

Acceleration of suprathermal particles by compressional plasma wave trains in the solar wind

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[1] This paper presents a calculation of particle acceleration by an idealized compressional plasma wave train. In this model, suprathermal particles, such as pickup ions, are continuously injected into a wave train consisting of a series of compression or rarefaction regions. The momentum distribution of particles will become broader and broader as they go through the wave train, which is very similar to diffusion in momentum space. The acceleration process is very fast: it does not take too many wave cycles even with a small compression amplitude to reach an asymptotic steady state momentum distribution. In the absence of large-scale adiabatic cooling, the asymptotic distribution is flat below the initial injection momentum, and above the injection momentum, it is proportional to a power law distribution with the slope of -3 . This distribution appears to be independent of any model parameters in this acceleration mechanism. If there is a prevailing large-scale adiabatic cooling by the expanding solar wind, the asymptotic steady state distribution remains to be flat below the injection momentum and it is a power law distribution but with a steeper slope above the injection momentum. The acceleration process alone does not automatically guarantee a p^{-5} power law. However, since the process can quickly build up pressure from the accelerated particles, it is expected that the amplitude of compressional plasma wave will be reduced. In the final state after nonlinear wave-particle interactions, the distribution of accelerated particles and plasma wave must achieve a balance between large-scale adiabatic cooling and the acceleration by compressional plasma waves. The particle distribution at the equilibrium will settle with a p^{-5} distribution given that the initial solar wind pressure is much larger than the initial pressure of newly injected particles.

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1. Introduction

[2] Suprathermal ions are charged particles of energies significantly above plasma thermal energy in the plasma reference frame. If particles have a Maxwellian distribution, we expect that their flux will soon drop to an undetectable level beyond a few times the thermal energy. In interplanetary space, we always see a strong tail particle population in energy spectrum. The origin of suprathermal particles can come from several possible sources [e.g., *Mewaldt et al.*, 2001] including pickup ions which are produced from interstellar neutral atoms through photoionization by solar radiation or charge exchange with solar wind ions.

[3] The production of pickup ions in the heliosphere is well understood [see, e.g., *Rucinski and Fahr*, 1991]. Since these particles mainly convect with the solar wind with little diffusion on the scale size of the heliosphere, the spatial

distribution of pickup ion density in the heliosphere is known to a good degree. However, what is much less settled is their distribution in velocity or momentum space, particularly at speed above their initial injection speed. The initial speed of pickup ions in the solar wind reference frame is equal to the solar wind speed. Observations of pickup ions in interplanetary space [*Gloeckler et al.*, 1994] show they can extend to much higher speeds. Clearly, they are accelerated in the solar wind. Similarly, suprathermal particles of solar or other origins will also undergo acceleration during their transport through interplanetary media. While a few suggested that interplanetary shocks can accelerate pickup ions and other suprathermal particles [e.g., *Giacalone et al.*, 1997], most litterateurs attribute the acceleration of these particles to stochastic wave-particle interactions in the form of diffusion in momentum space [*Isenberg*, 1987; *Schwadron et al.*, 1996; *le Roux and Ptuskin*, 1998; *Chalov et al.*, 2004, 2006]

[4] According to the theory of wave-particle interaction, the distribution of freshly injected pickup ions is unstable, providing free energy to excite plasma waves. The waves have been seen to heat up solar wind [*Smith et al.*, 2001],

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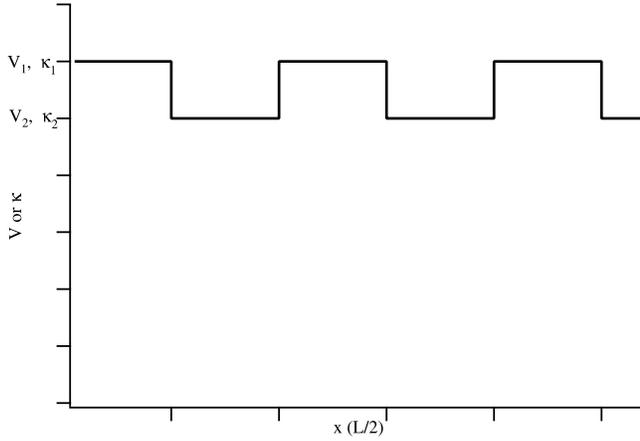


Figure 1. An idealized profile of plasma speed and particle diffusion coefficient in the reference frame moving with the wave train.

and in the mean time, isotropize and flatten the momentum distribution of pickup ions. The calculation by *Chalov et al.* [2004] showed that most of the waves energy is reabsorbed by pickup ions for acceleration to higher energies. It is thus hoped that the second-order stochastic Fermi acceleration could eventually lead to a ubiquitous $\sim p^{-5}$ power law distribution often observed in the heliosphere [*Fisk and Gloeckler*, 2006, 2008]. However, the calculation by *Chalov et al.* [2004, 2006] showed the distribution of pickup ions has a steep cutoff slightly beyond the initial injection momentum. Furthermore, *Fahr et al.* [2009], assuming that the waves are generated by pickup ions, dismissed the Fisk and Gloeckler's suggestion that the $\sim p^{-5}$ distribution can be formed in the equilibrium with hydromagnetic turbulence, and then put forward a theory for the power law tail as a result of modulation of higher energy particles such as anomalous cosmic ray.

[5] In this paper, we carry out a detailed calculation of particle acceleration by a train of compressional plasma wave in the solar wind. Our calculation differs from previous calculations in the following manners: (1) compressional plasma waves can pre-exist in the solar wind. They can come directly from solar corona, or be produced in solar wind stream interaction regions or shocks, and (2) we assume that the compressional plasma waves have wavelengths comparable or smaller than particle diffusion scale. Under these conditions, we can demonstrate that particles can quickly form a very flat power law distribution beyond their initial injection momentum. The power law tail can contain large amount of pressure that may modify the amplitude of compressional waves, leading to an equilibrium spectrum. Once the compressional waves die down at some large radial distances, the power law spectrum above the initial injection momentum remains as the entire population cools down adiabatically.

2. Model

[6] Our model assumes that there is a preexisting plane compressional plasma wave train propagating in the solar wind. The wave can be any type of propagating compressional waves, such as fast mode plasma wave. An example

of these waves can be seen in observations [e.g., *Burlaga and Turner*, 1976]. The waves have short wavelengths and they are relatively localized compared to the size of the heliosphere. We seek a steady state solution to the transport equation for the isotropic part of particle distribution function $f(x, p, t)$, which can be written as

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x} \kappa(x) \frac{\partial f}{\partial x} - V(x) \frac{\partial f}{\partial x} + \frac{1}{3} \nabla \cdot \mathbf{V}_{sw} \frac{\partial f}{\partial \log p} + Q(x), \quad (1)$$

where x is the coordinate in the wave reference frame, p particle momentum in the solar wind plasma frame in which convective electric field is zero. See Appendix A for derivation of particle transport equation in wave frame. Other models parameters are particle diffusion $\kappa(x)$, plasma speed in the wave frame $V(x)$, solar wind plasma velocity \mathbf{V}_{sw} in an inertial frame and pickup ion source injection rate $Q(x)$. We assume $\kappa(x)$ is not a function of momentum and the diffusion length κ/V is larger than the size of each compression or rarefaction region but smaller than the distance between them. This assumption is mainly for the purpose of mathematical simplicity for demonstrating the main properties of particle acceleration. Under these assumptions, we can approximate the compressional plasma wave train to be a series of square waves as shown in Figure 1. Each compression or rarefaction region is infinitely thin and it looks like a shock wave to the particles, although it is thermodynamically impossible to form rarefaction shocks. The solar wind velocity is a superposition of average solar wind velocity, wave propagation velocity and plasma velocity in the wave frame, i.e., $\mathbf{V}_{sw} = \bar{\mathbf{V}}_{sw} + \mathbf{V}_w + \mathbf{V}(x)$. Since the solar wind speed is much larger than its sound speed or Alfvén speed, the total solar wind velocity is approximately radial and its magnitude is dominated by the average solar wind speed. The divergence of the solar wind velocity becomes

$$\nabla \cdot \mathbf{V}_{sw} = \frac{2V_{sw}}{r} + \frac{\partial V}{\partial x}. \quad (2)$$

Since we considered particle acceleration by waves on small scales, r can be treated as a constant. Particles are assumed to be injected at each of the compression or rarefaction regions, i.e.,

$$\begin{aligned} Q(x) &= \sum_{k=1}^N Q_0 \delta(p - p_0) \delta(x - kL/2) \\ &= \sum_{k=1}^N \frac{n_0 \nu}{4\pi p_0^2} \delta(p - p_0) \delta(x - kL/2), \end{aligned} \quad (3)$$

where N is total number of wave cycles, $p_0 = mV_{sw}$ the initial particle momentum, ν pickup ion production rate over half wavelength L , and n_0 neutral atom density. This source term is equivalent to a continuous constant injection over the entire space where the wave is present.

[7] The transport equation can be solved using Fourier transform of variable $q = \log p$ space

$$\tilde{f}(x, \omega) = \int_{-\infty}^{\infty} f(x, e^q) e^{-i\omega q} dq, \quad (4)$$

$\tilde{f}(x, \omega)$ can be viewed as a characteristic function of the momentum distribution. Since we assume that the diffusion

length is much smaller than the gaps between the compression and rarefaction regions, particle acceleration or cooling in each of these regions can be treated separately. The calculation procedure becomes similar to the treatment of particle acceleration by an ensemble of shock waves done by *Achterberg* [1990]. For each of the compression or rarefaction regions, boundary conditions similar to those for shock waves relate the downstream (dn) and upstream (up) characteristic functions as follows (see Appendix B),

$$\tilde{f}_{dn} = \frac{1}{1 - i\omega/s} \tilde{f}_{up} + \frac{1}{1 - i\omega/s} \frac{Q_0}{p_0 V_{up}} e^{-i\omega q_0}, \quad (5)$$

where $q_0 = \log p_0$ and $s = 3V_{up}/(V_{dn} - V_{up})$ is the slope of power law spectrum produced by shock acceleration. For the compression region $s = s_- = -3V_1/(V_1 - V_2)$, and for the rarefaction region $s = s_+ = 3V_2/(V_1 - V_2)$. Notice that $s_+ + s_-$ is always equal to -3 . As a property of diffusive shock acceleration, the recursion relation (5) does not depend on κ , even though κ is not zero. In the region between the adjacent compression and rarefaction regions, which is much longer than the particle diffusion length, the particles undergo a constant adiabatic cooling while transporting downstream mainly through convection, so the recursion relation for the upstream and downstream characteristic functions is

$$\tilde{f}_{dn} = \tilde{f}_{up} e^{i\omega c}, \quad (6)$$

where $c = c_1 = V_{sw} L/(3rV_1)$ is the cooling rate in the region with convection speed V_1 or $c = c_2 = V_{sw} L/(3rV_2)$ for the other region. Then the solution to the downstream characteristic function after N wave cycles is (see Appendix B),

$$\tilde{f}_N = \frac{Q_0}{p_0} e^{-i\omega q_0} \left[\frac{1}{V_1(1 - i\omega/s_-)(1 - i\omega/s_+)} e^{i\omega c_2} + \frac{1}{V_2(1 - i\omega/s_+)} \right] \times \sum_{k=0}^{N-1} \left[\frac{1}{(1 - i\omega/s_-)(1 - i\omega/s_+)} e^{i\omega(c_1+c_2)} \right]^k. \quad (7)$$

Equation (7) contains a sum of a geometric series which can be evaluated easily, leading to

$$\tilde{f}_N = \frac{Q_0}{p_0} e^{-i\omega q_0} \left[\frac{1}{V_1(1 - i\omega/s_-)(1 - i\omega/s_+)} e^{i\omega c_2} + \frac{1}{V_2(1 - i\omega/s_+)} \right] \times \left[1 - \frac{1}{(1 - i\omega/s_-)(1 - i\omega/s_+)} e^{i\omega(c_1+c_2)} \right]^{-1} \times \left\{ 1 - \left[\frac{1}{(1 - i\omega/s_-)(1 - i\omega/s_+)} e^{i\omega(c_1+c_2)} \right]^N \right\}. \quad (8)$$

Substitute equation (7) or equation (8) into equation (4) and carry out the integration by completing a closed contour on the complex plane with a semi-circle of infinity radius above the $\omega = 0$ line for $q > q_0$ or below $\omega = 0$ for $q < q_0$. The final solution of particle distribution is a sum of residues contained inside the contour.

3. Results

3.1. Spectrum After Infinite Number of Wave Cycles

[8] If the compressional plasma wave train is long enough or N is very large, the term in equation (8) containing the

N th power becomes infinitely small on the entire complex plane of ω . Then

$$\tilde{f}_\infty = \frac{Q_0}{p_0} e^{-i\omega q_0} \left[\frac{1}{V_1} e^{i\omega c_2} + \frac{1}{V_2} (1 - i\omega/s_-) \right] \cdot \left[(1 - i\omega/s_+)(1 - i\omega/s_-) - e^{i\omega(c_1+c_2)} \right]^{-1}. \quad (9)$$

Corresponding article distribution function can be obtained with a inverse Fourier transform of the characteristic function, which can be evaluated using contour integration on the complex plane of ω (see Appendix C). This characteristic function has two simple poles, which can be found by setting the denominator equal to zero. If we further assume that adiabatic cooling per wave cycle $c_1 + c_2$ is small enough to ensure significant particle acceleration above the initial injection momentum p_0 , the two poles can be found analytically through Taylor expansion of the exponential function $e^{i\omega(c_1+c_2)}$ in (9). One of poles is at $\omega = 0$ and the other is at $\omega = -i\gamma$, where

$$\gamma = s_+ + s_- + s_+s_-(c_1 + c_2). \quad (10)$$

The first pole corresponds to a flat distribution function below p_0 with a phase space density,

$$f(p < \sim p_0) = f_0 \frac{s_+s_-}{\gamma} = \frac{Q_0}{p_0} \left[\frac{1}{V_1} + \frac{1}{V_2} \right] \frac{s_+s_-}{\gamma}, \quad (11)$$

and the second pole produces a power law distribution for $p > p_0$ with a slope γ . Some examples of these distribution functions are shown by the lines labeled as asymptotic power law in Figure 2.

[9] If adiabatic cooling by large-scale expansion of the solar wind is negligibly small compared to the acceleration effect of the compression or rarefaction regions, i.e., $c_1 + c_2 \ll (s_+s_-)^{-1}$, then the slope of the power law on the $p > p_0$ side is always equal to $s_+ + s_- = -3$. This slope is independent of any model parameters. The result is consistent with the calculation of particle acceleration by ensemble of identical shocks [*Achterberg*, 1990]. We have done calculations with different shapes for the waves including finite thickness for each of the compression or rarefaction regions. We have put in particle diffusion coefficients of various diffusion scales compared to the size of the compression/rarefaction region and wavelength. We have even put in waves with randomly distributed compression/rarefaction amplitudes. (For space and simplicity of presentation we leave these calculations out of this paper.) We found all of them produce the same asymptotic distribution function: a flat distribution below p_0 and p^{-3} power law distribution above p_0 . This universal distribution function can be established as long as there is enough cycles of compression waves. Details, such as the particle diffusion coefficient and the amplitude of wave compression ratio V_1/V_2 only affect the number of wave cycles needed to establish this asymptotic distribution at a given momentum. The p^{-3} distribution is a very flat spectrum. If it continues to infinite momentum, it contains a divergent number density of particles, which is a reasonable result because we have injected infinite numbers of particles over an infinitely long wave train. This kind of hard spectrum has never been

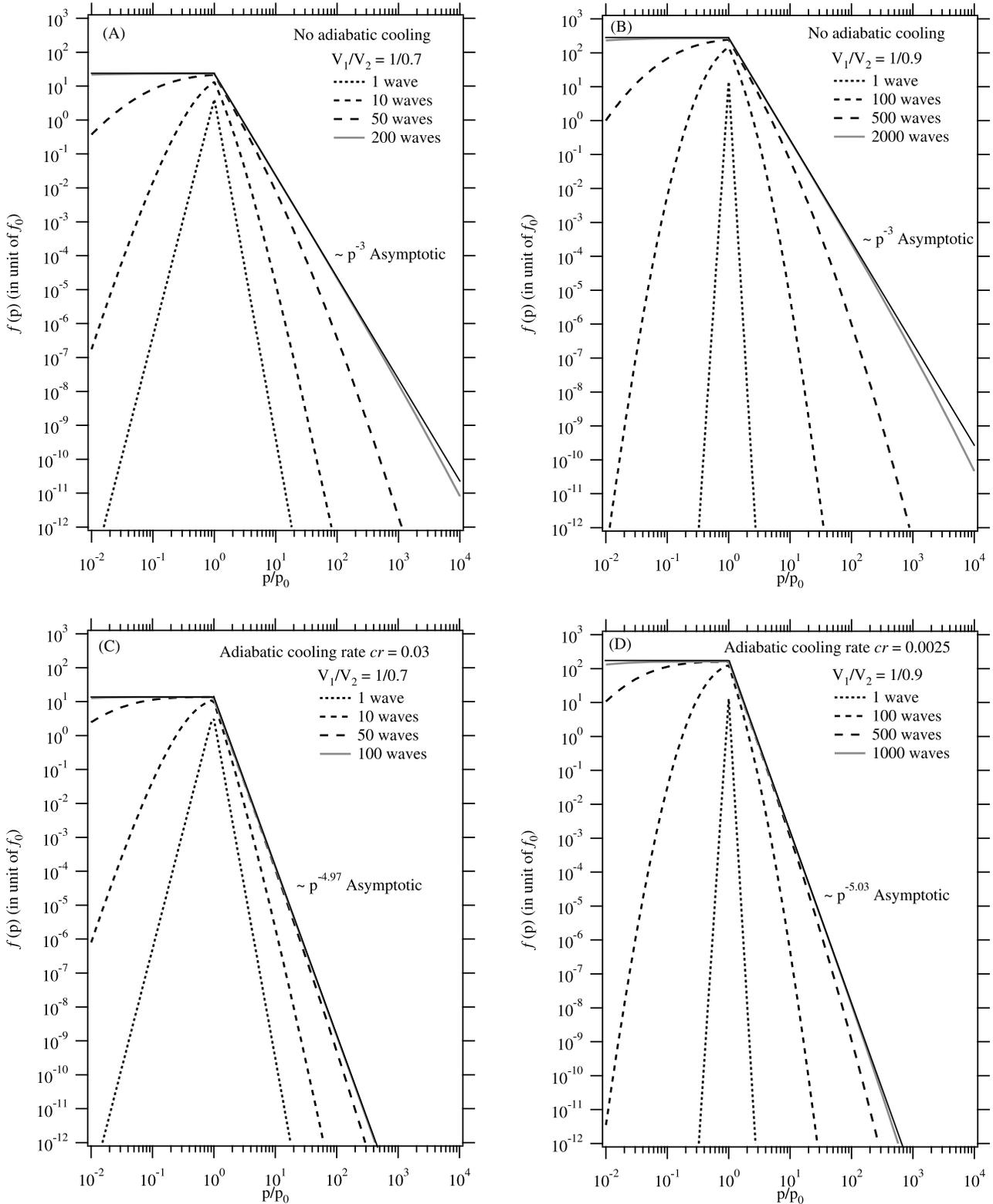


Figure 2. Momentum distribution function of particles after they are injected into and accelerated by certain number of waves cycles.

observed. We believe that other factors, such as finite wave cycles, large-scale adiabatic cooling, or back reaction of accelerated particles on to the plasma waves, must have acted to prevent it from happening.

[10] If adiabatic cooling by large-scale expansion of the solar wind gets a larger, the asymptotic distribution function on the $p > p_0$ side becomes steeper while the distribution function on the $p < \sim p_0$ side remains flat (Figures 2c and 2d).

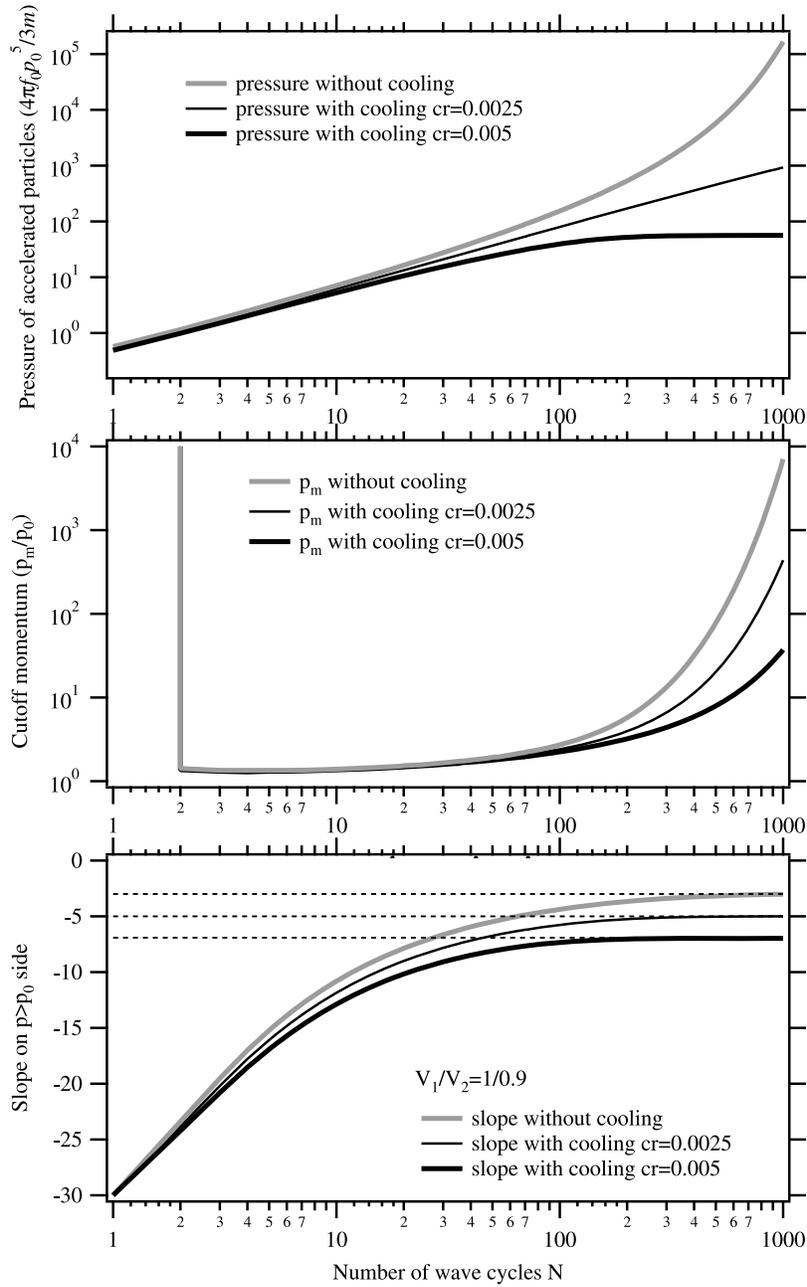


Figure 3. Evolution of spectral slope of particles at momentum slightly above p_0 , cutoff momentum of the power law slope p_m , and the pressure of accelerated particles with the number of wave cycles they have gone through.

The slope of the power law distribution generally depends on the rate of adiabatic cooling, distance scale L as well as the amplitude of wave compression ratio. The slope can be any number below -3 . The total number density of particles remain finite, even though the wave is infinitely long. This is because the volume gets larger in a flow with a finite expansion to the infinity. *Achterberg* [1990] put in particle leakage as a different loss mechanism. He found that the power law slopes of particle spectrum on both sides of p_0 become steeper than without loss, which is similar to our result on the $p > p_0$ side but different on the $p < p_0$ side.

3.2. Spectrum After Finite Number of Wave Cycles

[11] If the number of wave cycles N is finite, the inverse Fourier transform of the characteristic function (8) must be evaluated to derive the particle distribution function. This situation can longer be modeled by the ensemble approach of *Achterberg* [1990]. The characteristic function has two more poles at $\omega = -is_+$ and $\omega = -is_-$ in addition to the two poles in the case of $N \rightarrow \infty$. These two additional poles are high-order poles and their residues can also be calculated, but the result will be too complicated to write it in analytical form. Figure 2 shows a few examples of particle distribu-

tions after acceleration by a finite number of wave cycles. All these calculations start with mono-energetic injection. After one wave cycle, the distribution function becomes a double power law with a slope s_+ on the $p < p_0$ side and a slope s_- on the $p > p_0$ side. The distribution after one wave cycle is flatter with a larger plasma compression ratio. As the number of wave cycles increases, the slopes of the distribution function on both sides of $p/p_0 = 1$ become less steep. The broadening the spectrum mimics the process of particle diffusion in the momentum space. The distribution above p_0 starts approximately with a power law until some cut-off momentum. Eventually, after a sufficient number of wave cycles, the distribution function approaches to the above described asymptotic distribution of power laws, first starting at low momentum and then propagating to higher and higher momentum. Comparing Figure 2b with Figure 2a and Figure 2d with Figure 2c, we can see it takes few wave cycles to approach to the asymptotic power law distribution at a given momentum if the amplitude of plasma compression ratio is larger. When large-scale adiabatic cooling is present, the distribution function approaches to the asymptotic power law distribution in less number of wave cycles, partly because the asymptotic distribution is steeper. Figure 3 gives some examples that how the spectral slope of particle distribution function at the momentum slightly above p_0 varies with the number of wave cycles N . Even with a small plasma compression amplitude of 1/0.9, it just takes a few hundred wave cycles to establish the asymptotic power law distribution. Therefore, we conclude that particle acceleration by compressional plasma waves is very fast and efficient.

3.3. Back Reactions of Accelerated Particles to Plasma Waves

[12] When pickup ions are injected in to a train of compressional plasma wave, pressure from the accelerated particles builds up. If the initial amplitude of the compressional plasma wave train is larger enough to overcome the effect of adiabatic cooling by large-scale expansion of the solar wind and produce a distribution of a slope equal to or greater than -5 , the pressure will increase exponentially or faster. Figure 3 shows how the pressure of accelerated particles varies with the number of wave cycles. It may start with a negligible amount compared to initial solar wind thermal pressure. If we let it go on, eventually the pressure will become infinitely large. Under such a circumstance, the properties of plasma wave train will be modified.

[13] For simplicity, let's assume that the compressional plasma wave is a sound wave. Initially it propagates in the solar wind with a sound speed of $c_{p0} = \sqrt{5P_0/3\rho_0}$, where P_0 and ρ_0 are the thermal pressure and mass density of the solar wind. After pickup ions are injected and accelerated by sufficient number of wave cycles, pressure will increase rapidly, while total plasma density increases slowly (if photoionization) or remains constant (if charge exchange). The changes will result in increase of wave propagation speed,

$$c_p = \sqrt{\frac{5(P_0 + P_{pui})}{3(\rho_0 + \rho_{pui})}}, \quad (12)$$

where ρ_{pui} and P_{pui} are pickup ion density and pressure, which can be evaluated if we assume that the pickup ion distribution is flat $f = f(p_0)$ for $p < p_0$ and it is $f = f(p_0)(p/p_0)^\gamma$ for $p \geq p_0$ up to a maximum momentum $p_m \gg p_0$,

$$\rho_{pui} = 4\pi m f_0 p_0^3 \left[\frac{1}{3} - \frac{1}{\gamma+3} + \frac{1}{\gamma+3} \left(\frac{p_m}{p_0} \right)^{\gamma+3} \right], \quad (13)$$

$$P_{pui} = \frac{4\pi f(p_0) p_0^5}{3m} \left[\frac{1}{5} - \frac{1}{\gamma+5} + \frac{1}{\gamma+5} \left(\frac{p_m}{p_0} \right)^{\gamma+5} \right]. \quad (14)$$

Typically, the increase of total plasma density due to the injection of pickup ions is minimal compared to the background solar wind density, or $\rho_{pui} \ll \rho_0$, but the increase of total plasma pressure could be significant because (1) the initial injection speed of pickup ions is large compared to solar wind thermal speed and (2) the tail of distribution function beyond the initial momentum p_0 could be flatter than -5 to make its contribution to pressure increasingly larger at larger p . This will result in a dramatic enhancement of wave propagation speed.

[14] Using WKB approximation to the wave equation [e.g., *Filippi*, 1999, chap. 5.4] we will see two effects on the compressional wave. Both the wavelength and wave amplitude will change according to the following relation,

$$L = \frac{c_p}{c_{p0}} L_0, \quad (15)$$

$$V_1 - V_2 = \Delta V = \sqrt{\frac{c_{p0}}{c_p}} \Delta V_0, \quad (16)$$

where L_0 is the initial wavelength and ΔV_0 is initial wave amplitude. The scaling of the wave amplitude is a direct consequence of the conservation of wave energy. When the propagation speed increases with increasing pressure, the amplitude of wave has to decrease to conserve the energy flux of wave. Equation (17) can also be understood as an approximate dependence of shock compression ratio on Mach number at a weak shock of Mach of ~ 1 . The exact dependence of compressional wave amplitude to particle pressure may be different for different wave system, but in most physical cases, we expect that the amplitude should decrease with pressure. The argument is in line with a property of ideal gas; it is more difficult to compress the gas when its temperature is high.

[15] Figure 4 qualitatively depicts how the slope and pressure of the accelerated particles might happen when particles are continuously injected into a train of compressional plasma wave. The wave amplitude is allowed to change as a result of back reaction of accelerated particles while the cooling rate per wave cycle $cr = c_1 + c_2$ is kept constant. The initial solar wind pressure is set at a hypothetical value of 10^4 in the unit of $4\pi f_0 p_0^3 / (3m)$, which is roughly equal to the pressure of injected particles in one wave cycle. If the initial amplitude of wave is small, for example, $V_1/\Delta V = 15$, both the slope and pressure will increase along the line of constant $V_1/\Delta V$ curve until they achieve their maximum values. The maximum pressure is not sufficient compared to the solar wind pressure P_0 to

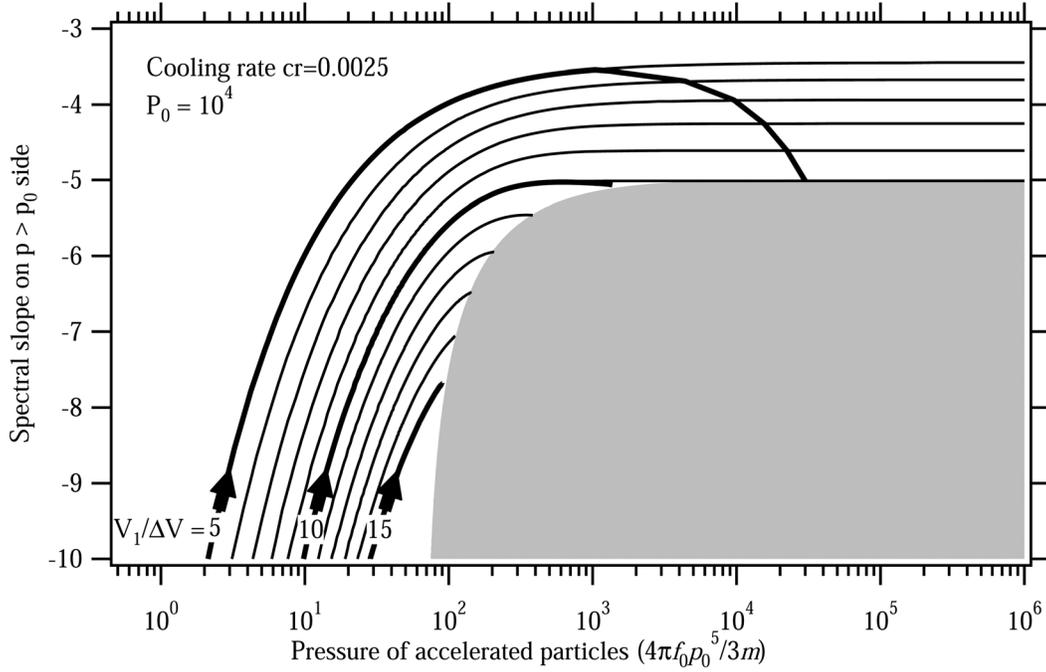


Figure 4. Trajectories of spectral slope of accelerated particles versus their pressure as the particles go through acceleration by compressional plasma waves. Thin lines are the trajectories with a constant plasma wave amplitude. The thick lines trajectories if the amplitude of compressional waves can get reduced by the pressure of accelerated particles. The gray area is forbidden from this model. When non-linear wave-particle interaction reaches an equilibrium, the particle spectral slope and the pressure will end up at some point along the boundary of the gray area depending on the initial amplitude of the wave.

affect the wave amplitude. If the initial wave amplitude is large (e.g., $V_1/\Delta V = 5$), the slope and the pressure of the accelerated particles will build up quickly along the constant $V_1/\Delta V$ curve. After only ~ 100 wave cycles, the pressure has reached a significant level compared to P_0 , the wave amplitude begins to decrease and the trajectory of slope versus pressure leaves its original track. By the time the pressure of the accelerated particles is a few times the solar wind pressure P_0 (intersection with the gray area), the wave amplitude has been reduced enough so that the particles can no longer gain net energy and the pressure reaches its maximum value. At this point, the system of particles and wave is in an equilibrium state and the final spectral slope is very close to -5 . This equilibrium spectral slope of -5 can be established as long as the initial solar wind pressure is bigger than a few thousands times the initial pressure of pickup ions injected in one wave cycle. In the process to reach the equilibrium state of wave amplitude and particle spectrum, we can see that the spectrum slope can exceed -5 momentarily in order to build up its pressure. This is short-lived and will disappear once enough particles are injected. If the initial wave amplitude is just right (e.g., $V_1/\Delta V = 10$) to make the final distribution approach to a -5 slope, the buildup of pressure is slower. At some point after many wave cycles, the pressure may be enough to modify the wave amplitude. A minute modification is just needed to stop pressure increase. The slope will again settle at approximately -5 .

3.4. Comparison With Diffusion in Momentum

[16] As shown in the above sections, when particles go through repeated acceleration or cooling in compression or rarefaction regions, their momentum distribution function becomes broader and broader. This is similar to diffusion in momentum space. We have adopted a perturbative approach to particle transport in random compressional plasma turbulence derived by *Bykov and Toptygin* [1982, 1993] and *Ptuskin* [1988]. The particle distribution averaged over an ensemble of plasma fluctuations obeys

$$\frac{df}{d\tau} = \frac{\partial f}{\partial t} + \bar{\mathbf{V}} \cdot \nabla f = \frac{1}{3} \nabla \cdot \bar{\mathbf{V}} p \frac{\partial f}{\partial p} + \frac{1}{p^2} \frac{\partial}{\partial p} \left(D p^2 \frac{\partial f}{\partial p} \right) + \frac{2Q_0}{L} \delta(p - p_0), \quad (17)$$

where $\bar{\mathbf{V}}$ is averaged velocity of large-scale plasma motion and D is the momentum diffusion coefficient that includes the effect of diffusive particle acceleration by small-scale compression or rarefaction regions. Since Q_0 is the particle injection rate over a half wave cycle in our model, $2Q_0/L$ is the particle injection rate density. We have neglected spatial diffusion over large scales for particles of suprathermal energies. According to *Bykov and Toptygin* [1993], the momentum diffusion coefficient can be written as

$$D = D_0 p^2 = \frac{p^2}{9} \overline{(\nabla \cdot \delta \mathbf{V})^2} \tau_c, \quad (18)$$

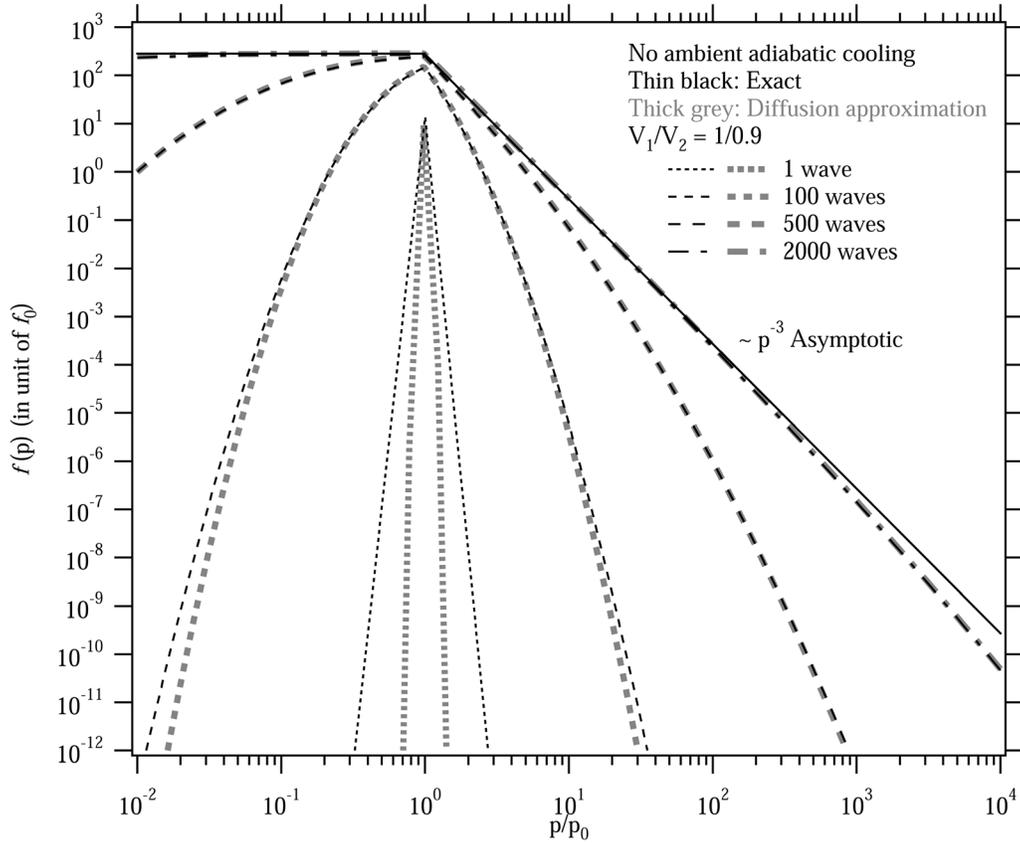


Figure 5. Comparison with particle distribution functions obtained by momentum diffusion approximation.

where $\delta\mathbf{V}$ is random fluctuation of plasma velocity, $(\nabla \cdot \delta\mathbf{V})^2$ is the ensemble square average of $\delta\mathbf{V}$, and τ_c is the correlation time of particle transport. Since it is complicated to evaluate D in Bykov and Toptygin's approach, which involves the calculation of Green's function for the characteristic motion including particle diffusion, we leave it as a parameter to be determined from our model.

[17] The steady state solution to equation (17), which corresponds to the particle distribution after a long time of acceleration process, can be easily obtained,

$$f(\tau = \infty) = \begin{cases} \frac{2Q_0}{-\gamma D_0 p_0 L} & : p \leq p_0 \\ \frac{2Q_0}{-\gamma D_0 p_0 L} \left(\frac{p}{p_0}\right)^\gamma & : p > p_0 \end{cases} \quad (19)$$

with

$$\gamma = -(3 + \nabla \cdot \bar{\mathbf{V}}/3D_0). \quad (20)$$

The large-scale divergence of solar wind velocity $\nabla \cdot \bar{\mathbf{V}} = 2V_{sw}/r$. If we let $(\nabla \cdot \delta\mathbf{V})^2 = (V_1 - V_2)^2/L^2$ and $\tau_c = 2L/(V_1 + V_2)$, or $D_0 = 2(V_1 - V_2)^2/9L(V_1 + V_2)$, we can exactly reproduce all the results in section of 3.1. This p dependence of the momentum diffusion D has not been derived with the method of quasi-linear perturbation [Bykov and Toptygin, 1982, 1993; Ptuskin, 1988].

[18] For the time-dependent solution of equation (17), we can perform a Fourier transform. The characteristic function of the momentum distribution can be written as

$$\tilde{f}(\tau, \omega) = \frac{Q_0}{\pi p_0 L} e^{-i\omega q_0} \frac{1}{D_0 \omega^2 - (\nabla \cdot \bar{\mathbf{V}}/3 + 3D_0) i\omega} \cdot \left\{ 1 - e^{-[D_0 \omega^2 - (\nabla \cdot \bar{\mathbf{V}}/3 + 3D_0) i\omega] \tau} \right\}, \quad (21)$$

where $\tau = N(L/2V_1 + L/2V_2)$ for our model. With $D_0 = 2(V_1 - V_2)^2/9L(V_1 + V_2)$ and $\nabla \cdot \bar{\mathbf{V}} = 2V_{sw}/r$, we can find that equations (21) and (8) are the same except that the exponential function $e^{(D_0 \omega^2 - 3D_0 i\omega)\tau}$ is in place of $(1 - i\omega/s_-)^N (1 - i\omega/s_+)^N$. Figure 5 shows a comparison between the exact solution to particle distribution function after acceleration by a wave train and the solution by a corresponding momentum diffusion approximation. The difference is particularly in the tail of particle distribution function when the particles have gone through few wave cycles. The diffusion approximation has a sharper cutoff partly because of Gaussian nature of probability function.

[19] The evolution of the pressure of accelerated particles can be calculated by multiplying equation (17) with $4\pi p^4/3m$ and integrating it over p from 0 to ∞ . Carrying out the integration by parts, we arrive at

$$\frac{dP_{pui}}{d\tau} = \left(10D_0 - \frac{5}{3}\nabla \cdot \bar{\mathbf{V}}\right) P_{pui} + \frac{8\pi Q_0 p_0^4}{3mL}. \quad (22)$$

The last term of (22) is the initial pressure increase rate from the injected pickup ions and it should be much smaller than that of accelerated particles once they have been sufficiently accelerated. If there is a mechanism that modifies that momentum diffusion coefficient D_0 through wave-particle interactions, the pressure from the accelerated particle will have a maximum value. At that time, it requires that $D_0 = \nabla \cdot \bar{V}/6$. When it is substituted back into equation (20), we get a slope of -5 for the particle distribution function on the $p > p_0$ side.

4. Conclusion and Discussion

[20] We have presented a calculation of particle acceleration by a compressional plasma wave train. The waves are small-scale waves whose wavelength is comparable to or smaller than particle diffusion length. Under this condition, the transport of particles behaves diffusive and particle acceleration by each of compression or expansion region in the waves is similar to shock acceleration. Particles get repeated acceleration or cooling by compression or rarefaction regions. The acceleration process exhibits the following properties:

[21] 1. Particle acceleration going through a train of compressional plasma wave will get either accelerated or cooled, resulting in a broader momentum distribution. The spectrum on both the acceleration side and cooling side typically obeys a power law. This spectral broadening is similar to diffusion in momentum space, but the tail distribution is heavier at momenta far away from initial injection momentum because of the power law rather than a Gaussian distribution.

[22] 2. Repeated acceleration or cooling inside compressional plasma waves will make the particle distribution get broader and broader very quickly even with a small amplitude of plasma compression ratio. A very heavy tail can develop and the pressure of accelerated particles can build up quickly.

[23] 3. After the particles go through enough number of wave cycles, their momentum distribution will approach to an asymptotic distribution: flat distribution on the cooling side ($p < p_0$) and a power law on the acceleration side ($p > p_0$). The slope of power law on the $p > p_0$ side is always -3 in the absence of large-scale adiabatic cooling by the solar wind. The p^{-3} distribution appear to be a universal distribution which is independent of any model parameters. If there is a significant large-scale adiabatic cooling, the power law distribution becomes steeper, its slope depending on the value of the adiabatic cooling rate as compared to the amplitude of plasma compression ratio.

[24] 4. Under the condition of weak large-scale adiabatic cooling, the pressure of accelerated particles will become divergent. We believe the pressure can eventually modify the amplitude and propagation speed of the compressional plasma wave. At some point when the pressure of accelerated particles is large enough compared to the initial solar wind plasma pressure, the amplitude of plasma wave has to be reduced enough to establish a balance between large-scale adiabatic cooling by the solar wind and particle acceleration by compressional plasma waves. The nonlinear interaction between particles and waves will reach an equilibrium state with a distribution of accelerated particles very close to p^{-5} .

[25] Although the compressional plasma waves and particle transport are prescribed in idealized forms so that an analytical solution to the particle transport equation becomes

possible, we can catch a few major features of this model that may be applicable to reality.

[26] 1. Compressional plasma waves are ubiquitous in the solar wind. When the solar wind emanates from solar corona, any fluctuation of solar wind speed will result in compressional plasma waves. Initial amplitude of the compressional wave may be large, but it will decay as it propagates out with the solar wind, particularly those high-frequency waves. At 1 AU, measurement of solar wind density and velocity can still see plenty of compressional waves in timescales ranging from minutes to days in the spacecraft reference frame [Marsch and Tu, 1990]. According to our calculation, these compressional waves should be able to accelerate particles.

[27] 2. Particle acceleration by compressional waves must compete with large-scale adiabatic cooling by the expanding solar wind. The cooling per wave cycle $cr = c_1 + c_2 = V_{sw} L r^{-1} (V_1^{-1} + V_2^{-1})/3$. So cr must be comparable or smaller than $|s_+ s_-|^{-1} = (V_1 - V_2)^2/9V_1V_2$ in order to have a particle distribution function that does not too steeply decrease with p . If we want a wave of compression ratio 1/0.9 to have a significant effect of particle acceleration, the cooling rate cannot greatly exceed 1.2×10^{-3} . That requires the $L(V_1^{-1} + V_2^{-1}) < 3.6 \times 10^{-3} r/V_{sw}$. $L(V_1^{-1} + V_2^{-1})$ is approximately equal to the wave period in the solar wind plasma frame. The cooling is too strong if very close to the sun, but the amplitude and frequency of the waves may become smaller at larger distances even though adiabatic cooling is slow there. So we expect that particle acceleration by compressional plasma wave to be most effective at radial distances anywhere from a fraction of AU to a few AU. At large radial distances after the waves die down, particles will be cooled adiabatically and the entire particle distribution shifts to lower and lower momentum, during which the spectral slope remains the same. Particle cooling in fast solar wind stream at high latitudes is stronger than in slow solar wind, so we will expect to see this type of acceleration more in low latitude slow solar wind.

[28] 3. Large-scale solar wind disturbances, such as corotating interaction regions or interplanetary shock waves, may generate compressional plasma waves at radial distances of several AU. At shock front, incompressional Alfvén wave can be converted to compressional magnetosonic wave, yielding fresh free energies to accelerate particles.

[29] 4. Particle injected for acceleration by this mechanism do not have to be pickup ions. In our model, we inject particles continuously into the waves from acceleration. This can happen to the solar wind ions too. For the majority of thermal solar wind ions, injection into the acceleration process is expected to be very slow. If there is a mechanism that accelerates thermal solar wind ions slowly, we can treat solar wind ions as being continuously injected. Thermal solar wind ions have low diffusion; to them each of the compression or rarefaction regions inside the wave may not be a shock-like structure. Our calculation in a separate paper with finite length of compression region relative to particle diffusion length shows that particle acceleration is slower. In order to investigate both injection and acceleration process a calculation with momentum-dependent diffusion coefficient may be needed to reveal their detailed behaviors. That is left for future development.

[30] 5. Our calculation is done for a steady state solution of particle transport equation. For a plasma wave of finite life-

time T_w , a time-dependent calculation is needed. Since the acceleration process of this model is driven by many small shock waves, we can use the diffusive shock acceleration theory to estimate the time of acceleration or energy cutoff for the steady state solution. Those particles with a diffusion coefficient $\kappa \ll V\Delta VT_w$ should be able to establish a steady distribution in this acceleration mechanism. The diffusion of suprathermal particles is probably small enough that their distribution can reach a steady state during their propagation with the solar wind through the heliosphere.

Appendix A: Transport Equation in Wave Frame

[31] If the distribution function is nearly isotropic, the equation governing the variation of particle distribution function $f(\mathbf{r}, p, t)$ obeys Parker's [1965] equation,

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \mathbf{r}} \cdot \left(\kappa \frac{\partial f}{\partial \mathbf{r}} \right) - \mathbf{v}_{sw} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{1}{3} \nabla \cdot \mathbf{v}_{sw} \frac{\partial f}{\partial \log p} + Q(\mathbf{r}), \quad (\text{A1})$$

where the spatial coordinate \mathbf{r} is in the solar reference frame and the particle momentum p is in the solar wind plasma frame where the convective electric field is zero and the assumption of isotropic particle distribution is good. Most probably in this reference frame, the wave prescribed will be propagating so the solution to the equation will be time dependent. If the wave propagates at a speed of \mathbf{V}_w relative to the ambient solar wind plasma that moves at an average speed of $\bar{\mathbf{V}}_{sw}$ relative to the sun, we can choose a new spatial coordinates that moves with the wave,

$$\mathbf{x} = \mathbf{r} - \int (\bar{\mathbf{V}}_{sw} + \mathbf{V}_w) dt. \quad (\text{A2})$$

In this wave reference frame, plasma speed is \mathbf{V} , so $\mathbf{V}_{sw} = \bar{\mathbf{V}}_{sw} + \mathbf{V}_w + \mathbf{V}$. Transformation of variables gives rise to a particle transport equation in the wave frame,

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \mathbf{x}} \cdot \left(\kappa \frac{\partial f}{\partial \mathbf{x}} \right) - \mathbf{V} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{1}{3} \nabla \cdot \mathbf{V}_{sw} \frac{\partial f}{\partial \log p} + Q(\mathbf{x}). \quad (\text{A3})$$

Note that p remains in the plasma reference frame. The divergence of plasma flow $\nabla \cdot \mathbf{V}_{sw}$ that controls the rate of particle acceleration or cooling comes from two parts: the divergence of ambient solar wind $\nabla \cdot \bar{\mathbf{V}}_{sw}$ and the variation of plasma speed in the wave frame $\nabla \cdot \mathbf{V}$. Typically the divergence of ambient solar wind is positive and slowly varying, resulting in a roughly constant large-scale cooling of particles. A one-dimensional form of (A3) shown in equation (1) is used for all the calculations in this paper.

Appendix B: Solution to Characteristic Function of Transport Equation

[32] Make a Fourier transform of particle transport equation (1), we get an equation for the characteristic function of particle distribution function,

$$\frac{\partial \tilde{f}}{\partial t} = 0 = \frac{\partial}{\partial x} \left(\kappa \frac{\partial \tilde{f}}{\partial x} \right) - V \frac{\partial \tilde{f}}{\partial x} + \frac{1}{3} \nabla \cdot \mathbf{V}_{sw} i\omega \tilde{f} + \tilde{Q}. \quad (\text{B1})$$

We seek a steady state solution of (B1).

[33] In an environment of shock, under either compression with $V_{up} > V_{dn}$ or rarefaction with $V_{up} < V_{dn}$,

$$\nabla \cdot \mathbf{V} = (V_{dn} - V_{up}) \delta(x), \quad (\text{B2})$$

where the shock is set at $x = 0$. If a mono-energetic particle population is injected $Q = Q_0 \delta(p - p_0) \delta(x)$,

$$\tilde{Q} = \frac{Q_0}{p_0} e^{-i\omega q_0 \delta(x)}. \quad (\text{B3})$$

A trial solution of equation (B1) for the upstream region of the shock

$$\tilde{f}(x < 0) = \tilde{f}_{up} + \tilde{f}_{tr} e^{\frac{V_{up} x}{\kappa}} \quad (\text{B4})$$

and downstream of shock

$$\tilde{f}(x > 0) = \tilde{f}_{dn}, \quad (\text{B5})$$

where $\tilde{f}_{up}, \tilde{f}_{tr}, \tilde{f}_{dn}$ are constant as a function of x . With the jump condition at the shock for f and $\partial f / \partial x$, i.e., as $\epsilon \rightarrow 0$,

$$\begin{aligned} \tilde{f}(x = -\epsilon) &= \tilde{f}(x = \epsilon), \\ 0 &= \kappa \frac{\partial \tilde{f}(x)}{\partial x} \Big|_{x=-\epsilon}^{x=\epsilon} + \frac{V_{dn} - V_{up}}{3} i\omega \tilde{f}(x = \epsilon) + \frac{Q_0}{p_0} e^{-i\omega q_0} \end{aligned} \quad (\text{B6})$$

we can obtain a relation between downstream and upstream solutions as

$$\tilde{f}_{dn} = \frac{1}{1 - i\omega/s} \tilde{f}_{up} + \frac{1}{1 - i\omega/s} \frac{Q_0}{p_0 V_{up}} e^{-i\omega q_0}, \quad (\text{B7})$$

where $s = 3V_{up} / (V_{dn} - V_{up})$ is the slope of power law spectrum produced by shock acceleration or cooling.

[34] In the region between adjacent compression and rarefaction regions, since the diffusion scale is short than the distance, particle transport is dominated by convection. We have

$$0 = -V \frac{\partial \tilde{f}}{\partial x} + \frac{1}{3} \nabla \cdot \mathbf{V}_{sw} i\omega \tilde{f}. \quad (\text{B8})$$

There is a roughly constant adiabatic decompression $\nabla \cdot \mathbf{V}_{sw} = 2V_{sw}/r$ in this region. Integrate it, we obtain a relation between downstream and upstream solutions,

$$\tilde{f}_{dn} = \tilde{f}_{up} e^{i\omega \frac{2V_{sw} x}{3V}}. \quad (\text{B9})$$

Equations (B7) and (B9) serve as recursion equation for the calculation of characteristic function of particle distribution throughout the wave sequence. Apply them to any one wave cycle, where the compression and rarefaction regions are separated by two convective-cooling regions, we obtain a recursion formula from one cycle to the next,

$$\begin{aligned} \tilde{f}_N &= \frac{1}{(1 - i\omega/s_-)(1 - i\omega/s_+)} e^{i\omega(c_1+c_2)} \tilde{f}_{N-1} \\ &+ \frac{1}{(1 - i\omega/s_-)(1 - i\omega/s_+)} e^{i\omega c_2} \frac{Q_0}{p_0 V_1} e^{-i\omega q_0} \\ &+ \frac{1}{(1 - i\omega/s_+)} \frac{Q_0}{p_0 V_2} e^{-i\omega q_0}, \end{aligned} \quad (\text{B10})$$

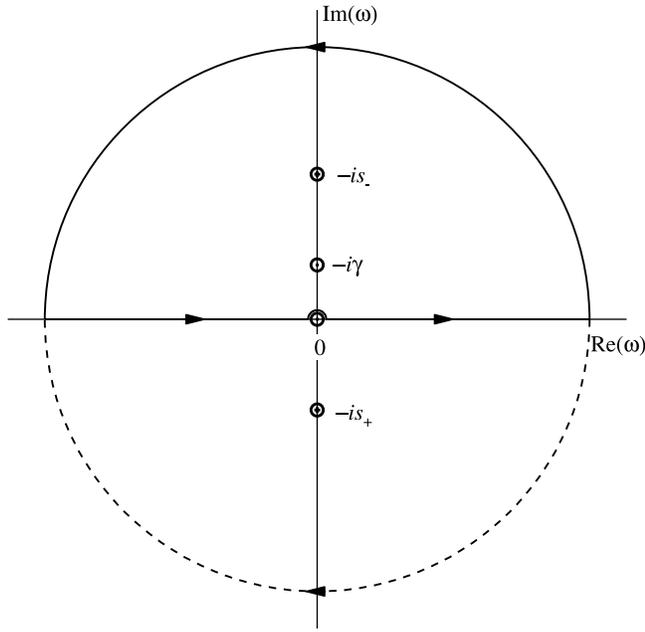


Figure C1. Illustration of contours for the integration of inverse Fourier transform.

where $s_+ = 3V_2/(V_1 - V_2)$, $s_- = -3V_1/(V_1 - V_2)$, $c_1 = V_{sw}L/(3rV_1)$ and $c_2 = V_{sw}L/(3rV_2)$. The last two terms of (B10) are the contribution of newly injected particles at the compression and rarefaction regions. Using this recursion equation (B10) repeatedly for all wave cycles, we can arrive at equation (7). The upstream value of the first wave cycle has been neglected, because the sum of injection particles over many wave cycles should be much larger than it.

Appendix C: Calculation of Inverse Fourier Transform

[35] Particle distribution function can be obtained by inverse Fourier transform of the characteristic function,

$$f(q) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(\omega) e^{i\omega q} d\omega. \quad (C1)$$

Rewrite the solution of characteristic function in (8) as

$$\tilde{f}_N(\omega) = \frac{Q_0}{p_0} e^{-i\omega q_0} \left[\frac{1}{V_1} e^{i\omega c_2} + \frac{(1 - i\omega/s_-)}{V_2} \right] \frac{1}{(1 - i\omega/s_-)(1 - i\omega/s_+) - e^{i\omega(c_1+c_2)}} \times \left\{ 1 - \left[\frac{1}{(1 - i\omega/s_-)(1 - i\omega/s_+)} e^{i\omega(c_1+c_2)} \right]^N \right\}. \quad (C2)$$

The integration of (C1) can be carried out by contour integral on the complex plane of ω . Figure C1 shows appropriate contours according to the sign of the exponential exponent in the integrand $\tilde{f}_N(\omega) e^{i\omega q}$. If the exponent is positive (approximately $q > q_0$), we choose the upper contour, so that the integration along the large arch is zero.

Then the $f(q)$ is equal to $2\pi i$ times the residue of the integrand in the upper half of the plane. There are two poles in the upper half plane: one at $\omega = -is_-$ and the other at $\omega = -i\gamma = -i[s_+ + s_- + s_+ s_- (c_1 + c_2)]$. The late pole is a simple first-order pole, its residue corresponding to the asymptotic solution of particle distribution on $p > \sim p_0$ side. The pole at $\omega = -is_-$ is a high-order pole and its residue gives the difference of particle distribution to the asymptotic distribution when there are finite number of wave cycles. Similarly, the distribution function on the $p < \sim p_0$ can be obtained using the lower contour and the residues in the lower half plane.

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