consequently, the streamer initiation. Further investigation is required to identify the main parameters that contribute to the differences between the two cases.
Figure 4.15: Cross-sectional views of electron density and electric field of streamers formed from: (top) a dielectric hydrometeor; case (1), and (bottom) an ionization column hydrometeor; case (2).
Chapter 5

Summary and Future Work

5.1 Summary of Results

How lightning originates inside thunderclouds and propagates through air is a longstanding fundamental problem in the field of atmospheric electricity. Lightning researchers have been trying to answer this question since Benjamin Franklin discovered the electrical nature of lightning through his famous kite experiment [e.g., Rakov and Uman, 2003, p. 121, and references cited therein]. The theoretical efforts of this dissertation research has aimed to advance the current knowledge of lightning initiation. The first step in improving our understanding of lightning initiation is to study the precursors of lightning leaders, streamers. We have focused our efforts on studying streamer initiation and propagation from thundercloud hydrometeors and have established that streamers can be initiated from thundercloud hydrometeors in the maximum thundercloud electric field measured so far, which is well below the conventional breakdown threshold field $E_k$. 
For the first part of this study, we have further investigated the idea proposed by Liu et al. [2012b] to study streamers from ionization column hydrometeors in thundercloud conditions. According to our simulation results, positive streamers can be initiated from columniform hydrometeors subject to the maximum observed thundercloud electric fields of $\sim 0.3E_k$. We have performed simulations for ambient electric field values of $0.5E_k$ at thundercloud altitudes, and obtained successful streamer formation, confirming the work of Liu et al. [2012b]. Reducing the ambient electric field to as low as $0.3E_k$ affects the streamer’s ability to propagate stably. In this case the streamer will either not form, or branch during initiation. The branching of the streamer interrupts the stable streamer propagation and changes the dynamics of the streamer. According to our results, introducing a proper background ionization distribution stabilizes the initiation and propagation of the streamer. We have investigated the streamer branching behavior and characteristics, and a theory to explain the mechanism of this phenomenon has been formulated and tested.

In order to verify whether an ionization column is a proper representation of a dielectric hydrometeor, for the second part of this dissertation we modify the streamer discharge model to accommodate an isolated dielectric particle representing the hydrometeor inside the computational region. Since the governing equations of the plasma discharge model are discretized on a uniform rectangular grid, the boundary cells do not conform to the shape of the dielectric. We utilize a boundary-cut cell method proposed by Ye et al. [1999] to handle these cut cells, and properly evaluate the corresponding fluxes. A boundary condition capturing method by Liu et al. [2000] has been used to properly enforce the electrostatic boundary conditions on the dielectric hydrometeor. This model
enables us to accurately simulate the discharges around dielectric hydrometeors with various shapes and physical states. Streamer discharge results obtained from the dielectric hydrometeor have been presented and compared with the results obtained from the first part of this work.

5.2 Implications to Lightning Initiation

The modeling study presented here has direct implications to lightning initiation, since streamers are known to be the precursors of lightning leaders. The obtained results demonstrate that hydrometeors present in measured thundercloud electric field values can give rise to streamer discharges. This is the first detailed theoretical modeling study that has been able to confirm the experimental work previously done on this topic [e.g., Crabb and Latham, 1974, Griffiths and Latham, 1974, Petersen et al., 2006]. The maximum of the observed thundercloud electric field values hundreds of meters away from the lightning initiation point just before lightning discharges are typically between 300 and 450 kV/m (at ground pressure) [Marshall et al., 1995], while higher values as large as 929 kV/m (at ground pressure) have also been measured [Stolzenburg et al., 2007]. The minimum ambient electric field value used in this study is equivalent to 960 kV/m at ground pressure. For this electric field, the initiation and stable propagation of the streamer has been shown to depend on the amount and distribution of background ionization present inside the cloud. Confirmation of streamer formation from hydrometeors inside thundercloud electric fields positively impacts lightning initiation studies.

This study also has implications to lightning related phenomena such as the
relativistic feedback discharges that are thought to produce Terrestrial Gamma-ray Flashes (TGFs) [Dwyer, 2012, Dwyer et al., 2012, Liu and Dwyer, 2013]. Terrestrial gamma-ray flashes are bursts of extremely high energy radiation coming from thunderclouds and have been observed from space [e.g., Fishman et al., 1994] and from ground [Dwyer et al., 2012]. One theory to explain TGF production from thunderclouds is relativistic feedback discharges [Dwyer, 2012, Liu and Dwyer, 2013]. According to Dwyer [2012] and Liu and Dwyer [2013], TGF pulses can be produced naturally by the relativistic feedback discharge that develops in thundercloud electric fields. These studies introduced a self-propagating discharge resulting from the relativistic feedback discharge mechanism, called the relativistic feedback streamer. The relativistic feedback streamer tends to propagate unstably, and the electric field in the streamer head continually increases. This head field can reach up to one-third to one-half of the conventional breakdown threshold field, raising the possibility that if a hydrometeor is present inside the relativistic feedback streamer head field, a conventional streamer is able to form.

From our study, for electric field values lower than $0.3E_k$, it is unlikely that the conventional streamer is initiated from realistic hydrometeors. At an early stage of the relativistic feedback streamer, when the peak electric field in the head is smaller than $0.3E_k$, conventional streamer activity may therefore be ignored. Figures 6a and 10 from the study by Liu and Dwyer [2013] present axial profiles of the electric field of a relativistic feedback streamer propagating in a uniform and nonuniform thundercloud electric field, respectively. From their Figure 6a, the relativistic feedback streamer propagates for about 12 ms before the electric field at the head exceeds $0.3E_k$. During this time, the first
two TGF pulses have already been produced (Figure 7 in Liu and Dwyer [2013]). The relativistic feedback streamer propagating in a nonuniform field (Figure 10 in Liu and Dwyer [2013]) demonstrates a similar behavior. The electric field in the head of the feedback streamer exceeds $0.3E_k$ after about 15 ms, during which at least three TGF pulses have been produced. This means that the relativistic feedback discharge streamer can produce TGF pulses before any conventional streamers might be initiated from positive hydrometeors.

Based on what has been demonstrated here, when the electric field in the relativistic feedback streamer head reaches and exceeds $0.3E_k$, with the presence of hydrometeors around the relativistic feedback streamer head, it is possible that conventional streamers may form. Liu and Dwyer [2013] have estimated the residence time of the high electric field of the relativistic feedback streamer to be 200 $\mu$s, which is sufficient time for polarization of water or ice hydrometeors. Therefore, it may be expected that bursts of streamers are produced in the later stage of the relativistic feedback discharge. The importance of these conventional streamer discharges in the dynamics of the feedback streamer has yet to be investigated.

5.3 Suggestions for Future Work

The ultimate objective of the project we have started in this dissertation research is to improve our understanding of how lightning is initiated in the thundercloud and how it can propagate such long distances through air. This study offers solid results for continuing this path to improve the knowledge of lightning. There are three critical steps in the lightning initiation and propagation
problem: the initiation and propagation of streamers in observed thundercloud electric fields, the transition of these streamers to bright, hot, and highly conductive leaders, and the propagation of leaders.

The results of this study have shown that streamers can be initiated from thundercloud hydrometeors in maximum thundercloud electric fields. The next step for continuing this research would be to understand how the streamer-to-leader transition occurs. Whether the streamers originating from thundercloud hydrometeors will be able to generate a leader is still unknown. There are a number of scientific questions that must be resolved in this area. For example, what are the effects of Joule heating in the streamer channels originating from thundercloud hydrometeors? What are the thundercloud conditions that the streamer-to-leader transition depends on? We believe that the contributing results of this dissertation research will have a significant impact on following this path to understanding lightning.
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