Infrared Electric Field Enhancement via a Hyperbolic-Metalens-Coupled Nanoantenna

by

Mohamed Jebril Kyamo

A dissertation
submitted to the College of Engineering and Science at
Florida Institute of Technology
in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy
in
Electrical Engineering

Melbourne, Florida

May, 2022
We the undersigned committee hereby approve the attached dissertation,

“Infrared Electric Field Enhancement via a Hyperbolic Metalens Coupled to a Nanoantenna,”

by

Mohamed Jebril Elmadani Kyamo

____________________  ____________________
Brian A. Lail, Ph.D.   Syed Murshid, Ph.D.
Professor            Professor
Computer Engineering and Sciences  Computer Engineering and Sciences
Major Advisor

____________________  ____________________
Ersoy Subasi, Ph.D.   Carlos E. Otero, Ph.D.
Assistant Professor  Associate Professor
College of Aeronautics  Computer Engineering and Sciences

___________________________
Philip Bernhard, Ph.D.
Associate Professor and Department Head
Computer Engineering and Sciences
Abstract

Title: Infrared Electric Field Enhancement via a Hyperbolic Metalens Coupled to a Nanoantenna

Author: Mohamed Jebril Elmadani Kyamo

Major Advisor: Brian A. Lail, Ph.D.

The development of near- and mid-infrared applications requires high enhancement of the electric field intensity (EFI) and a high absorption cross-section of the electric field. The high enhancement of EFI can be obtained by concentrating optical energy to areas much smaller than the diffraction limit or by using surface plasmon polaritons (SPPs). Noble metals such as gold (Au) or silver (Ag) are not feasible for infrared (IR) applications because of high losses, lack of tunability, low resolution, and low-intensity enhancement. Our methods to solve these problems are based on appropriate metamaterial selection and on coupling systems with optimization of their geometries. The high performance was achieved by introducing alternative metamaterials (AMM), a Fresnel zone plate (FZP), and nanoantennas.

This dissertation demonstrates a novel metalens design in the near-infrared band from 1.5 μm to 3 μm, consisting of an FZP, which is called a plasmonic waveguide coupler (PWC), situated on a slab of type I hyperbolic metamaterial (HMM) that lies
on a silicon substrate that has a silver nanodipole embedded within. The PWC is made of rings of indium tin oxide (ITO), and the type I HMM is constructed using a periodic stack of ITO and silicon layers that, through effective medium theory (EMT), act as a slab of type I HMM. Together the PWC and the HMM slab serve to focus incoming radiation onto a focal point marked by the location of the silver (Ag) nanodipole. The Ag nanodipole allows for high subwavelength confinement of optical modes because of the in-focal point component of the electromagnetic field vector coupled to the plasmonic resonance of the dipole.

This dissertation also demonstrates a novel metalens in the mid-infrared range from 6.4 µm to 10 µm, consisting of a doped indium arsenide (InAs) which works as a plasmonic waveguide coupler (PWC) on a slab of type I hyperbolic metamaterial (HMM) on the top of a substrate of undoped InAs with an Au nanoantenna at the other end. The metalens focuses both propagating and evanescent waves into a focal point at which the field confinement and enhancement are improved using the plasmonic nanoantenna. Finally, ultrahigh mid-infrared electric field enhancement with a high absorption cross-section and a focusing resolution are obtained via gap nanoantenna plasmonic resonance coupled with the novel hyperbolic metalens. By combining the focal spot of the metalens with the excited plasmons of a gap nanoantenna, the resulting structure exhibits an ultrahigh field-intensity enhancement with a high absorption cross-section and a focusing resolution.
Table of Contents

Abstract ........................................................................................................................................ iii
List of Figures .......................................................................................................................... viii
List of Tables ............................................................................................................................. xii
List of Symbols ......................................................................................................................... xiii
List of Abbreviations .............................................................................................................. xvi
Acknowledgment ...................................................................................................................... xix
Dedication ................................................................................................................................... xx

Chapter 1: Introduction .............................................................................................................. 1
  1.1 Background: Introduction to Infrared (IR) ................................................................. 1
  1.2 Motivation ....................................................................................................................... 3
  1.3 Objectives ....................................................................................................................... 4
  1.4 Techniques ...................................................................................................................... 5
    1.4.1 Finite Element Method (FEM) .............................................................................. 6
    1.4.2 Meshing ................................................................................................................... 6
    1.4.3 Asymptotic Solvers, Integral Equations (IE), and Advanced Hybrid Methods ... 7
  1.5 Dissertation Contribution ............................................................................................... 7
  1.6 Outline of the Dissertation ............................................................................................. 8

Chapter 2: Background of Electromagnetic and Optical Materials ........................................ 10
2.1 Maxwell’s and EM Wave Equations .......................................................... 10
  2.1.2 Electromagnetic (EM)Wave Modes ...................................................... 11
2.2 Introduction to Optical Materials ............................................................ 12
  2.2.1 Introduction to Permittivity and Permeability of Materials .................. 12
  2.2.2 Introduction to Plasmonic Materials .................................................. 16
2.3 Introduction to Metamaterials (MMs) ....................................................... 19
  2.3.1 Hyperbolic Metamaterials (HM) ......................................................... 22
  2.3.2 The Use of Effective Medium Theory (EMT) for Anisotropic Materials ... 23
  2.3.3 Hyperbolic Metasurfaces (HMS) ......................................................... 27
2.4 Infrared Plasmonic Materials ................................................................. 30
  2.4.1 Near-Infrared Plasmonic Materials ...................................................... 32
  2.4.2 Hyperbolic Metamaterials of ITO/Silicon .......................................... 34
  2.4.3 Mid-Infrared Plasmonic Materials ..................................................... 37
  2.4.4 Hyperbolic Metamaterials of Doped InAs/Undoped InAs ...................... 40
2.5 Plasmonic Fresnel Zone Plates ............................................................... 46
2.6 Plasmonic Nanoantenna ........................................................................... 50

Chapter 3: A Novel NIR Metalens Using Type I Hyperbolic Metamaterial ........ 53
3.1 Design and Optimization ........................................................................ 53
3.2 Results and Discussion .......................................................................... 56
  3.2.1 The Electric Field and Diffraction Limit for a Structure without a Nanoantenna
       ............................................................................................................. 56
  3.2.2 Electric Field and Diffraction Limit for a Structure with a Nanoantenna .... 57
  3.2.3 The Electric-Field Enhancement of the Structure ................................ 58
3.3 Summary .....................................................................................................................61

Chapter 4: Novel Mid-Infrared Metalens Using a Hyperbolic ........................................62

4.1. Design and Optimization .........................................................................................62

4.2 Results and Discussion ............................................................................................65

4.3 Summary .................................................................................................................67

Chapter 5: Ultrahigh MIR Electric Field Enhancement via Nanoantenna
Plasmonic Resonance Coupled to a Novel Hyperbolic Metalens ...................................68

5.1 Design and Optimization .........................................................................................68

5.2 Numerical Results and Discussion ............................................................................76

5.3 Summary .................................................................................................................85

Chapter 6: Conclusion ...................................................................................................87

References ......................................................................................................................89

Appendix .......................................................................................................................107
List of Figures

Figure 1. Electromagnetic Spectrum .................................................................2
Figure 2. Meshing Element in ANSYS HFSS and Meshing Structure ...............6
Figure 3. Permittivity (ε) and permeability (µ) diagram ....................................13
Figure 4. Dispersion relation of surface plasmon polaritons (SPPs) [53] ..........17
Figure 5. SPPs waves at the interface between a metal and a dielectric with surface
charges and the penetration depths inside metal and dielectric [57] [58]. ..........18
Figure 6. Metamaterials map with their parameters, carrier concentration carrier
mobility, and interband losses for plasmonic [32] .............................................21
Figure 7. (a) Multilayer HMM structure consisting of alternating metallic and
dielectric layers. (b) Nanowire HMM structure consisting of metallic nanorods
embedded in a dielectric host ...........................................................................23
Figure 8. (a) An isotropic dielectric. (b) A uniaxial medium Type I HMM. (c) A
uniaxial medium Type II HMM for TM mode [74] .............................................26
Figure 9. Design MIR metalens operating at 4 µm wavelength. (a) Configuration of
MIR metalens and, (b) Structure of the NP unit cell with Radius (R), height (h), and
period (a) [80]. .........................................................................................28
Figure 10. (a) Metasurface lens (b) Schematic of incident Gaussian rays at an angle
θ on the metasurface lens and reflected waves to the focal point .....................29
Figure 11. Drude model of real and imaginary permittivity for GZO, ITO, and
AZO. .............................................................................................................33
Figure 12. The layer structure of the periodic arrangement of dielectric and the
metal layers. The effective medium has a different direction parallel and
perpendicular to the layers .............................................................................34
Figure 13. The HMM dispersion of ITO/silicon with various filling fractions of the
ITO over NIR wavelengths showing (a) the effective real parallel permittivity, (b)
the effective imaginary parallel permittivity, (c) the effective real perpendicular permittivity, and (d) the effective imaginary perpendicular permittivity of the structure.

Figure 14. Optical phase diagrams for the effective medium theory prediction of effective behavior, Type I, Type II, metal, and dielectric based on the fill fractions of the ITO.

Figure 15(a) The Drude model of real and (b) imaginary parts of the permittivity of InAs with different carrier concentrations over mid-IR wavelength.

Figure 16. Stacked layers of doped InAs/undoped InAs and the homogeneous medium with effective permittivities.

Figure 17. The HMM dispersion of doped InAs/undoped InAs with various filling fractions of the doped InAs over NIR wavelengths, (a) the effective real parallel permittivity, (b) the effective imaginary parallel permittivity, (c) the effective real perpendicular permittivity, and (d) the effective imaginary perpendicular permittivity of the structure.

Figure 18. Effective medium theory (EMT) shows that a doped InAs/InAs multilayer system can act as an effective dielectric, effective metal, Type I HMM, Type II HMM, metal, and dielectric basing on fill fraction of the metal in the mid-IR region.

Figure 19. Optical properties of the noble metals Au, Ag, and Al, described by the Drude model, (a) real (solid curve) and (b) imaginary (dashed curve) permittivity.

Figure 20. Diagrams of PFZPs lenses.

Figure 21. A schematic diagram of the radial Fresnel zone plates diffracts and incident wave rays at a focal distance f.

Figure 22. (a) The electric field distribution |E| and (b) the magnetic field distribution of the focusing PFZP lens.
Figure 23. Some different kinds of plasmonic nanoantennas [137] [138]...........51
Figure 24. Metalens structure.................................................................53
Figure 25. (a) The real and imaginary part of permittivity of ITO and (b) Effective
medium analysis of an ITO/Si.................................................................56
Figure 26. Magnitude of electric field distribution at the focal point without a
nanoantenna.........................................................................................57
Figure 27. The electric field at the focal point of the structure: (a) is the magnitude
of electric field distribution and (b) is the field intensity profile of the x-axis and y-
axis. .............................................................................................................58
Figure 28. Electric field enhancement with different thicknesses. .......................59
Figure 29. The electric field enhancement of the focal point with different lengths
(Ls) of the nanoantenna.........................................................................60
Figure 30. (a) The electric field enhancement as a function of frequency with fixed
radius (Rs=5 nm) and (b) shows the electric field enhancement with different radii
of the nanoantenna................................................................................61
Figure 31. Metalens structure.....................................................................63
Figure 32. Effective permittivity as a function of wavelength.........................64
Figure 33. The magnitude of electric field distribution at the focal point: (a) without
a nanodisk and (b) with a nanodisk..........................................................65
Figure 34. The electric field enhancement of the focal point with different values
for separation between the disks. ............................................................66
Figure 35. Field intensity profile of the x-axis and y-axis.................................67
Figure 36. (a) Three-dimensional representation of the structure, (b) XZ view, (c)
XY top view of the PFZP’s, (d) XY bottom view of the structure, and (e) a dipole
nanoantenna and stub-loaded gold dipole nanoantenna (SLD nanoantenna). An Au
nanoantenna permittivity (εAu) at (λ = 7.8 μm) is εAu = −2806.57 + j720. .....69
Figure 37. The real and imaginary parts of the relative permittivity of undoped InAs (green line) and doped InAs (red line) are plotted as a function of the wavelength from 6 to 15 µm. The marked black dot indicates the start of the metallic behavior of the doped InAs.

Figure 38. The effective permittivities (real and imaginary parts) for HMMs (undoped InAs/doped InAs) with normal and parallel direction.

Figure 39. Comparison of EFI enhancement for different configurations: i- PFZP/HMM/spacer/SLD ANT. (red curve), ii- PFZP/Hom/spacer/SLD ANT (cyan curve), iii- PFZP/air/spacer/SLD ANT. (magenta curve), iv- PFZP/HMM/spacer/Dip ANT. (green curve), and v- PFZP/HMM/spacer (dashed blue curve).

Figure 40. (a) |E|−field distributions of the metalens without a nanoantenna at the wavelength of 7.8 µm, Case (v) in Figure 37, (b) |E|−field distributions of the hybrid structure at the wavelength of 7.89 µm for a dipole nanoantenna with a dipole-gap of 10 nm, Case (iv) in Figure 40, and (c) |E|−field distributions of the hybrid structure at the wavelength of 7.8 µm for an SLD nanoantenna with different gaps of 8.3, 10, and 11.4 nm, Case (i) in Figure 39.

Figure 41. (a) Absorption cross-section (ACS) and scattering cross-section (SCS) (dashed lines) of the hybrid metalens and SLD gold nanoantenna system with PFZPs rings heights, h = 0.07 µm (blue), 0.42 µm (green), and 2.93 µm (red). (b). The ratio of the scattering and absorption (c). EFI of the hybrid system with the PFZPs ring heights.
List of Tables

Table 1. Calculation of Radii of the PFZPs .......................................................... 75
Table 2. State-of-the-Art NIR Intensity Enhancement with Different Systems in MIR. ................................................................................................................... 81
# List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$ [rad/m]</td>
<td>angular frequency</td>
</tr>
<tr>
<td>$\vec{k}$</td>
<td>complex propagation constant</td>
</tr>
<tr>
<td>$\varepsilon_d$</td>
<td>dielectric constant of the dielectric</td>
</tr>
<tr>
<td>$\varepsilon_m$</td>
<td>dielectric constant of the metal</td>
</tr>
<tr>
<td>$J$ [C/m$^3$]</td>
<td>electric current density</td>
</tr>
<tr>
<td>$\varepsilon_{eff}$</td>
<td>effective permittivity</td>
</tr>
<tr>
<td>$n_{eff}$</td>
<td>effective refractive index</td>
</tr>
<tr>
<td>$n_{sp}$</td>
<td>effective refractive index of the spacer</td>
</tr>
<tr>
<td>$\lambda_{eff}$</td>
<td>effective wavelength</td>
</tr>
<tr>
<td>$\vec{E}$ [V/m]</td>
<td>electric field intensity</td>
</tr>
<tr>
<td>$\vec{D}$ [C/m$^2$]</td>
<td>electric displacement</td>
</tr>
<tr>
<td>$\vec{B}$ [wb/m$^2$]</td>
<td>magnetic induction.</td>
</tr>
<tr>
<td>$P$</td>
<td>filling fraction of the metal</td>
</tr>
<tr>
<td>$\varepsilon''$</td>
<td>imaginary part of relative permittivity</td>
</tr>
<tr>
<td>$\vec{H}$ [A/m]</td>
<td>magnetic field intensity</td>
</tr>
<tr>
<td>[A/m$^2$]</td>
<td>magnetic flux density</td>
</tr>
<tr>
<td>$\varepsilon_{//}$</td>
<td>parallel permittivity</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\delta_m$</td>
<td>penetration depth into a dielectric</td>
</tr>
<tr>
<td>$\delta_d$</td>
<td>penetration depth into metal</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>permeability</td>
</tr>
<tr>
<td>$\mu_0$ [H/m]</td>
<td>permeability of a vacuum</td>
</tr>
<tr>
<td>$\varepsilon_\infty$</td>
<td>permittivity at an infinite frequency</td>
</tr>
<tr>
<td>$\varepsilon_0$ [F/m]</td>
<td>permittivity of a vacuum</td>
</tr>
<tr>
<td>$\varepsilon_\perp$</td>
<td>perpendicular permittivity</td>
</tr>
<tr>
<td>$\phi(r)$</td>
<td>phase profile at the position</td>
</tr>
<tr>
<td>$\omega_p$ [rad s$^{-1}$]</td>
<td>plasma frequency</td>
</tr>
<tr>
<td>$\omega_{sp}$</td>
<td>plasmon resonant frequency</td>
</tr>
<tr>
<td>$L_{spp}$</td>
<td>propagation length of the SPPs</td>
</tr>
<tr>
<td>$q$</td>
<td>phase correction step number</td>
</tr>
<tr>
<td>$r_n$</td>
<td>radius of the nth zone</td>
</tr>
<tr>
<td>$\varepsilon'$</td>
<td>real part of relative permittivity</td>
</tr>
<tr>
<td>$n$</td>
<td>refractive index</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>relative permittivity</td>
</tr>
<tr>
<td>$\Gamma$ [rad s$^{-1}$]</td>
<td>relaxation constant</td>
</tr>
<tr>
<td>$w_0$</td>
<td>spot-width</td>
</tr>
<tr>
<td>$t_d$</td>
<td>thickness of the dielectric</td>
</tr>
<tr>
<td>$t_m$</td>
<td>thickness of the metal</td>
</tr>
</tbody>
</table>
$\rho [c/m^3]$ volume charge density

$\lambda [m]$ wavelength

$k_{spp} [rad\cdot m^{-1}]$ wavevector of SPP
### List of Abbreviations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACS</td>
<td>absorption cross-section</td>
</tr>
<tr>
<td>AMM</td>
<td>alternative metamaterial(s)</td>
</tr>
<tr>
<td>AlZnO</td>
<td>aluminum doped zinc oxide</td>
</tr>
<tr>
<td>AlInAs</td>
<td>aluminum indium arsenide</td>
</tr>
<tr>
<td>AZO</td>
<td>aluminum zinc oxide</td>
</tr>
<tr>
<td>EMT</td>
<td>effective medium theory</td>
</tr>
<tr>
<td>EM</td>
<td>electromagnetic wave</td>
</tr>
<tr>
<td>FEM</td>
<td>finite element method</td>
</tr>
<tr>
<td>FWHM</td>
<td>full width half maximum</td>
</tr>
<tr>
<td>GZO</td>
<td>gallium doped zinc oxide</td>
</tr>
<tr>
<td>Ga: ZnO</td>
<td>Gallium doped ZnO</td>
</tr>
<tr>
<td>GaN</td>
<td>gallium nitride</td>
</tr>
<tr>
<td>GaP</td>
<td>gallium phosphide</td>
</tr>
<tr>
<td>GSP</td>
<td>gap-surface plasmon</td>
</tr>
<tr>
<td>HFSS</td>
<td>high-Frequency Structural Simulator</td>
</tr>
<tr>
<td>HMM</td>
<td>hyperbolic metamaterials</td>
</tr>
<tr>
<td>HMSs</td>
<td>hyperbolic metasurfaces</td>
</tr>
<tr>
<td>InAs</td>
<td>indium arsenide</td>
</tr>
<tr>
<td>InGaAs</td>
<td>indium gallium arsenide</td>
</tr>
<tr>
<td>Acronym</td>
<td>Full Form</td>
</tr>
<tr>
<td>---------</td>
<td>-----------</td>
</tr>
<tr>
<td>InP</td>
<td>indium phosphide</td>
</tr>
<tr>
<td>ITO</td>
<td>indium tin oxide</td>
</tr>
<tr>
<td>IE</td>
<td>integral equations</td>
</tr>
<tr>
<td>LSP</td>
<td>localized surface plasmon</td>
</tr>
<tr>
<td>LIR</td>
<td>long-wavelength infrared</td>
</tr>
<tr>
<td>MgF2</td>
<td>magnesium fluoride</td>
</tr>
<tr>
<td>MM</td>
<td>metamaterial(s)</td>
</tr>
<tr>
<td>MoM</td>
<td>method of moments</td>
</tr>
<tr>
<td>MEMS</td>
<td>micro-electro-mechanical system</td>
</tr>
<tr>
<td>MIR</td>
<td>mid-wavelength infrared</td>
</tr>
<tr>
<td>NIR</td>
<td>near-infrared</td>
</tr>
<tr>
<td>PO</td>
<td>physics optics</td>
</tr>
<tr>
<td>PFZPs</td>
<td>plasmonic Fresnel zone plate</td>
</tr>
<tr>
<td>PWC</td>
<td>plasmonic waveguide</td>
</tr>
<tr>
<td>SCS</td>
<td>scattering cross-section</td>
</tr>
<tr>
<td>SWIR</td>
<td>short-wavelength infrared</td>
</tr>
<tr>
<td>SiC</td>
<td>silicon carbide</td>
</tr>
<tr>
<td>SPP</td>
<td>surface plasmonic polariton</td>
</tr>
<tr>
<td>SRR</td>
<td>split-ring resonators</td>
</tr>
<tr>
<td>SrTiO3</td>
<td>strontium titanium oxide</td>
</tr>
<tr>
<td>SLD</td>
<td>stub-loaded gold dipole</td>
</tr>
</tbody>
</table>
SPR | surface plasmonic resonances
SPs | surface plasmons
TiN | titanium nitride
TCOs | transparent conducting oxides
TE | transverse electric
TEM | transverse electromagnetic mode
TM | transverse magnetic
UV | ultraviolet
ZnO | zinc oxide
ZrN | zirconium nitride
Acknowledgment

First and foremost, I acknowledge Alhamdulillah, for giving me the strength, knowledge, ability, and opportunity to undertake this research study and to complete the dissertation.

I acknowledge my supervisor Professor. Brian A. Lail for his guidance, support, encouragement, and patience in the last six years of my Ph.D.

I would like to acknowledge my other Ph.D. committee members, Professor Murshid, Syed H, Assistant Professor Ersoy Subasi, Assistant Professor Carlos E. Otero, for their time and effort in my research and dissertation.

I am also thankful to all my lab partners in the Applied Computational Electromagnetic Laboratory, particularly Michael F. Finch, Navaneeth Premkumar, Shenjie Miao, Yuchen Yang, and Abdelgader A. A. Alsalhin for their successful collaboration, constructive proposals, worthwhile discussions, and friendship.

I would also like to thank all the kind staff in the Electrical Engineering Department for their support in every aspect.
Dedication

I dedicate this dissertation to my supervisor, who was the guiding light every step during my research, and to all my teachers, who inspired me to learn. I dedicate my work to my parents, who gave me moral lessons on discipline, and to my sisters, brothers, and friends who are waiting for the moment I earn a doctorate. I also dedicate this dissertation to my wife and my children for their love and support throughout the entire doctorate program.
Chapter 1: Introduction

1.1 Background: Introduction to Infrared (IR)

Infrared (IR) light is the part of electromagnetic radiation (EM) with frequencies lower than visible light. The infrared ray region is between the red radiation in the visible rays and the short wave (microwaves) spectrum of the electromagnetic field. The EM spectrum is shown in Figure 1.

The human eye cannot see infrared light with 3 to 30 microns wavelength but can feel the heat because of the molecules' vibration when the molecules absorb the IR energy. Blackbody radiation from objects close to room temperature is practically all at infrared frequencies [1] [2]. Its peak blackbody [3] emission can be detected for most mechanical and biological investigations [4]. Thermal imaging devices assist in visualizing in the dark by detecting IR radiation [5]. For instance, transparent materials such as glass and water absorb infrared radiation. In 1800, Herschel [6], discovered that IR could be absorbed or emitted by rotational-vibrational molecules. When the particles of material absorb infrared radiation, excitation of the material atoms occurs. The absorbed energy displays as vibration, which leads to a periodic change in the chemical bonds and angles of the molecules. The wavelength at which the absorption occurs depends on relative mass and bond strength [7]
Infrared radiation can be classified according to the wavelength: near-infrared (NIR), short-wavelength infrared (SWIR), mid-wavelength infrared (MIR), and long-wavelength infrared (LIR). NIR starts from 0.75 to 1.4 µm. SWIR, which starts from 1 to 3 µm, is used for biomolecules, water absorption at 1.45 µm [8], and telecommunication at 1.3 to 1.56 µm [9]. MIR starts from 3 to 15µm [10], using wavelengths 3-5 µm for guided rocket technology [11] and 8-15 µm thermal imaging such as the human body, sun, and moon [12]. Far infrared (FIR) starts from 15 to 1000 µm. IR has many applications such as in night vision devices [14], thermography (infrared camera), spectroscopy (to identify molecules and absorption)
[15], hyperspectral imaging (spectroscopy with NIR, SWIR, MIR, and LIR spectral regions) [16], tracking (infrared homing) [17], communications Infrared Data Association (IrDA devices, emitting diodes, lens detector, and optical fiber) [18], and metrology (infrared images for weather) [19]. Different techniques were used to design such IR devices, yet many limitations affect the performance of the applications. However, the introduction of nanostructures and plasmonic materials can open the door to scientific techniques that would give more permission to control IR waves on nanoscales.

1.2 Motivation

Lenses are the most important and extensively used elements in optical design. They bend light to focus light beams to a point in space using refraction as defined by Snell’s law [20] [21]. The resolution of conventional lenses is limited by the diffraction limit, which is about a half wavelength \( \sim \lambda/(2n) \), where \( \lambda \) is the working wavelength and \( n \) is the refractive index, so by such optical elements, there is no way to go beyond the diffraction limit. However, many techniques are available to overcome the limitation of diffraction and realize a higher resolution. One is having negative permittivity and permeability to propagate waves toward a path against that of the flow of energy.

Lately, in [21], a flat lens was designed using a slab of material with a negative index
of refraction to create a metalens for visible light. Other essential techniques utilize a plasmonic Fresnel zone plate (PFZP) for the sub-diffraction limit and optical focusing [22] as well as hyperbolic metamaterials for higher wavevectors [23][24][25][26][25][27].

Noble metals, especially gold and silver nanostructures that can exhibit surface plasmonic resonances (SPR), are used in the visible spectrum [28]. However, controlling SPR properties and concentrating optical energy to areas much smaller than the diffraction limit with high field enhancement in NIR and MIR using noble metals is not feasible because of high losses, lack of tunability, low resolution, and low-intensity enhancement [29][30][31][32]. Selecting appropriate metamaterial coupling systems such as a hyperbolic metalens and a nanoantenna and optimizing their geometries can be useful to overcome these obstacles and to exhibit high performance. Moreover, recent advances in nanotechnology and IR metamaterials allow the building of new nanostructures with excellent resolution and high field intensity. They open new paths for working around the diffraction limit with very well-tunable properties.

1.3 Objectives

The objectives of this research include the following:
• Control and scale down the wavelength of IR radiation to realize the effective shorter wavelength in the media with smaller optical losses.

• Investigate appropriate plasmonic materials with a negative refractive index for NIR and MIR with low losses.

• Control the IR electromagnetic wave by selecting proper materials and using effective medium theory of metamaterials.

• Design a novel hyperbolic metalens that confines the NIR beyond the diffraction limit with high electric field enhancement.

• Design a novel hyperbolic metalens that confines the mid-IR beyond the diffraction limit with high electric field enhancement.

• Design a structure that is coupled between the electric field enhancement of a hyperbolic metalens and a resonant MIR optical nanoantenna to create an ultrahigh near-field enhancement, achieving critical coupling for some applications such as sensitive detection applications, biosensing, solar cell structures, and photonic devices,

1.4 Techniques

ANSYS High-Frequency Structural Simulator (HFSS) software is a 3-D industry standard for computational full-wave electromagnetic field analysis. It exhibits high performance with accuracy and efficiency. HFSS utilizes algorithms based on the
finite element method (FEM), integral equations (IE), asymptotic solvers, and advanced hybrid methods [33].

1.4.1 Finite Element Method (FEM)

HFSS utilizes the FEM model by applying an adaptive mesh and compliant solver. Users define the compliant mesh, mesh refinement, and convergence criteria in a frequency domain.

The FEM can be treated as a model for differential and integral Maxwell's equations.

![Meshing Element in ANSYS HFSS and Meshing Structure](image)

Figure 2. Meshing Element in ANSYS HFSS and Meshing Structure

1.4.2 Meshing

Meshing is the process by which the 3D geometries are subdivided into tetrahedral elements as shown in Figure 2. The meshing in the ANSYS is based on the following:
• initial mesh process (automatically constructed without a user)
• adaptive mesh process takes numerous factors (geometry factors, field solution factors, and the percent refinement number)
• mesh considerations and impact on solutions (The mesh is used to determine the numerical field solution and produce all secondary results, such as field calculations.)

1.4.3 Asymptotic Solvers, Integral Equations (IE), and Advanced Hybrid Methods

The methods are based on using the shooting, bouncing ray (SBR+) method and physics optics (PO) analyzes radiation patterns. These methods can operate in large environments by a combination simulation approach with far-field or near-field sources. IE use the 3D Method of Moments (MoM) to solve scattering issues and unclosed radiation. Advanced hybrid methods: hybrid solvers are matrixes that consist of the finite element method (FEM) with the MoM and PO [34]

1.5 Dissertation Contribution

A novel metalens, was designed in the NIR band from 1.5μm to 3μm. The metalens is made with attentive plasmonic materials that overcome the limitations of diffraction and optical losses, achieving high sub-diffraction focusing with a high electric field intensity (EFI) enhancement beyond the diffraction limits.
A novel metalens in the MIR range from 6.4μm to 10μm is designed. A Type I hyperbolic metamaterial (HMM) and a plasmonic waveguide coupler (PWC) are used to create this metalens. The PWC and HMM, made of alternative plasmonic materials in MIR to overcome optical losses, are controlled MIR waves by exciting SPP and focusing both propagating and evanescent waves into a focal point. The electric field was confined beyond the diffraction limit, and the EFI enhancement is improved.

Coupling between a novel hyperbolic metalens and nanoantenna plasmonic resonance is obtained by combining the metalens' focal spot with the excited plasmons of a nanoantenna. The proposed structure exhibits ultrahigh EFI enhancement with a high focusing resolution. The design provides the possibility of controlling the system scattering cross-sections (SCS) or absorption cross-sections (ACS), obtaining critical coupling, under coupling, or over coupling. These features can efficiently fit practical applications such as guiding MIR waves, imaging, and sensing.

1.6 Outline of the Dissertation

Chapter 1 views the background and introduction to IR waves and then outlines the main points of the motivation of the work, objectives, techniques, dissertation contribution, and outline of the dissertation.
Chapter 2 introduces the background of electromagnetic waves and optical materials, describes electromagnetic waves in free space, providing a view of Maxwell’s equations, and then introduces optical materials, starting with a brief description of permittivity and permeability of materials classified as metals, dielectrics, and semiconductors. The chapter also introduces metamaterials, providing hyperbolic metamaterials, effective medium theory (EMT), and hyperbolic metasurfaces (HMS), and gives the infrared plasmonic material, indicating materials for NIR and MIR. The chapter also introduces plasmonic Fresnel zone plates. The last section focuses on the plasmonic nanoantenna.

Chapter 3 presents a novel NIR metalens using type I hyperbolic metamaterial and focuses on the plasmonic waveguide coupler (PWC), the negative behavior of type I hyperbolic metamaterial in the propagation direction, and localized surface plasmon resonances.

Chapter 4 demonstrates another novel metalens using a hyperbolic metamaterial in MIR.

Chapter 5 extends our study on the novel hyperbolic metalens, and we show coupling between the metalens and plasmonic nanoantenna and show how our work compares to other works.

Chapter 6 gives conclusions of this dissertation and discusses future work. Last is the Appendix.
Chapter 2:
Background of Electromagnetic and Optical Materials

2.1 Maxwell’s and EM Wave Equations

Maxwell’s time-domain equations of macroscopic electromagnetism are the following:

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (2.1), \]
\[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (2.2), \]
\[ \nabla \cdot \vec{D} = \rho \quad (2.3), \]
\[ \nabla \cdot \vec{B} = 0 \quad (2.4), \]
\[ D = \varepsilon \varepsilon_0 \vec{E} \quad (2.5), \]
and \[ \vec{B} = \mu \mu_0 \vec{H} \quad (2.6), \]

where \( \vec{E} \) is the electric field density (V/m), \( \vec{H} \) is the magnetic field density (A/m), \( \vec{B} \) is the magnetic flux density (Wb/m\(^2\)), \( \vec{D} \) is the electric flux density (Wb/m\(^2\)), \( \varepsilon \) is relative permittivity, \( \varepsilon_0 = 8.854 \times 10^{-12} \) (F/m) is the permittivity of a vacuum, \( \mu \) is the relative permeability, \( \mu_0 = 4\pi \times 10^{-7} (H/m) \) is the permeability of a vacuum.
vacuum, $\rho$ is the external charge density (C/m$^3$), and $J$ is current densities (A/m$^3$) [35].

EM Wave Equations are the following:

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \tag{2.7}$$

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \tag{2.8}$$

The waves in 3 D are $E(\vec{r}, t) = \vec{E}_0 e^{j(\omega t - \vec{k} \cdot \vec{r})}$, where $E(\vec{r}, t)$ is the complex field, $\vec{E}_0$ is the amplitude, and $\vec{k}$ is the wave vector.

2.1.2 Electromagnetic (EM)Wave Modes

In transverse electromagnetic mode (TEM) for nonmagnetic dielectric materials, assuming wave propagation in the $z$ direction, both $E_z$ and $H_z$ are zeros, and the electric field and magnetic field are transverse waves. Maxwell’s equations can split TEM into two independent transverse magnetic (TM) modes and transverse electric (TE) modes. Their equations can be found by applying the boundary conditions [35].

In transverse magnetic (TM$^z$) modes, an electric field is only along the direction of propagation in the $z$-direction [35]:

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = j\omega \mu_0 H_y \tag{2.9}.$$
\[
\frac{\partial H_y}{\partial z} = j\omega \varepsilon_0 E_x \quad \text{(2.10)},
\]
and
\[
\frac{\partial H_y}{\partial x} = -j\omega \varepsilon_0 E_z \quad \text{(2.11)}.
\]

In transverse electric (TE\(_Z\)) modes, a magnetic field is only along the direction of propagation [35]

\[
\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = -j\omega \varepsilon_0 E_y \quad \text{(2.12)},
\]
\[
\frac{\partial E_y}{\partial z} = -j\omega \mu_0 H_x \quad \text{(2.13)},
\]
and
\[
\frac{\partial E_y}{\partial x} = j\omega \mu_0 H_z \quad \text{(2.14)}.
\]

### 2.2 Introduction to Optical Materials

#### 2.2.1 Introduction to Permittivity and Permeability of Materials

Depending on the electromagnetic properties, materials can be classified according to permittivity and permeability, including right-handed isotropic dielectrics (\(\varepsilon > 0\) and \(\mu > 0\)) with forward-wave propagation, metals (\(\varepsilon < 0\) and \(\mu > 0\)) at optical frequencies with a plasmonic wave (\(\omega < \omega_p\)) and an evanescent wave, ferrimagnetic materials (\(\varepsilon > 0\) and \(\mu < 0\)) with an evanescent wave, and Veselago’s materials or left-handed with backward-wave propagation [36].
In the 1960s, Veselago was first showed negative-index materials, showing transmitting of light through the metamaterials (MM). He proved that the phase velocity could be opposite to the direction of the Poynting vector [37], which is contrary to the natural property’s propagation of waves. Veselago’s materials, artificial materials with negative permittivity and permeability, are designed by periodic elements, and the length of each element is much smaller than the operating wavelength [38]. Ferrimagnetic materials are magnetic materials with negative
permeability and positive permittivity. The split-ring resonators (SRR) structure shows an effective negative permeability [39].

a) Optical Properties of Metals

Optical properties of metals such as magnesium, aluminum, gold, nickel, and silver are indicated as free-electron metals. The metals occupy valence bands and part of conduction bands, and their optical response is described by the dielectric function epsilon $\varepsilon(\omega)$ with the unity of magnetic permeability ($\mu=1$) [40]. The special noble metals (gold and silver) show interband transitions from or into the conduction band [41]. The optical properties of free-electron metals described by the Drude-Lorentz model [42] are as indicated:

$$\varepsilon(\omega) = \varepsilon_{\infty} - \frac{\omega_p^2}{\omega^2 - i\Gamma}\omega = \varepsilon'(\omega) + j\varepsilon''(\omega),$$

where $\omega_p$ is the Drude plasma frequency, $\Gamma$ is the relaxation constant, and $\varepsilon_{\infty}$ is the permittivity at infinite frequency. The real part $\varepsilon'(\omega)$ of permittivity describes the polarization, and the imaginary part $\varepsilon''(\omega)$ of permittivity indicates the energy dissipation [43]. If $\omega < \frac{\omega_p}{\sqrt{\varepsilon_{\infty}}}$, the real part $\varepsilon'(\omega)$ is negative and the field is totally reflected, and if $\omega > \frac{\omega_p}{\sqrt{\varepsilon_{\infty}}}$, the metal becomes transparent. The Fermi energy lies within a band.

b) Optical Properties of a Dielectric
The complex dielectric function $\varepsilon(\omega)$ is related to the energy band of the materials [44]. From optical fields, $E(\vec{r}, t) = E_0 e^{i(\omega t - \vec{k} \cdot \vec{r})}$, where $\vec{k}$ is a complex propagation constant, $\vec{r}$ is $x\hat{i} + y\hat{j} + z\hat{k}$, and $\omega$ is the angle frequency. If there are no losses in a material, the dispersion relation is $k_0 = \frac{\omega}{c} \sqrt{\varepsilon \mu}$ and if there are losses the dispersion relation is $k_0 = \frac{\omega}{c} \sqrt{\varepsilon(\omega)\mu}$, where $c$ is the speed of light in a vacuum and $\varepsilon(\omega) = \varepsilon'(\omega) + j\varepsilon''(\omega)$. The complex index of the refraction of the dielectric is $\bar{n}(\omega) = \frac{\varepsilon(\omega)}{\lambda(\omega)} = n - ik$. For isotropic materials, the real part and the imaginary part of the permittivity are $\varepsilon' = n^2 - k^2$ and $\varepsilon'' = 2nk$ [45], respectively. The Fermi energy of the dielectric lies in a large bandgap.

c) Optical properties of semiconductors

The optical properties of semiconductors lie between metals and insulating dielectrics. Their Fermi energy (collection of electron energy) also lies in a small bandgap less than 1 eV [46]. The optical properties of semiconductors can be understood by the Drude theory, as shown in the following equation:

$$\varepsilon(\omega) = \varepsilon_\infty \left(1 - \frac{\omega_p^2}{\omega^2 - i\Gamma\omega}\right),$$  

(2.16)

where $\Gamma$ is the plasma collision frequency, $\varepsilon_\infty$ is the permittivity at an infinite frequency, and $\omega_p$ is the plasma frequency.
2.2.2 Introduction to Plasmonic Materials

A plasmonic material is a material that provides surface plasmon polariton (SPPs) when the light hits its surface and interacts with free electrons. As a result, the free electrons oscillate collectively, creating surface plasmons (SPs) [47] at a metal-dielectric interface. The resonant oscillation of free electrons is called surface plasmon resonance (SPR) [48] which is the vibration resonance between photons in a dielectric and free electrons in a metal (negative permittivity) [49]. When the real part of the permittivity becomes equal to zero, the resonance occurs at the plasmon frequency. The size of the materials tunes the resonance frequency. For instance, the SPR of gold in visible light can be adjusted to NIF by reducing the size of the structure [50]. The surface plasmon (SP) produces EM fields outside and inside the metals known as localized surface plasmons (LSPs) [51] or a SPP [51]. The plasmonics phenomenon shows propagating surface plasmons (SPP) along the metals' surfaces or localized surface plasmons (LSPs) [52]. The surface plasmons (SPs) explain the motion of a charge in the metal while the polariton explains the motion of charge in the dielectric [52]. The wavelength of SPPs is much smaller than the wavelength of the incident wave [53]. Thus, the high EM field enhancement and subwavelength spatial confinement can be obtained [54]. The SPPs can be used to overcome the diffraction limit [55]. The dispersion relation of the SPPs is found by [56]
\[ k_{spp}(\omega) = \frac{2\pi}{\lambda} \sqrt{\frac{\varepsilon_m(\omega)\varepsilon_d(\omega)}{\varepsilon_m(\omega) + \varepsilon_d(\omega)}} = k'_{spp}(\omega) + jk''_{spp}(\omega) \]  

Figure 4. Dispersion relation of surface plasmon polaritons (SPPs) [53].

Figure 4 shows the excitation of SPPs on the air-metal interface. The magenta line shows a light line \((\omega = ck_x)\), where \(c\) is the light speed in a vacuum and \(k_x\) is the wave vector in the \(x\)-axis. The red curve matches to the SPPs dispersion (the Drude model) without damping. At a low \(k_{spp}\), the dispersion behaves like a light line, but it moves away from the light line when the frequency rises and reaches a surface plasmon resonant frequency \((\omega_{sp})\). The yellow dashed curve illustrates the SPP with
damping, producing a back-bending on the SPP and limited value of $\omega_{sp}$. The Green curve is the volume plasmons.

Figure 5. SPPs waves at the interface between a metal and a dielectric with surface charges and the penetration depths inside metal and dielectric [57] [58].

Figure 5 shows the excitation of the SPPs on the air-metal interface. SPPs propagate along the metal surface while the SPPs’ power is decayed and absorbed by the metal over a distance of $1/e$. The propagation length of the SPPs is found by [56]

$$L_{spp} = \frac{1}{2k_{spp}''}$$

(2.18).

The penetration depth of the SPPs can be calculated by [59]

$$\delta_d = \frac{\lambda}{2\pi} \left( \frac{\varepsilon_m' + \varepsilon_d}{\varepsilon_d^2} \right)^{1/2}$$
$$\delta_m = \frac{\lambda}{2\pi} \left( \frac{\varepsilon_m' + \varepsilon_d}{\varepsilon_m'^2} \right)^{1/2}$$

(2.19).
where $\delta_d$ is the penetration depth into the dielectric and $\delta_m$ is penetration depth into the metal. The SPPs create an evanescent field on both sides of the interface [60]. The amplitude intensity of the evanescent field fades exponentially over a distance of the interface surface, and its field wave normally penetrates space over $1/e$ [61].

### 2.3 Introduction to Metamaterials (MMs)

The term *metamaterial* is a Greek word that means beyond, a material with extraordinary properties that are not noticed in nature [62]. They are formed from combinations of various elements designed from materials such as metals and dielectrics. The materials are organized in duplicating forms at scales that are much smaller than the wavelengths. Metamaterials have a new property in the whole designed structures. Changing the form's size, shape, or geometry can be controlled to absorb, scatter, enhance, and focus the EM fields to obtain high performances.

Metamaterials (MMs) are artificial, modified materials with nanostructured blocks. These materials can be fitted for practically any application due to their phenomenal reaction to electromagnetic, acoustic, and thermal waves that rise above common materials' properties. Negative-index metamaterials show a negative index of refraction at specific wavelengths. These behaviors drive potential applications such as imaging [63], sensing [64], focusing, and superlensing [65].
Conventional metals have been the matter of choice for the nanostructure, yet they experience high optical losses. Even the noble metals silver and gold show high losses at optical frequencies that limit the improvement of applications. The progress of new materials for low-loss MM IR applications is required. Plasmonic MMs meet the challenge related to reducing the losses. To overcome the losses with a gain medium is not enough to compensate for the significant losses. An alternate methodology would be to identity plasmonic materials with dielectric permittivity having a negative real part [32]. The loss issue and the capacity to adjust the real part of the dielectric permittivity are together variables to consider while looking for elective plasmonic materials.

Materials can be categorized based on two significant parameters that decide the optical properties of conducting materials, carrier mobility and carrier concentration. Lower carrier mobilities convert to higher material losses. Extra losses due to interband changes are exceptionally unfavorable and seriously limit possible applications. The carrier concentration must be sufficiently high to give a negative real permittivity. However, it additionally must be adjustable so that bigger negative quantities would be available with additional expansions in the carrier concentration [32]. Figure 6 shows the plasmonic materials with low loss. Materials are arranged by considering three significant restrictions like carrier concentration, carrier mobility, and interband losses. The high carrier concentration materials offer a
negative real part of the permittivity, and high carrier mobility materials offer lower material losses. The figure also shows the potential application for EM from UV to infrared [32].

Figure 6. Metamaterials map with their parameters, carrier concentration carrier mobility, and interband losses for plasmonic [32].

The expected materials to supplant silver and gold in plasmonic applications include alkali metals, different alloys [66], graphene [67], and transparent conducting oxides (TCOs) [68]. In the visible frequency, silver, gold, and their alloys with somewhat improved properties (8) are the best materials [68]. Numerous different metals display higher losses while alkali metals have attractive properties yet are precluded by their extreme substance reactivity [32].
2.3.1 Hyperbolic Metamaterials (HM)

Hyperbolic metamaterials (HMMs) can be achieved with subwavelength structures that control light propagation. These materials display numerous uncommon properties that do not exist in nature. They exhibit hyperbolic dispersion, which acts as a dielectric with positive permittivity in one direction and as a metal in other directions, and vice versa [69]. HMMs are anisotropic metamaterials since their principal terms of permittivity or permeability tensor have negative and positive signs. HMMs convert evanescent waves with a large wavevector, which has significant results for super-resolution imaging and strong enhancement [70].

The use of effective medium theory (EMT) [71] [72] has described the structures of HMMs in different spectral areas with subwavelength unit cells including metal-dielectric layers, metal-dielectric and nanowire structures. The dispersion relations of the hyperbolic metamaterials can be expressed by the relative permittivity tensors of an anisotropic material that are found by using the following formula [70]

\[
\varepsilon = \begin{pmatrix}
\varepsilon_{xx} & 0 & 0 \\
0 & \varepsilon_{yy} & 0 \\
0 & 0 & \varepsilon_{zz}
\end{pmatrix},
\]

where \(\varepsilon_{xx}=\varepsilon_{yy}=\varepsilon_{||}, \varepsilon_{zz}=\varepsilon_{\perp}\). The \(\perp\) and \(||\) show the perpendicular and parallel components to the x, y plane, respectively.
2.3.2 The Use of Effective Medium Theory (EMT) for Anisotropic Materials

The effective medium theory for the periodic array of artificial structures with subwavelengths can be considered an effective homogeneous medium characterized by homogeneous permittivity. By designing suitable artificial structures such as metal/dielectric multilayers [Figure 7(a)] and nanowire metal-dielectric arrays [Figure 7(b)], the EMT can be applied.

Figure 7. (a) Multilayer HMM structure consisting of alternating metallic and dielectric layers. (b) Nanowire HMM structure consisting of metallic nanorods embedded in a dielectric host.
EMT in multilayers can be approached by using the Maxwell Garnett mixing formula, analyzing the material in two directions, parallel ($\varepsilon_\parallel$) and perpendicular ($\varepsilon_\perp$). The parallel ($\varepsilon_\parallel$) EMT can be found with the following equations [73]:

$$p = \frac{t_m}{t_m + t_d}$$ \hspace{1cm} (2.20),

where $p$ is the filling fraction of the metal, $t_m$ is the thickness of the metal, and $t_d$ is the thickness of the dielectric, and

$$\vec{D} = \varepsilon_{\text{eff}} \vec{E},$$ \hspace{1cm} (2.20),

where $\varepsilon_{\text{eff}}$ indicates the effective permittivity, which is a tangential component of the electric field ($\vec{E}$) in metal and dielectric. The electric field can be found with the following formula:

$$E_\parallel^d = E_\parallel^m = E_\parallel$$ \hspace{1cm} (2.21),

where $E_\parallel$ is the electric field of the MM subwavelength, $E_\parallel^m$ is the electric field in dielectric layers, and $E_\parallel^m$ is the electric field in metallic layers.

The average displacement from metallic and dielectric can be obtained found with the following equations:
\[ D^\parallel = pD^\parallel _m + (1 - p)D^\parallel _d \]  
(2.22).

By replacing the (2.20) and (2.21) in (2.22), we get:

\[ \varepsilon^\parallel _{eff}E^\parallel = p\varepsilon_mE^\parallel + \varepsilon_d(1 - p)E^\parallel \]  
(2.23),

The final equation can be defined by

\[ \varepsilon^\parallel = p\varepsilon_m + \varepsilon_d(1 - p) \]  
(2.24).

The electric displacement vector is continuous at the interface between the metal and dielectric by boundary conditions. The perpendicular (\(\varepsilon\perp\)) EMT can be found with the following equations:

\[ D^\perp = D^\perp _m = D^\perp _d \]  
(2.25).

The perpendicular electric field can be found with

\[ E^\perp = pE^\perp _m + E^\perp _d (1 - p) \]  
(2.26),

where \(E^\perp\) indicates the total perpendicular component of the electric field in a unit cell, \(E^\perp _m\) indicates perpendicular the electric field in the metal region, and \(E^\perp _d\) shows the perpendicular component of the electric field in the dielectric region. The perpendicular permittivity of the MM\(s\) can be found with

\[ \varepsilon^\perp = \frac{\varepsilon_m\varepsilon_d}{p\varepsilon_d + \varepsilon_m(1 - p)} \]  
(2.27).
The effective parameters of the metal nanowire array structure can be expressed with

\[
\varepsilon_{\parallel} = \frac{[(p + 1)\varepsilon_m + (1 - p)\varepsilon_d] \varepsilon_d}{(1 - p)\varepsilon_m + (p + 1)\varepsilon_d} \tag{2.28}
\]

and

\[
\varepsilon_{\perp} = p\varepsilon_m + (1 - p)\varepsilon_d \tag{2.29},
\]

where \(\varepsilon\) is the permittivity and the subscripts m and d are the metal and dielectric, respectively. Also, \(p = s/s_0\) is the filling ratio between the metal nanowire and dielectric, where \(s\) is the cross-sectional areas of the metal wires and \(s_0\) is the cross-sectional areas of the host dielectric. The subscripts \(\perp\) and \(\parallel\) indicate the perpendicular and parallel regions [70].

![Image of dielectric and HMMs](image)

**Figure 8.** (a) An isotropic dielectric. (b) A uniaxial medium Type I HMM. (c) A uniaxial medium Type II HMM for TM mode [74].
According to the signs of the component of the tenser, the HMM can be classified as metal-like ($\varepsilon_\parallel < 0, \varepsilon_\perp < 0$), dielectric ($\varepsilon_\parallel > 0, \varepsilon_\perp > 0$), Type I HMM ($\varepsilon_\parallel > 0, \varepsilon_\perp < 0$) in Figure 8(b), and Type II HMM ($\varepsilon_\parallel < 0, \varepsilon_\perp > 0$) in Figure 8(c).

2.3.3 Hyperbolic Metasurfaces (HMS)

Metasurfaces are considered 2D-analogs of metamaterials, and they provide exceptional adjustment over light transmission, refraction, and reflection [75]. The SPP can be manipulated by the HMS in-plane and display hyperbolic dispersion. Metasurfaces can support the localized waves, [76] and the TM and TE momentum of surface waves can be controlled [77].

HMS are planar optical parts made of a single layer [78] or multilayers [79] of phase-shifting nanostructures. The single-layer metasurfaces are dependent on their distinctive phase mechanism systems and differentiation among the plasmonic metals and dielectric on the surface [76]. Genevet et al. reported achromatic reaction at chosen wavelengths. In planer metalenses, the scattered wavefront was dependent on a metasurface through the outer surface of the second spherical wave produced from the optical components [78].

Multilayer gap-surface plasmon metasurfaces [73] were designed with a metal/insulator/metal (MIM) configuration based on a gap-surface plasmon (GSP) resonator. In such a design, perfect absorption can be obtained.
The authors of [80] designed IR metasurface lenses with an amorphous silicon (α-Si: H) nanowire supported by an MgF2 substrate, as shown in Figure 9. They created a focal spot close to the diffraction limit with 78% efficiency [80].

The following equations can find the phase profile at the position of the phase distribution

\[ \phi(r) = \frac{2\pi}{\lambda} \left( f - \sqrt{r^2 + f^2} \right), \]  

(2.30),

where \( f \) is the focal length, \( r \) is the lattice position to the metal center, and \( \lambda \) is the operating wavelength.

Figure 9. Design MIR metalens operating at 4 \( \mu \)m wavelength. (a) Configuration of MIR metalens and, (b) Structure of the NP unit cell with Radius (R), height( h), and period (a) [80].
Another technique is called the micro-electro-mechanical system (MEMS) (Figure 10). The MEMS can be utilized to design a flat lens metasurface to focus mid-IR using Lidar scanners [81] or MEMS [82] to produce a 2D dynamic ray scanning MEMS, which controls the metalens angle on two orthogonal axes. The MEMS can build and correct aberrations of the incoming light but does not influence the mechanical exhibition in low angular relocations. Additionally, the system protects the focused spot profile and full width half maximum (FWHM).

![Figure 10](image)

Figure 10. (a) Metasurface lens (b) Schematic of incident Gaussian rays at an angle $\theta$ on the metasurface lens and reflected waves to the focal point [82].

Metasurface optical devices can shape the wavefront light. The MEMS have the novel benefits of rapid development, excellent optical quality, low visual loss
frequency, and broad polarization [82]. Roy et al. [82] designed (Figure 10) metasurface optical devices to shape the wavefront of a light metasurface lens at \( \lambda = 4.6 \, \mu m \) of the wavelength. They placed gold discs with a thickness of 50 nm on the top of the 400 nm thickness of the silicon dioxide layer with the gold layer on the bottom. Their design results showed that the focusing performance was improved and that the varying radius of the disc-controlled reflectance and phase for only one unit. Also, they utilized different radii of the discs to obtain the hyperbolic phase profile. The potential applications of metasurfaces include metalenses, beam deflectors, holograms, wavefront controls, and imaging systems [78].

### 2.4 Infrared Plasmonic Materials

Semiconductor materials have properties that qualify them to use as IR plasmonic materials instead of noble metals. The plasmonic properties of semiconductors are suitable for MM applications at NIR and MIR [83] [84]. Doped semiconductors have been chosen as new plasmonic materials to substitute noble metals [79] because of having several advantages, including few free electrons in doping, lower losses than metals [86] resistance to high temperatures [87], tunability [88], and support of propagating surface plasmon polaritons (SPPs) in MIR [29].

There are significant disadvantages of metals in their optical characteristics since they cannot be adjusted or tuned without any problem. For instance, to tune the
carrier concentration of metals much with MIR optical fields is not easy. Subsequently, in applications where the tuning of the optical characteristics is fundamental, metals are not a good choice. With the defects of traditional plasmonic materials, specialists have been persuaded to look for better options and they have recommended materials that overcome downsides referenced previously [30]. Additionally, the actual losses in traditional metals are high to such an extent that a massive increase is fundamental to repay the losses. Furthermore, adding offset gains complicates manufacturing and increases noise to the system. Even though the material cannot be totally without losses, to use a metal with fewer losses than traditional metals losses would be more reasonable. The carrier-damping losses (γ) and plasmonic frequency (ω_p) in the Drude model can be reduced to scale down the losses. However, ω_p ought not to be decreased an excessive amount to hold a metallic reaction in the frequency range.

Two prospects in creating elective plasmonic materials are dependent on the Drude model dispersion. One of the prospects is doping semiconductors intensely and making sufficient free carriers to convert the material's optical characteristic to metallic [30] [89]. The other alternative is eliminating free carriers from metals to lessen the carrier concentration to the ideal worth [30], [90]. Both of these strategies have advantages from the viewpoint of material design. Fitting the Drude metals by
adjusting the materials’ plasma frequencies is a method that does not represent any significant technical restriction, and hence, is a beneficial methodology.

2.4.1 Near-Infrared Plasmonic Materials

Due to the requirement for massive density of free electrons, much consideration has been directed toward strongly doped semiconductors. Researchers have developed materials for IR wavelengths such as transparent conducting oxides [(AlZnO and indium tin oxide (ITO)], transition metal nitrides (TiN and ZrN), silicides, doped semiconductors, InAs, InP, graphene, GaN, GaP, SiC, and the perovskite SrTiO3 [87][90][91][92][93][94][95][96]. These materials have low scattering rates with the small density of states around the Fermi level, which leads electrons to scatter [29]. Transparent conducting oxides (ITO, Ga: ZnO, and Al: ZnO ) have been utilized as alternative materials to noble metals in the NIR [90]. Such materials have small real part of the permittivity and very small imaginary part of the permittivity in NIR and visible light as shown in Figure 11. They have huge benefits including low losses, SPP waves, and tunability.

The NIR materials are achieved by heavy doping (carriers $10^{21}$ cm$^{-3}$) that can be accomplished in TCOs, ITO, or zinc oxide doped (AZO) with aluminum or gallium (GZO) [97]. In the MIR, materials are achieved by using silicon carbide, gallium arsenide, and different semiconductors.
Since the losses related to metals partially grow from free-electron densities, which are too enormous, a general procedure is to lessen the electron density in metals or increment the electron density in semiconductors. The carrier concentration in metals is expanded by blending with nonmetals (such as nitrides, germanides, oxides, and metallic composites), leading to intermetallic materials [30][53][98].

Low-loss plasmonic optical materials are used in NIR applications. Nonstoichiometric oxides like titanium, vanadium, and aluminum oxides are acceptable candidates in the NIR plasmonic materials [32]. Doping semiconductors act like metals ($\varepsilon' < 0$) because of their carrier concentration and response. Since TCOs can be doped very heavily, TCOs display high DC conductivity [99]. This property shows metallic behavior in the NIR range. Due to having a large bandgap

Figure 11. Drude model of real and imaginary permittivity for GZO, ITO, and AZO.
of these semiconductors visible to the NIR range, they become transparent. Oxide semiconductors, for example, indium oxide, cadmium oxide, and zinc oxide, are highly doped to act like metallic films [89] [100].

2.4.2 Hyperbolic Metamaterials of ITO/Silicon

HMM of ITO/silicon is created using a periodic stack of ITO and silicon layers. The effective medium theory is used to obtain the effective permittivity as shown in Figure 12.

![Figure 12](image.png)

Figure 12. The layer structure of the periodic arrangement of dielectric and the metal layers. The effective medium has a different direction parallel and perpendicular to the layers.

The dispersion of ITO/silicon can be tuned by controlling the fraction of metal \( p \), as shown in Figure 13. The increase in the fraction of metal drives a real parallel
permittivity of $\varepsilon_x$ and $\varepsilon_y$ to a metallic behavior, as shown in Figure 13(a). For instance, when $p < 0.5$, the MM behaves like a dielectric, but when $p \geq 0.5$, the MM acts like metal in a specific range of wavelengths.

Figure 13. The HMM dispersion of ITO/silicon with various filling fractions of the ITO over NIR wavelengths showing (a) the effective real parallel permittivity, (b) the effective imaginary parallel permittivity, (c) the effective real perpendicular permittivity, and (d) the effective imaginary perpendicular permittivity of the structure.
The increase also leads to a perpendicular real permittivity of \( \varepsilon_z \) to change from the dielectric to metallic behavior at the plasmonic wavelength. The parallel [Figure 12(c)] and perpendicular [Figure 12(d)] imaginary permittivities of \( \varepsilon_x \) and \( \varepsilon_z \) are increased by increasing the \( p \).

In Figure 14, the optical phase diagram illustrates the effective medium response of ITO/silicon hyperbolic metamaterials with various fill fractions of ITO and corresponding wavelengths. The diagram can be utilized to select operating points for specific applications in NIR region \([26]\). The graph offers Type I and Type II hyperbolic behaviors. Type I is achieved from 0.1 to 1 of the fractions at 1.5 to 2.69 of the wavelengths while Type II occurs from 0.4 to 0.6 of the filling fractions at 2.7 to 3\( \mu \)m of the wavelength. The diagram also shows effective metal and dielectric behaviors. When \( p = 0.5 \), the graph shows the plasmon resonant where the four different behaviors meet.
Figure 14. Optical phase diagrams for the effective medium theory prediction of effective behavior, Type I, Type II, metal, and dielectric based on the fill fractions of the ITO.

2.4.3 Mid-Infrared Plasmonic Materials

An alternative material such as III-V doped semiconductors such as InGaAs, SiC, AlInAs, graphene, and doped InAs can act at the MIF range of a plasmonic material [87]. Doping concentration drives these materials to work as a metal at MIR wavelengths (~3 to 20 μm) with the ability to tune the plasmonic wavelength [71]. InAs can be considered the best of several semiconductors to give the small plasmon wavelength because it has the smallest mass. [94]. Law et al.[101] proved that a carrier concentration in InAs, which demonstrated metal-like behavior for wavelengths longer than 6 μm, is about $7.5 \times 10^{19}$ cm$^{-3}$. 
Figure 15(a) and (b) show the real and imaginary parts of permittivity of InAs from 2 to 10 μm for different electron concentrations. The Drude model describes different concentrations of doped InAs (si) in the MIR in the following equation:

\[
\varepsilon(\omega) = \varepsilon_{\infty} - \frac{\omega_p^2}{\omega(\omega + i\gamma)}
\]  

(2.31),

where \(\omega_p\) is the plasma frequency, \(\varepsilon_{\infty}\) is the relative permittivity of undoped InAs, \(\gamma\) is the damping rate. The plasma frequency \(\omega_p = \sqrt{\frac{ne^2}{m^*\varepsilon_0}}\) and the damping rate \(\gamma = \frac{e}{(m^*\mu)}\). where \(n\) is the carrier concentration, \(m^*\) is the effective mass, \(\varepsilon_0\) is the free space permittivity, \(e\) is the electron charge, and \(\mu\) is the mobility [102]. The damping constants \(\Gamma\) are 1.13 \(\times\) 10\(^{13}\) s\(^{-1}\), 1.13 \(\times\) 10\(^{13}\) s\(^{-1}\), 2.83 \(\times\) 10\(^{13}\) s\(^{-1}\), 4.93 \(\times\) 10\(^{13}\) s\(^{-1}\), and 6.3 \(\times\) 10\(^{13}\) s\(^{-1}\), at 14.9 μm, 9.1 μm, 6.5 μm, 5.6 μm, and 4.5μm, respectively. Since the plasma frequency \(\omega_p\) is contrarily relative to the effective mass, a more significant change in the permittivity's real part may occur for a substrate with a small effective mass.

Figures 15(a) and (b) illustrate the permittivities' real and imaginary parts for different carrier concentrations of highly doped InAs with the Drude model. It is notable in Figure 15(a) that by changing doping concentrations of InAs, the transition from dielectric to a metallic behavior can be tuned. Doped InAs has high electron density, making the metallic property stronger, acting like metal when the real part
of the permittivity is negative. The band edge of conduction in InAs is less than the band edge of the valence in other semiconductors.

Figure 15(a) The Drude model of real and (b) imaginary parts of the permittivity of InAs with different carrier concentrations over mid-IR wavelength.

Figure 15(a) also shows that the red curve transition from positive to negative occurs at 4.5 μm the plasmonic wavelength and 7.5 x10^{18} (cm^{-3}) of the concentration. Likewise for blue, green, black and yellow curves, the transitions occur at 5.6 μm, 6.5 μm, 9.1 μm, 14.9 μm and 4.4 x10^{19} (cm^{-3}), 3.3 x10^{19} (cm^{-3}) ,10 x10^{18} (cm^{-3}) and 2.7 x10^{18} (cm^{-3}) of the concentrations (n), respectively [103],[104].

Figure 15(b) represents the imaginary part of the InAs permittivities over IR wavelength for different carrier concentrations of highly doped InAs. The magnitude of imaginary parts of InAs is smaller than that of noble metals. However, noble
metals show resonances of interband absorption near the plasma frequency of the metals [105]. However, the imaginary part of permittivity grows quickly, and it turns lossy at longer wavelengths.

2.4.4 Hyperbolic Metamaterials of Doped InAs/Undoped InAs

Doped InAs/undoped InAs metamaterial is a uniaxial metamaterial consisting of stacked alternating layers of dielectric and metallic materials, as seen in Figure 17(a). Metallic materials (doped InAs) with permittivity \( \varepsilon_m = \varepsilon_m' + j\varepsilon_m'' \) and thickness \( (t_m) \), and dielectric (undoped InAs) with permittivity \( \varepsilon_d = \varepsilon_d' + j\varepsilon_d'' \) and thickness \( (t_d) \) and a unit cell thickness is \( T = t_m + t_d \) (\( T \ll \lambda \) is subwavelength) and the filling fraction of the metal is \( p = \frac{t_m}{T} \).

Figure 16. Stacked layers of doped InAs/undoped InAs and the homogeneous medium with effective permittivities.
This MM can be described as a homogeneous uniaxial metamaterial by applying the effective medium theory and obtaining the effective permittivities.

For different filling fraction values (p), the effective permittivities can be tuned over MIR (Figure 17). By properly tuning the filling fraction of doped InAs, a distinctive wavelength can be chosen to accomplish a high enhancement and resolution, exhibiting the prospect of tuning the metamaterial to work at any ideal wavelength. Such metamaterials can achieve a Type I HMM, a Type II, metal, and dielectric behavior; selecting p = 0.5 (cyan curves) shows the Type I HMM between 6.4 μm to 9.3 μm. It has positive permittivity in the x-axis and the y-axis and a negative permittivity in the z-axis with low optical losses.

Still, the dispersion shows Type II HMM between 9.4μm to 12μm with a negative permittivity on the x-axis and the y-axis and a positive permittivity on the z-axis. Dielectric behavior appears from 1μm to 6.3μm. Similarly, the yellow curves (p = 0.1), black curves (p = 0.2), and a blue curve (p = 0.3) represent just two behaviors, Type I and dielectric, for slightly different ranges of wavelengths. However, the red curves (p = 0.6) only represent four cases of behaviors: Type I (6.4 μm to 8.52 μm), Type II (10.55 μm to 12 μm), dielectric (<5 μm to 6.4 μm), and metallic (8.52 μm to 10.55 μm). In contrast, magenta curves (p = 0.7) represent Type I (6.4 μm to 7.8 μm), dielectric (<5 μm to 6.4 μm), and metallic (8.7 μm to 12 μm) behaviors at different wavelengths.
Figure 17. The HMM dispersion of doped InAs/undoped InAs with various filling fractions of the doped InAs over NIR wavelengths, (a) the effective real parallel permittivity, (b) the effective imaginary parallel permittivity, (c) the effective real perpendicular permittivity, and (d) the effective imaginary perpendicular permittivity of the structure.
The dispersion relation of doped InAs/InAs HMM with different concentrations were reported by Desouky et al. [104]. They showed that IR focusing in InAs based doped InAs/undoped InAs HMM obtains a resolution of 0.045 $\lambda$.

Figure 18 illustrates the diagram of optical phase for doped InAs/undoped InAs HMM where EMT is used with a fill doped InAs fraction and the wavelength and displays the various optical isofrequency surfaces in different sections based on the wavelength of the operation and the fill fraction of the metal.

Figure 18. Effective medium theory (EMT) shows that a doped InAs/undoped InAs multilayer system can act as an effective dielectric, effective metal, Type I HMM, Type II HMM, metal, and dielectric basing on fill fraction of the metal in the mid-IR region.
The Drude model parameters for doped InAs and undoped InAs in Figure 18 were taken from Peragut et al. [106]. For undoped InAs, $\omega_p = 3 \times 10^{13} \, rad \cdot s^{-1}$, $\gamma = 10^{12} \, rad \cdot s^{-1}$ and $\varepsilon_\infty = 12.3$, which corresponds to a residual doping concentration of $8 \times 10^{16} \, cm^{-3}$. Likewise, for doped InAs, $\omega_p = 2.95 \times 10^{14} \, rad \cdot s^{-1}$, $\gamma = 10^{13} \, rad \cdot s^{-1}$, and $\varepsilon_\infty = 10.4$, which corresponds to a concentration level (silicon) of $3.5 \times 10^{19} \, cm^{-3}$.

In this diagram, the magenta and red areas denote an anisotropic effective Type I and Type II, respectively. The yellow area indicates metal where propagating waves are not possible, and the green area represents dielectric [107]. On the borders, changed colors show the transitions between different surfaces [26] and indicate the plasmon resonance of the doped InAs/undoped InAs or transition point, where the four effective materials meet at a wavelength of 9.3 μm and a fill fraction of 0.5. The wavelength dispersion changes the metamaterial's structure from an effective dielectric to HMM, HMM to metal or dielectric to metal, and vice versa at the phase boundaries. The dispersion subsequently changes from an ellipsoid (dielectric) to a hyperboloid (HMM) at the phase boundaries. Figure 19 can assist in selecting a working point for some mid-infrared applications [26].
Figure 20. Optical properties of the noble metals Au, Ag, and Al, described by the Drude model, (a) real (solid curve) and (b) imaginary (dashed curve) permittivity [87].

Figure 19 shows that traditional plasmonic metals which have small permittivity (real and imaginary part) in the visible light range. At the same time, they have massive negative real parts and massive imaginary parts in the NIR and MIR permittivity. For these conditions, noble metals are often utilized for plasmonic materials in the visible light [108]. However, compared to an alternative plasmonic material (Figure 11 and Figure 16), the real permittivity of TCO materials is small and negative, and the imaginary part is very small.
2.5 Plasmonic Fresnel Zone Plates

Fresnel zone plates (FZPs) are one kind of diffractive lens [109], which are annular diffraction gratings of substitute nontransparent and transparent zones and are generally utilized as centering focal points in some applications such as detecting and imaging [110]. The FZP can produce a high image transmission quality [111] and an increased field power at the focal plane [112]. The PFZP with adjusting metallic zones supports a smaller full-width half-maximum width (FWHM) spot with a small diffraction limit and maximizes a resolution more than a traditional dielectric lens [113] [114].

Because of SPP waves and the constructive interference of diffracted waves, the PFZPs can achieve a strong coupling mechanism and turn the SPP waves from far-field propagating waves to near-field SPP waves [115]. The PFZP has shown that plasmonic planar focal points can reconstruct the light power dispersion [116], and destructive interferences can occur between propagating SPP waves and a plasmon focal spot as a result of the mismatch between incoming waves and the subwavelength zone of the lens. In order to overcome destructive interferences, the radius of zones can be calculated by the following equation [117]:
\[ r_n = \sqrt{n\lambda f + \frac{n^2\lambda^2}{4}} \]  

(2.32),

where \( r_n \) is the radius of the \( n \)th zone, \( n \) is the zone number, \( f \) is the focal length, and \( \lambda \) is the wavelength of the incident light. The material of the PFZPs lens, as shown in Figure 20, is the metallic film, and the outside is dielectric (air) with fixed thickness and tuning width \((r_n)\). The metal zone’s width \((r_n - r_{n-1})\) is less than the operating wavelength, so it works like a plasmonic waveguide \([118][117]\).

![PFZP](image)

**Figure 21.** Diagrams of PFZPs lenses.

When an incoming electromagnetic wave hits the zone, it will be changed into SPP modes and sent along the waveguide. These waves will be separated back to an
electromagnetic mode with a phase delay after leaving the zone [119]. The focusing spot is created by coupling between SPPs and diffractive waves, and each zone works as a nano resonance waveguide [117]. Figure 21 shows a schematic diagram of the incident wave rays hitting the radial Fresnal plates, and then the rays diffracting and focusing at a focal f of distance.

![Schematic Diagram](image)

**Figure 22.** A schematic diagram of the radial Fresnel zone plates diffracts and incident wave rays at a focal distance f.

Di Feng has designed a gold bowtie nanoantenna in the focal plane of a PFZP lens, and he shows that the strongly focused beam spot confinement with high resolution and small full width at half-maximum (FWHM) beamwidth [22]. Gonzalez et al. depicted utilizing the gathering properties of FZPLs to focus the optical transition
that arrives at an infrared antenna and showed how an FZPL can improve the reactions of the infrared antenna [120]. Galil et al. presented three distinctive metasurfaces that can obtain subwavelength concentration of the irradiation in the infrared range. The designs accomplished high resolution and high bandwidth efficiency about 60% to 75% at MIR. Thus, they can fulfill the diverse necessities of various infrared applications. They achieved 0.5λ of the resolution at 9 μm of the wavelength [121].

![Figure 23. (a) The electric field distribution |E| and (b) the magnetic field distribution of the focusing PFZP lens.](image)

Figure 22 shows the simulation results when the polarized incident waves hit the Fresnel zone plate (made of doped InAs). The waves diffract around radial zones and focus at the focal spot in the air. Figure 22(a) illustrates the electric field distribution...
|E|, and Figure 22(b) shows the magnetic field distribution |H| of the focusing PFZP lens.

2.6 Plasmonic Nanoantenna

Optical antennas are devices that transmit or receive EM waves [122] and convert propagating far-field into localized surface plasmon (LSP) near field when the incident light interacts with a nanoantenna, providing effective waves (the SPPs) to couple photons with plasmonic in nanoscale objects. Optical antennas concentrate and manipulate optical fields to sub-wavelength dimensions, smaller than the diffraction limit and the operating wavelength [123]. Plasmonic nanoantennas couple waves efficiently between propagating and localizing waves. They are not perfect conductors at optical wavelength but can provide LSP and SPP [124]. The optimum optical antenna utilization is required strong field confinement, high field enhancement, and enormous absorption cross-section [124]. Plasmonic waves on noble metal nanostructures provide strong subwavelength localized field enhancement in the nanoscale in visible light.

The electric field enhancement can be increased in the gap of nanoantennas. LSP can be confirmed in the small gap between pairs of rods, dipoles, or nanoantennas and produced high confined fields.
A wide range of optical antenna structures have been concentrated over recent years, such as the dipole nanoantenna [127] [128], bow-tie nanoantennas [129], [130], slot nanoantennas [131] [132], monopole antennas [133], Yagi–Uda nanoantennas [134], nanodisk [135], and nanoparticles. Nanoantennas are used for biomolecular sensing, where the individual antenna resonance (LSPR) is utilized to distinguish between such resonant and shifted resonance with the surrounding materials [136]. Adato et al. [137] showed that infrared nanoantennas offer strong near-field enhancement and high SNR spectra in collecting protein binding interaction. Bow-tie infrared
nanoantennas is used in biological imaging, spectroscopy, and chemical high-resolution [138].
Chapter 3:
A Novel NIR Metalens Using Type I Hyperbolic Metamaterial

3.1 Design and Optimization

ANSYS HFSS software is used to design and simulate near-infrared metalens. A normally incident TM plane wave excites the structure. Figure 24 shows the structure of the metalens, which consists of a plasmonic waveguide coupler (PWC), a cylindrical slab of hyperbolic metamaterial (HMM), and a nanoantenna embedded in a dielectric substrate.

Figure 25. Metalens structure.
The PWC is made of annular rings of ITO which have a thickness of 50 nm, with inner ring widths of 100 nm, spacing between the rings of 100 nm, and an outer ring width of 350 nm. The PWC is placed on a cylindrical slab of HMM, which consists of periodically placed ITO and silicon layers.

The effective bulk material properties of the metamaterial cylindrical slab are computed using EMT for wavelengths of 1μm to 3μm. The Drude model is used to generate the data of the constituent material properties for the ITO. Complex permittivity for the ITO is found by using \( \omega_p = 440 \times 10^{12} \text{ rad.s}^{-1}, \gamma = 19.7 \times 10^{12} \text{ rad.s}^{-1}, \) and \( \varepsilon_\infty = 4.08, \) while permittivity of silicon is 12.25; where \( \omega_p \) is the plasmonic resonance frequency, \( \gamma \) is electron relaxation, and \( \varepsilon_\infty \) is the relative permittivity [24]. The ITO permittivity is described using a Drude Model:

\[
\varepsilon = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 - i\Gamma \omega}
\]  

(3.1). The ITO permittivity of the real and imaginary part is shown in Figure 25. The PWC, a perfect solution to adapt a phase compensation mechanism, is also used to generate point sources with the same phase based on the Huygens-Fresnel principle. The incident plane wave originates in the free space above the PWC. Each spacing in the PWC works as a nanoscale waveguide, so a strong subwavelength focusing effect can be found in the structure through constructive interference. In this work, the phase compensation is realized by modifying and optimizing the geometric parameters. For the PWC, the thickness is set to be \( T = 50 \text{ nm} \), and the width of the
inner rings is w=100 nm while the width of the outer ring wou = 300 nm. The HMM can support the propagation of high wavevectors. There are the two most common HMM structures: a multilayer structure made of alternating metallic and dielectric layers and a nanowire structure fabricated by embedding metallic nanowires in a dielectric medium. In this paper, the multilayer variant of the structure is used. The metamaterial is a uniaxial material with permittivities \( \epsilon_\\parallel \) and \( \epsilon_\\perp \). The dispersion relation of a uniaxial material is given by

\[
\frac{k_\\parallel^2}{\epsilon_\\perp} + \frac{k_\\perp^2}{\epsilon_\\parallel} = \frac{\omega^2}{c^2}.
\]

The effective permittivity is calculated by the effective medium theory (EMT) \cite{22}:

\[
\epsilon_\\parallel = \epsilon_{xx} = \epsilon_{yy} = p\epsilon_m + (1-p)\epsilon_d
\]

and

\[
\epsilon_\\perp = \epsilon_z = \frac{\epsilon_m \cdot \epsilon_d}{p\epsilon_d + (1-p)\epsilon_m}, \quad p = \frac{t_m}{t_m + t_d},
\]

where \( t_m \) is the thickness of the metallic layer ITO, and \( t_d \) is the thickness of the dielectric layer silicon, with \( t_m/t_d = 1/4 \). The permittivity of the ITO is \( \epsilon_m \), and \( \epsilon_d \) is the permittivity of silicon. Figure 26 shows real and imaginary parts of the permittivity of ITO and the perpendicular and parallel permittivities of the effective medium analysis for ITO/Si. The resulting material is a Type I HMM with a diagonal
dielectric tensor described as $\epsilon_{xx} = \epsilon_{yy} > 0$, $\epsilon_z < 0$ in the wavelength range of 1.4 μm to 1.8 μm, and as dielectric $\epsilon_{xx} = \epsilon_{yy} > 0$, $\epsilon_z > 0$.

Figure 26. (a) The real and imaginary part of permittivity of ITO and (b) Effective medium analysis of an ITO/Si.

### 3.2 Results and Discussion

#### 3.2.1 The Electric Field and Diffraction Limit for a Structure without a Nanoantenna

Figure 26 shows the magnitude of electric field distribution for the metalens without a nanoantenna at the focal point with an x-polarized normal plane wave at 1.52 μm operating wavelength. The focal length is 1.05 μm. The FWHM values along the x-axis and the y-axis are 250 nm, which is 1/6 of the wavelength and its electric field intensity enhancement on the order of $10^2$. 

56
3.2.2 Electric Field and Diffraction Limit for a Structure with a Nanoantenna

Figure 27 shows the magnitude of the electric field distribution for the whole structure. Figure 27(a) is the magnitude of electric field distribution and plasmon resonances (LSPRs) around the resonator’s edges on the interface between the Ag nanodipole and the substrate, and (b) is the electric field profile at the focal point, which is plotted on the x-axis (green) and y-axis (blue). The full-width half maximum (FWHM) is 0.016 μm on the x-axis and 0.0215 μm on the y-axis. The FWHM values along both the x-axis and the y-axis are smaller than the diffraction limit of the 0.5λ,
thus allowing us to break the diffraction limit to sub-wavelength to about 1/1000 times the wavelength.

Figure 28. The electric field at the focal point of the structure: (a) is the magnitude of electric field distribution and (b) is the field intensity profile of the x-axis and y-axis.

3.2.3 The Electric-Field Enhancement of the Structure

Figure 28 illustrates the variation of electric-field enhancement at the focal point versus frequency for different thicknesses of the PWC. The electric field enhancement is on the order of $10^6$. The wavelength ranges from 1.392 µm to 1.8 µm while the thicknesses of the PWC are H=110 nm, 130 nm, 150 nm, and 190 nm. The effect of PWC thickness in the electric enhancement shows that 110 nm, 130 nm, and 150 nm thicknesses generate similar electric field enhancements ($1.4 \times 10^6$ – 1.8
× 10^6) at 1.58 μm. In comparison, a thickness of 190 nm shows a slight increase in the electric field enhancement (2.3 × 10^6) at 1.45 μm.

Figure 29 demonstrates the variation of electric-field enhancement as a function of frequency at the focal point with variation in the nanoantenna (L) length. The localized surface plasmon resonances (LSPRs) alter by changing the dipole length (Ls). The highest electric field enhancement is 2.64 × 10^6, when Ls = 26 nm at 1.52 um while an enhancement of 2.32 × 10^6 enhancement is seen at 1.51 μm when Ls = 25 nm and 0.9 × 10^6 enhancement is seen at 1.64 μm when Ls = 30 nm.
Figure 30. The electric field enhancement of the focal point with different lengths ($L_s$) of the nanoantenna.

Figure 30(a) shows the electric field enhancement as a function of frequency. Figure 30(b) shows the electric field enhancement corresponding to the variation of the nanoantenna radius, $R_s = 4$ nm, 5 nm, and 6 nm. The electric field enhancement at $R_s = 6$ nm from 1.43 µm to 1.512 µm was high, reaching $2.4 \times 10^7$ electric field enhancement at 1.44 µm while it reaches $0.4 \times 10^7$ with $R_s = 4$ nm at 1.77 µm and $0.26 \times 10^7$ at 1.58 µm with $R_s = 5$ nm. The sharp corner of the end of the nanoantenna has the maximum enhancement of the electric field.
Figure 31. (a) The electric field enhancement as a function of frequency with fixed radius (Rs=5 nm) and (b) shows the electric field enhancement with different radii of the nanoantenna.

3.3 Summary

Metamaterials have been employed to realize useful behavior in the control of electromagnetic waves. A novel metalens is designed in the NIR band from 1.5 µm to 3 µm. It consists of a stack made up of a plasmonic waveguide coupler (PWC), a hyperbolic metamaterial slab, and a silicon substrate with an embedded silver nanodipole. It is used instead of traditional plasmonic metals in the PWC. Highly sub-diffraction focusing is achieved with up to 0.01667 λ maximum achieved resolution, and an electric field intensity enhancement of 10⁶ is achieved around the silver nanodipole.
Chapter 4:

Novel Mid-Infrared Metalens Using a Hyperbolic

4.1. Design and Optimization

The metalens was designed and simulated using ANSYS HFSS software, and the structure is excited by an incident plane wave. Figure 31 shows the structure of the metalens, which consists of a PWC with annular rings of doped InAs, a slab of HMM [106], and a nanoantenna situated on a dielectric substrate. The PWC has a thickness of 70 nm and ring width of 20 nm, and it is illuminated by normally incident TM plane waves. The PWC is placed on the cylindrical slab of the HMM, which consists of undoped InAs and doped InAs. The Drude model is used to generate the data of the constituent material properties for the InAs. Complex permittivity for undoped InAs is found by using $\omega_p = 3 \times 10^{13}$ rad.$s^{-1}$, $\gamma = 10^{12}$ rad.$s^{-1}$, and $\epsilon_r = 12.3$, while the doped InAs parameters are $\omega_p = 2.95 \times 10^{14}$ rad.$s^{-1}$, $\gamma = 10^{12}$ rad.$s^{-1}$, and $\epsilon_r = 10.4$; where $\omega_p$ is the plasmonic resonance frequency, $\gamma$ is electron relaxation in radians and $\epsilon_r$ is the relative permittivity. The metamaterial is a uniaxial material with permittivities $\epsilon_\parallel$ and $\epsilon_\perp$. The dispersion relation of a uniaxial material is given by

$$\frac{k_\parallel^2}{\epsilon_\perp} + \frac{k_\perp^2}{\epsilon_\parallel} = \frac{\omega^2}{c^2}$$

(4.1)
The effective permittivity is calculated by the effective medium theory (EMT) [71]:

\[ \epsilon_\parallel = \epsilon_{xx} = \epsilon_{yy} = p\epsilon_m + (1 - p)\epsilon_d \]  

(4.2)

and

\[ \epsilon_\perp = \epsilon_z = \frac{\epsilon_m \cdot \epsilon_d}{p\epsilon_d + (1 - p)\epsilon_m}, \quad p = \frac{t_m}{t_m + t_d}, \]

(4.3),

where \( t_m \) is the thickness of the metallic layer (doped InAs) and \( t_d \) is the thickness of the dielectric layer (undoped InAs). \( \epsilon_m \) is the permittivity of doped InAs, \( \epsilon_d \) is the permittivity of undoped InAs, \( t_m \) is 290 nm, and \( t_d \) is 370 nm.

Figure 32. Metalens structure.
The perpendicular and parallel permittivities are shown in Figure 32. The resulting material is a Type I HMM with a diagonal dielectric tensor described as \( \epsilon_{xx} = \epsilon_{yy} > 0, \epsilon_z < 0 \) in the wavelength range of 6.366 \( \mu \)m to 8.837 \( \mu \)m, and a Type II HMM with \( \epsilon_{xx} = \epsilon_{yy} < 0, \epsilon_z > 0 \) in the wavelength range 10.4 \( \mu \)m to 12 \( \mu \)m. The dispersion also acts as an anisotropic metal in the range 8.838 \( \mu \)m to 10.03 \( \mu \)m. We use the Type I HMM.

Figure 33. Effective permittivity as a function of wavelength.
4.2 Results and Discussion

Figure 33 shows the magnitude of electric field distribution for a metalens with and without nanodisks at the focal point of the x,y plane with an x-polarized normal plane wave at 7.739 μm. The focal length is 2.248 μm. The FWHM values along the x-axis and the y-axis without the nanodisks are 800 nm, and its electric field enhancement is on the order of $10^2$. The electric field distribution is enhanced in the gap of two gold nanodisks. The enhancement is controlled by varying the height and radii of the disks.

![Figure 34. The magnitude of electric field distribution at the focal point: (a) without a nanodisk and (b) with a nanodisk.](image)

Figure 34 shows the variation of electric field enhancement at the focal point with variation in the gap between the two disks, and the electric field enhancement is on the order of $10^6$. 

65
Figure 35. The electric field enhancement of the focal point with different values for separation between the disks.

In Figure 35, the electric field at the focal point is plotted on the x-axis (blue) and y-axis (red). The full-width half maximum (FWHM) is 8.18 nm and 13.64 nm on the x-axis and y-axis, respectively. The FWHM values along the x-axis and the y-axis are smaller than the diffraction limit of the 0.5λ, allowing us to image particles that are 1/1000 times the wavelength.
4.3 Summary

As shown, by using a PWC made using doped InAs, an HMM (doped InAs/undoped InAs), and an Au plasmonic nan disk, a new metalens was designed in the MIR range capable of super-resolution focusing with high electric-field enhancement. Using InAs as an alternative to metals reduces losses. The HMM allows the lens to support the propagation of waves with high wave vectors. The electric field at the focal point is enhanced when using the Au nanodisk. The FWHM of the field at the focal point is 8.18 nm x 13.64 nm on the x-axis and y-axis, respectively. The FWHM allows us to image particles that are 1/1000 times the wavelength. Such metalens may be used for sub-wavelength imaging and sensing with super-resolution and high electric field enhancement.
Chapter 5:

Ultrahigh MIR Electric Field Enhancement via Nanoantenna Plasmonic Resonance Coupled to a Novel Hyperbolic Metalens

5.1 Design and Optimization

The hybrid system of a hyperbolic metalens and a nanoantenna is structured and simulated by using ANSYS HFSS software. The structure is similar to the metalens in [139], [140] [141] (Figure 36). The PFZP rings are placed on a cylindrical slab of the HMMs, which is located on a spacer slab with a dielectric constant of $\epsilon_r = 2.3$ ($n_{sp} = 1.52$). A nanoantenna is situated at the bottom of the structure. The electric field intensity, $E_0 = 1$ (V/m), of the incident plane wave with operating wavelength ranges from 7.3 µm to 8.84 µm is polarized linearly with the x-axis to match the nanoantenna polarization [142].

The Drude model is used to generate the permittivity for the undoped InAs and doped InAs at different doping concentrations as the following equation [143] [144][145] shows:
Figure 37. (a) Three-dimensional representation of the structure, (b) XZ view, (c) XY top view of the PFZP’s, (d) XY bottom view of the structure, and (e) a dipole nanoantenna and stub-loaded gold dipole nanoantenna (SLD nanoantenna). An Au nanoantenna permittivity ($\varepsilon_{Au}$) at ($\lambda = 7.8 \mu m$) is $\varepsilon_{Au} = -2806.57 + j720$.

$$\varepsilon = \varepsilon_{\infty} \left( 1 - \frac{\omega^2_p}{\omega(\omega + i\gamma)} \right) = \varepsilon' + j\varepsilon''$$  \hspace{1cm} (5.1),

where $\omega_p$ is the plasma frequency, $\gamma$ is the electron relaxation, and $\varepsilon_{\infty}$ is the high-frequency permittivity limit. The Drude model parameters for doped InAs and undoped InAs were taken from Peragut et al. [106]. For undoped InAs, $\omega_p = 3 \times 10^{13} \text{rad} \cdot \text{s}^{-1}$, $\gamma = 10^{12} \text{rad} \cdot \text{s}^{-1}$ and $\varepsilon_{\infty} = 12.3$, which corresponds to a
residual doping concentration of $8 \times 10^{16}$ cm$^{-3}$. Likewise, for doped InAs, $\omega_p = 2.95 \times 10^{14}$ rad s$^{-1}$, $\gamma = 10^{13}$ rad s$^{-1}$, and $\varepsilon_\infty = 10.4$, which corresponds to a concentration level (silicon) of $3.5 \times 10^{19}$ cm$^{-3}$ [104], [106]. The real ($\varepsilon'$) and imaginary ($\varepsilon''$) parts of the permittivities of undoped InAs and doped InAs are shown in Figure 37.

The HMM provides a dispersion that has a relationship between the frequency and the wavevector. Typically, the dispersion relation of SPP propagating along the interface between a metal and dielectric can be found by surface mode solutions of Maxwell’s equations [56]:

$$k_{SPP} = \frac{\omega}{c} \sqrt{\frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}}$$  \hspace{1cm} (5.2),

where $k_{SPP}$ is the SPP wavevector, $\omega$ is the angular frequency of the wave, $c$ is the speed of light, $\varepsilon_d$ is the relative permittivity of the dielectric, and $\varepsilon_m$ is the relative permittivity of the metal [146][56].

HMMs can be achieved by alternating thin metal layers and dielectrics. The thickness of the layer is much less than the operating wavelength. HMMs are considered uniaxial metacrystals with a dielectric response given by a dielectric tensor $\varepsilon = \text{diag} [\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}]$. Such metamaterials exhibit hyperbolic dispersion when one component has the opposite sign in permittivity to the other two principal
components. Characteristic properties of HMMs include strong enhancement, a larger density of electromagnetic states, negative refraction, and enhanced focusing effects [147], [69]. Type I HMMs have one term of the dielectric tensor that is negative \([\varepsilon_{xx}, \varepsilon_{yy} > 0, \varepsilon_{zz} < 0]\) whereas Type II HMMs have two terms of the dielectric tensor that are negative \([\varepsilon_{xx}, \varepsilon_{yy} < 0, \varepsilon_{zz} > 0]\) [147]. Moreover, the HMMs behave like metals when all three terms of the

![Figure 38](image)

**Figure 38.** The real and imaginary parts of the relative permittivity of undoped InAs (green line) and doped InAs (red line) are plotted as a function of the wavelength from 6 to 15 µm. The marked black dot indicates the start of the metallic behavior of the doped InAs.
dielectric tensor are negative \([\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz} < 0]\), and like a dielectric, when all three terms are positive \([\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz} > 0]\) [148].

The dispersion relation of a uniaxial material is found by using the following equation:

\[
\frac{k_\parallel^2}{\varepsilon_\perp} + \frac{k_\perp^2}{\varepsilon_\parallel} = \frac{\omega^2}{c^2} \tag{5.3}
\]

where \((\perp)\) denotes parallel components and \((\parallel)\) represents perpendicular components.

According to an EMT, \(\varepsilon_\perp\) and \(\varepsilon_\parallel\) are calculated by the following equations [147]:

\[
\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{\parallel} = p\varepsilon_m + (1 - p)\varepsilon_d = \varepsilon_{xx}' + j\varepsilon_{xx}'' \tag{5.4}
\]

and

\[
\varepsilon_{zz} = \varepsilon_{\perp} = \frac{\varepsilon_m\varepsilon_d}{p\varepsilon_d + (1 - p)\varepsilon_m} = \varepsilon_{zz}' + j\varepsilon_{zz}'' \tag{5.5},
\]

where \(\varepsilon_{xx}'\) is the real part of the parallel permittivity and \(\varepsilon_{zz}'\) is the real part of the perpendicular permittivity, and \(\varepsilon_{xx}''\) is the real imaginary of parallel permittivity and \(\varepsilon_{zz}''\) is the real imaginary of the perpendicular permittivity. The thicknesses of a metallic layer doped InAs and undoped InAs are \(t_m\) and \(t_d\), respectively. The fill fraction \((p)\) is equal to \(p = \frac{t_m}{t_m + t_d}\). The permittivity of doped InAs is \(\varepsilon_m\) and the permittivity of undoped InAs is \(\varepsilon_d\). In this study, we use \(t_m = 290 \text{ nm}\) and \(t_d = 370 \text{ nm}\), referring to [106]. Figure 38 shows three different behaviors, Type I in the
wavelength range 6.4 to 8.84 µm, metal-like in the range 8.85 to 10.04 µm, and Type II HMM in 10.04 to 15 µm.

Determining the effective refractive index \( (n_{\text{eff}}) \) of the HMM based on EMT mentioned above is achieved by calculating the angle \( (\theta) \) between the propagation direction and the optical axis [149]:

\[
\theta = \arctan \left( \sqrt{\frac{\varepsilon'_{xx}}{\varepsilon'_{zz}}} \right) \tag{5.6}
\]

\[
n_{\text{eff}} = \left( \sqrt{\frac{\cos^2(\theta)}{\varepsilon'_{xx}} + \frac{\sin^2(\theta)}{\varepsilon'_{zz}}} \right)^{-1} \tag{5.7}
\]

The permittivity of the HMM at the wavelength \( (\lambda_0 = 7.8 \, \mu\text{m}) \) is \( \varepsilon_{xx} = 2.478 + j0.36 \) and \( \varepsilon_{zz} = -13.44 + j2.52 \). \( \lambda_{SPP} = 5.34 \, \mu\text{m}, \theta = 23.35^\circ \), and the \( (n_{\text{eff}}) \) at the same wavelength is \( n_{\text{eff}} = 1.7 \) and \( \varepsilon_{\text{eff}} = (n_{\text{eff}})^2 = 2.9 \). The effective wavelength can be computed by using \( \lambda_{\text{eff}} = \frac{\lambda_0}{n_{\text{eff}}} = 4.63 \, \mu\text{m} \). The PFZPs’ rings in Figure 37(c) are made of rings of doped InAs, and their outer diameter \( D \) is 28.27 µm, and their thickness is 2.93 µm. The permittivity of doped InAs is \( \varepsilon_{\text{doped InAs}} = -5.07 + j0.64 \). Therefore, the focal point and radius of PFZPs can be calculated [146],[150],[151]:

73
Figure 39. The effective permittivities (real and imaginary parts) for HMMs (undoped InAs/doped InAs) with normal and parallel direction.

\[ r_m = \sqrt{\left(\frac{m\lambda_{\text{eff}}}{q}\right)^2 + \frac{2mf\lambda_{\text{eff}}}{q}} \]  

where \( r_m \) is the radius of the PFZPs for ring number \( m =1 \) to 6, \( f \) is the focal length \( (f = 13.43 \mu m) \) inside the HMM, and \( q \) is the phase correction step number (in this calculation \( q = 4 \), indicating a quarter wavelength) [150].

It should be mentioned that Type I HMM is used to design the structure and the radii of the PFZPs in Figure 37 for optimized values.
Table 1. Calculation of Radii of the PFZPs

<table>
<thead>
<tr>
<th>No.</th>
<th>r1</th>
<th>r2</th>
<th>r3</th>
<th>r4</th>
<th>r5</th>
<th>r6</th>
</tr>
</thead>
<tbody>
<tr>
<td>r(µm)</td>
<td>3.14</td>
<td>4.73</td>
<td>6.13</td>
<td>7.45</td>
<td>8.72</td>
<td>9.96</td>
</tr>
</tbody>
</table>

To minimize the reflection and maximize the transmission of the wave on the interface between an HMM and a spacer, as shown in Figure 37(b), we use the constructive interference theory with refractive index $n_{sp} \approx 1.52$ and thickness ($Lu$) of $\frac{\lambda_{sp}}{2n_{sp}} = 1.77 \, \mu m$ for the spacer [152].

We compared the performance of the five configurations:

i. Hyperbolic metalens with SLD nanoantenna (PFZP/HMM/spacer/SLD ANT.).

ii. Homogenous metalens-undoped InAs with SLD nanoantenna (PFZP/Hom/spacer/SLD ANT.).

iii. Homogenous metalens-Air with SLD nanoantenna (PFZP/air/spacer/SLD ANT). 


v. Hyperbolic metalens without antenna (PFZP/HMM/spacer).
This study focuses on two different nanoantennas, as seen in Figure 37(e). Both dipole (Dip) and stub-loaded dipole (SLD) nanoantennas consist of two identical Au arms, and an air gap separates each nanoantenna (g), which equals 0.01 µm. The length of the arm (L1 = 1.7 µm) is approximately equal to $\frac{\lambda_{SPP}}{2n_{sp}}$, and its diameter is d = 0.06 µm. Additionally, the SLD nanoantenna has two perpendicularly placed stubs (d1 and d2) in each arm.

### 5.2 Numerical Results and Discussion

We emphasize the investigation of the underlying strength of electric-field intensity (EFI) enhancement of the nanoantenna. This enhancement is defined by the ratio of the maximum intensity of the electric field ($|E|_{max}^2$) to the intensity of the incident wave ($|E_0|^2$) [153][154].

The simulation results in Figure 39 demonstrate that the largest EFI (12 orders of magnitude) is achieved by our proposed structure, case (i), at the operating wavelength ($\lambda_o = 7.8 \, \mu m$) as compared to the other configurations (ii ~ v). Since the incident wave was scattered by the PFZP into the HMM, consisting of propagating and evanescent modes, the HMM supports the propagating waves and converts the evanescent waves into propagating waves. Therefore, the HMM improves the EFI as noticed by comparing the (i) and (ii) configuration. This feature for HMM is attributed to uniaxial metacrystals Type I [$\varepsilon_{xx}, \varepsilon_{yy} > 0, \varepsilon_{zz} < 0$]. Likewise, a
A comparison between the (i), (ii), and (iii) configurations shows the difference between the HMM and other isotropic homogeneous media which support propagating waves only without evanescent waves.

Figure 40. Comparison of EFI enhancement for different configurations: i- PFZP/HMM/spacer/SLD ANT. (red curve), ii- PFZP/Hom/spacer/SLD ANT (cyan curve), iii- PFZP/air/spacer/SLD ANT. (magenta curve), iv- PFZP/HMM/spacer/Dip ANT. (green curve), and v- PFZP/HMM/spacer (dashed blue curve).
Then, the homogenous media are contributed less than HMMs to enhance the EFI. In terms of nanoantennas, our suggested antenna (SLD) illustrates better enhancement compared to the regular Dip as seen in curves (i) and (iv). The curve (v) also depicts that the enhancement without any antenna is at least $10^8$ times less than the EFI with an antenna. In fact, the contribution of plasmonic excitations on the antenna is the main reason behind this massive improvement of the EFI. Besides, the comparison between (i), (iv), and (v), as shown in Figure 39, is enriched with the electric field distributions, as shown in Figure 40. The scenario of field distribution, $E$, inside the HMM without any antenna, as shown in Figure 40(a), illustrates how the focusing phenomena, based on the propagating far-field takes place before the localized hotspot interacts with the plasmons of the nanoantenna at the spacer/air interface.

On the other hand, the existence of a nanoantenna with the metalens-structure is represented in Figure 40(b) and (c) for dipole- and SLD-antennas, respectively. For instance, the interaction between the hotspot and the SLD nanoantenna arises from the near-field coupling between the two nanoantenna arms’ SPPs. This coupling occurs at the focal spot with spatial extent comparable to the gap of the antenna ($\frac{FWHM}{\lambda} = \frac{1}{1000}$). FWHM of the electric field distribution in the cross-section through the gap of the nanoantenna [155] [156]. Moreover, changing the gap between the SLD nanoantenna's arms can affect the relevant interaction, as seen in Figure 39.
Figure 41. (a) $|E|$–field distributions of the metalens without a nanoantenna at the wavelength of $7.8 \mu m$, Case (v) in Figure 37, (b) $|E|$–field distributions of the hybrid structure at the wavelength of $7.89 \mu m$ for a dipole nanoantenna with a dipole-gap of $10 nm$, Case (iv) in Figure 40, and (c) $|E|$–field distributions of the hybrid structure at the wavelength of $7.8 \mu m$ for an SLD nanoantenna with different gaps of $8.3$, $10$, and $11.4 nm$, Case (i) in Figure 39.
As a result, the electromagnetic field is concentrated in the gap between the nanoantennas’ arms, which leads to high field enhancement. The HMM metalens, (see Figure 40(a), has a focal length of \( Lh + \left( \frac{Lu}{2} \right) = 13.43 \mu m \), where \( Lh \) (12.55 \( \mu m \)) is the height of the HMM and \( Lu \) (1.77 \( \mu m \)) is the height of the spacer. Also, HMM has an optical spot along the x-axis, with a spot-width spot-width \( (w_0) \) of 2 \( \mu m \). Notably, PFZP’s height is 2.93 \( \mu m \) in all cases in Figure 40.

Table 2 shows a comparison of EFIs with numerous investigations. Notably, introducing the doped InAs and undoped InAs in the metalens with two discs shows six orders of EFIs. Our study shows that the coupling system can be exploited to maximize the EFI. It reached eight orders for the dipole nanoantenna and 12 orders of EFIs for the SLD nanoantenna with a 0.01 \( \mu m \) gap. The significant improvement of the matching of far-field with the near-field of nanoantennas makes the EFI extremely high. The coupling system confines the electric field below the diffraction limit, which is defined by the gap nanoantenna. The EFI is enhanced by coupling the focal spot fields of the HMM metalens and SPP of the nanoantenna, resulting in a super EFI. The HMM metalens converges the propagating and evanescent waves into the gap of the nanoantenna. The plasmonic nanoantennas can converge this electric field into deep nanoscales and create strong local electric near field enhancements at the gap resulting from LSPR [157], [158]; consequently, the hybrid system shows an ultrahigh EFI enhancement in the air gap.
Table 2. State-of-the-Art Intensity Enhancement with Different Systems in MIR.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>$\lambda_0$ ($\mu m$)</th>
<th>Structure</th>
<th>Orders EFI</th>
<th>Ref. (year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Au, Si substrate</td>
<td>5.88</td>
<td>Nanoantenna</td>
<td>3</td>
<td>[137] (2015)</td>
</tr>
<tr>
<td>Doped InAs, InAs, Au</td>
<td>7.8 -8.8</td>
<td>Two Discs</td>
<td>6</td>
<td>[139] (2019)</td>
</tr>
<tr>
<td>Doped InAs, InAs Metalens, Au</td>
<td>7.8</td>
<td>Dipole</td>
<td>8</td>
<td>Our Work</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nanoantenna</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doped InAs, n=3.46, Metalens, Au</td>
<td>7.8</td>
<td>SLD</td>
<td>10</td>
<td>Our Work</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nanoantennas</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doped InAs, InAs, Metalens, Au</td>
<td>7.8</td>
<td>SLD</td>
<td>12</td>
<td>Our Work</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nanoantennas</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The proposed hybrid system with such high enhancement (EFI) can provide a wide range of potential applications in the nanoscale field for light-matter interaction. Applications such as photovoltaics, solar cells [161] [162], nanoscale imaging, and spectroscopy [163] could be targeted based on scattering and absorption properties.

We demonstrate the possibility of controlling the SCS and (ACS) with three different heights of the PFZP \(h=0.07, 0.42, \) and \(2.93 \mu m\). The generated diffractive plasmonic waves from the PFZP excite the HMM before the focal spot, and localized modes are created to interact with the Au nanoantenna.

The dimensions are optimized in a way that (SCS > ACS), (SCS < ACS), or (SCS = ACS) to maintain the high field intensity (EFI). The dimensions increase or decrease the scattering weight and bring the system in specific features of both cross-sections, obtaining a critical coupling, an under coupling, and an over coupling. These features can be efficiently fit for practical applications. For most guiding applications, the plasmonic structures do not require high absorption. In contrast, in other applications, such as imaging, it is significant that the IR light is needed to be absorbed instead of scattered.

Moreover, when the scattering and absorption cross-sections are equal, we expect that the system can also be utilized in sensitive sensing applications. The
fundamental optical properties of our system, namely the ACS, the SCS (dashed lines) in the case of the hybrid metalens, and the SLD nanoantenna system are presented in Figure 41(a), where we plotted the absorption and scattering cross-sections of different heights of the PFZPs. The scattering cross-section is found as

$$ scs = \frac{1}{I_0} \iint (n \cdot S) \, ds $$

(5.9),

where $I_0$ is the intensity of the wave, $n$ is the normal vector pointing, and $S$ is the Poynting vector. The absorption cross-section is defined as

$$ AcS = \frac{1}{I_0} \iiint Q \, dV $$

(5.10)

where $Q$ is the power-loss density, integrated over the volume of the structure [164] [165].

The structure in Figure 41(a) displays a strong scattering (dashed red curves) and low absorption (red curves), which indeed is the expected of the optical response for 2.93 $\mu m$ of the PFZP heights due to the scattering of the incident wave, which increases with the increase of the height of the PFZP. This case is called under coupling. On the contrary, the blue curves ($h = 0.07 \mu m$) in Figure 41(a), indicate high peak absorption (ACS = 780 $\mu m^2$) and low scattering (SCS= 450 $\mu m^2$) at the operating resonant wavelength (7.8 $\mu m$). The system, in this case, is designed to operate by over coupling. Moreover, in a situation where the crucial coupling is assumed, when
the PFZPs height is equal to 0.42 $\mu m$, the SCS and ACS become the same (SCS = ACS = 660 $\mu m^2$) at 7.81 $\mu m$ of the operating resonant wavelength, and the SCS shows two peaks (1200 $\mu m^2$ and 900 $\mu m^2$ at different wavelengths of 7.4 and 8.1 $\mu m$).

The ratio between the cross-sections depicted in Figure 42(b) can be applied as a measure to assess the three different coupling behaviors, allowing us to indicate the best potential applications [164]. When the cross-sections are equal (SCS/ACS = 1) at the resonant wavelength of 7.8 $\mu m$, the critical coupling occurred, as shown in the green curve. Furthermore, although the coupling (SCS/ASC < 1) is over coupling at the resonant wavelength in the blue curve in Figure 41 (b), the three crucial coupling points (SCS/ASC=1) shown are related to the four different wavelengths ($\lambda = 7.8, 8, 8.3,$ and $8.6 \mu m$). Furthermore, in the under coupled case, the ratio of the scattering and absorption is higher than 1 (SCS/ASC > 1), and scattering dominates as the red curve [Figure 41(b)].

The EFI is presented in Figure 41(c), where we plot the EFI for three different PFZP when heights (green curve when $h = 0.07 \mu m$), EFI (red curve when $h = 0.42 \mu m$), and EFI (blue curve when $h = 2.93 \mu m$). Remarkably, the EFI gives ultrahigh enhancement, reaching a peak resonance of $1.21 \times 10^{12}$ at $h = 0.07 \mu m$, $1 \times 10^{12}$ at $h = 2.93 \mu m$, and $6.5 \times 10^{11}$ at $h = 0.42 \mu m$. 

84
Figure 42. (a) Absorption cross-section (ACS) and scattering cross-section (SCS) (dashed lines) of the hybrid metalens and SLD gold nanoantenna system with PFZPs rings heights, $h = 0.07 \, \mu m$ (blue), $0.42 \, \mu m$ (green), and $2.93 \, \mu m$ (red). (b). The ratio of the scattering and absorption (c). EFI of the hybrid system with the PFZPs ring heights.

5.3 Summary

A hybrid system of a nanoantenna and metalens increases the capacity of creating an ultrahigh near-field enhancement by using PFZPs as plasmonic waveguides, HMMs
(doped and undoped InAS) for high wavevectors, and substrate SLD nanoantennas as plasmonic nanoantennas. The system uniquely manipulates MIR properties at the nanoscale and overcomes the diffraction limit to 1/1000 of the operating wavelength. Furthermore, the consequence of the geometric parameters of the SLD nanoantenna with a metalens on the enhancement was ultrahigh, achieving $10^{12}$. Our system's field enhancement is an essential parameter for potential applications such as sensitive detection and surface-enhanced Raman spectroscopy. Moreover, further applications will be found in ultrasensitive sensors, biosensing, and photonic devices, including a promising way to improve the efficiency of solar cell structures.
Chapter 6:
Conclusion

In infrared applications, using conventional lenses to focus the waves and noble metals to guide the propagating and evanescent waves is limited because of the diffraction limit, low resolution, high optical losses, lack of tunability, and low intensity enhancement. In both NIR and MIR, alternative plasmonic metamaterials (AMMs), plasmonic waveguide couplers (PWCs) or plasmonic Fresnel zone plates PFZPs, HMMs and nanoantennas are used.

AMMs are tunable materials which have been employed to realize useful behavior in the control of electromagnetic waves, and to reduce the optical losses. The plasmonic waveguide coupler (PWCs) and the Plasmonic Fresnel zone plates (PFZPs) have been designed for sub-diffraction limit and excitation of SPPs. The HMM allows us to break the diffraction limit and convert evanescent waves to propagating waves. Together the PWC or PFZPs and the HMM slab make metalens that serve to focus incoming radiation beyond a diffraction limit onto a focal spot. We achieve high field intensity enhancement by optimizing the structure and coupling between the electric field spot of the metalens and SPPs of nanoantenna.

In the NIR band from 1 \( \mu m \) to 3 \( \mu m \), a novel metalens is designed which consists of a stack made of a PWC (ITO), a slab of HMM (ITO/Si), and a silver nanodiple
embedded in a substrate of silicon. Electric field intensity enhancement of $10^6$ is achieved around the silver nanodipole with highly sub-diffracational focusing is achieved with up to $0.01667 \lambda$ maximum achieved resolution.

In the MIR at 8 $\mu$m, a novel metalens is designed, which consists of a PWC (doped InAs), an HMM (doped InAs/undoped InAs), and an Au plasmonic nanodisk. The electric field at the focal point is enhanced with the FWHM of 8.18 nm and 13.64 nm on the x-axis and y-axis, respectively. This allows us to image particles that are 1/1000 times the wavelength. Such metalens may be used for sub-wavelength imaging and sensing with super-resolution and high electric field enhancement.

At 7.8 $\mu$m, a hybrid system of a nanoantenna and metalens increases the capacity of creating an ultrahigh near-field enhancement. The system uniquely manipulates MIR properties at the nanoscale and overcomes the diffraction limit to 1/1000 of the operating wavelength. Furthermore, the consequence of the geometric parameters of the SLD nanoantenna with a metalens on the enhancement was ultrahigh, achieving $10^{12}$. Our system's field enhancement is an essential parameter for potential applications such as sensitive detection and surface-enhanced Raman spectroscopy. Moreover, further applications will be found in ultra-sensitive sensors, biosensing, and photonic devices, including a promising way to improve the efficiency of solar cell structures.
References


Aizpurua, “Resonant plasmonic and vibrational coupling in a tailored nanoantenna

[159] E. Sakat, L. Wojszvzyk, J.-P. Hugonin, M. Besbes, C. Sauvan, and J.-J. Greffet,
“Enhancing thermal radiation with nanoantennas to create infrared sources with

[160] L. V Brown, X. Yang, K. Zhao, B. Y. Zheng, P. Nordlander, and N. J. Halas, “Fan-
shaped gold nanoantennas above reflective substrates for surface-enhanced infrared

nanoantennas for higher harmonic generation,” ACS Nano, vol. 6, no. 4, pp. 3537–
3544, 2012.

[162] A. Chekini, S. Sheikhaei, and M. Neshat, “Nanoantenna arrays as diode-less

“Nanoplasmonic mid-infrared biosensor for in vitro protein secondary structure

[164] N. Maccaferri et al., “Hyperbolic meta-antennas enable full control of scattering

[165] A. A. Alsalhin, M. F. Finch, and B. A. Lail, “Coupling between metallic structure
and phonon polaritons for sensing applications,” in Metamaterials, Metadevices,
Appendix

Optical properties of metals

clc
close all

Lmba1=1e-6:.0001e-6:11e-6;
F=3e8./(Lmba1);
W1=2*pi.*F,
Wn=W1./(3*2*pi*1e10)
Wpwn=7.25e4
gwn=2.16e2
Wp1=(Wpwn)*2*pi*3e10
g1=(gwn)*2*pi*3e10
wf=806
Edo1=wf-((Wp1.^2)./(W1.^2+1i.*W1.*g1));Au=Edo1
Edown=wf-((Wpwn.^2)./(Wn.^2+1i.*Wn.*gwn));Au_wn=Edown

figure(1)
title('Au')
xlabel('W'); ylabel('Permittivity ');
plot(Lmba1,real(Au),'b',Lmba1,imag(Au),'b--','LineWidth',3);legend('Real Au',' Imaginary Au ')
EE= [Lmba1' real(Au)' imag(Au)']

figure(2)
title('Au')
xlabel('W'); ylabel('Permittivity ');
plot(Wn,real(Edown),'b',Wn,imag(Edown),'b--','LineWidth',3);legend('Real Au_wn',' Imaginary Au_wn ')
EE= [Wn' real(Edown)' imag(Edown)']

107
gwn=2.78e2
    Wp1=(Wpwn)*2*pi*3e10
    g1=(gwn)*2*pi*3e10
    wf=2000
    Edo1=wf-((Wp1.^2)./(W1.^2+1i.*W1.*g1));Cu=Edo1
    Edown=wf-((Wpwn.^2)./(Wn.^2+1i.*Wn.*gwn));Cu_wn=Edown

figure(3)
title('Cu')
xlabel('W'); ylabel('Permittivity');
plot(Lmba1,real(Edo1),'b',Lmba1,imag(Edo1),'b--','LineWidth',3);legend('Real Cu', 'Imaginary Cu ')
EE=[Lmba1' real(Cu)' imag(Cu)']

figure(4)
title('Cu')
xlabel('W'); ylabel('Permittivity');
plot(Wn,real(Edown),'b',Wn,imag(Edown),'b--','LineWidth',3);legend('Real Cu_wn', 'Imaginary Cu_wn ')
EE=[Wn' real(Cu_wn)' imag(Cu_wn)']

figure(5)
title('Al')
xlabel('W'); ylabel('Permittivity');
plot(Lmba1,real(Edo1),'b',Lmba1,imag(Edo1),'b--','LineWidth',3);legend('Real Al', 'Imaginary Al')
EE=[Lmba1' real(Al)' imag(Al)']

figure(6)
title('Al')
xlabel('W'); ylabel('Permittivity');
plot(Wn,real(Edown),'b',Wn,imag(Edown),'b--','LineWidth',3);legend('Real Al_wn',' Imaginary Al_wn')

EE= [Wn' real(Al_wn)'  imag(Al_wn)']

Lmba1=1e-6:0.001e-6:11e-6;
F=3e8./(Lmba1);
W1=2*pi.*F,
Wn=W1./(3*2*pi*1e10)
Wpwn=6.20e4
gwn=1.45e3
    Wp1=(Wpwn)*2*pi*3e10
g1=(gwn)*2*pi*3e10
    wf=50
    Edo1=wf-((Wp1.^2)./(W1.^2+1i.*W1.*g1));Pd=Edo1
    Edown=wf-((Wpwn.^2)./(Wn.^2+1i.*Wn.*gwn));Pd_wn=Edown
figure(7)
title('Pd')
xlabel('W'); ylabel('Permittivity ');
plot(Lmba1,real(Pd),'b',Lmba1,imag(Pd),'b--','LineWidth',3);legend('Real Pd',' Imaginary Pd')

EE= [Lmba1' real(Pd)'  imag(Pd)']
figure(8)
title('Pd')
xlabel('W'); ylabel('Permittivity ');
plot(Wn,real(Edown),'b',Wn,imag(Edown),'b--','LineWidth',3);legend('Real Pd_wn',' Imaginary Pd_wn')

EE= [Wn' real(Pd_wn)'  imag(Pd_wn)']
clc close all

Lmba=.5e-6:.005e-6:12e-6;
F=3e8./(Lmba);
W=2*pi*F;
Wp=2.3765e15;
g=9.710e13;
EAZO=3.8-((Wp.^2)./(W.^2+1i.*W.*g));

gg=1.4929e14;
Wpg=2.3867e15;
EGZO=3.8-((Wpg.^2)./(W.^2+1i.*W.*gg));

gI=1.2836e14;
WpI=2.3822e15;
EITO=3.8-((WpI.^2)./(W.^2+1i.*W.*gI));

kk=char(949)
figure(1)
plot(1e6.*Lmba,real(EAZO),'r',1e6.*Lmba,real(EGZO),'b',1e6.*Lmba,real(EITO),'g','LineWidth',3);legend( 'AZO','GZO','ITO')
xlabel('Wavelength ['\ mum ']'); ylabel('Real Permittivity')

kk=char(949)
figure(2)
plot(1e6.*Lmba,imag(EAZO),'--r',1e6.*Lmba,imag(EGZO),'--b',1e6.*Lmba,imag(EITO),'--g','LineWidth',3);legend('AZO','GZO','ITO')
xlabel('Wavelength ['\ mum ']'); ylabel('Imaginary Permittivity')
hold on

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
doi:10.1088/0022-3727/40/22/043
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Lmba1=0.2e-6:.005e-6:12e-6;
F1=3e8./(Lmba1);
W1=2*pi.*F1,

Wp1=1.3064e+16\%1.2412e+16 rad/sec
\[ g_1 = 1.1274 \times 10^{14} \text{ rad/sec} \%
\]
\[ \text{wf} = 1.030 \%
\]
\[ \text{DE} = 2.07122 \%
\]
\[ \text{WL} = 4.66171 \times 10^1 \%
\]
\[ g_L = 7.20958 \times 10^{13} \%
\]
\[ E_{do1} = \text{wf} - \left( \frac{Wp1.2}{W1.2 + \text{i} \cdot W1 \cdot g_1} \right) - \left( \frac{\text{DE} \cdot \text{WL}^2}{W1.2 - \text{WL}^2 + \text{i} \cdot g_L \cdot W1} \right); \]
\[ A_u = E_{do1} \]
\[ \text{Edo1} = \text{wf} - \left( \frac{Wp1.2}{W1.2 + \text{i} \cdot W1 \cdot g_1} \right), A_u = \text{Edo1} \]

figure(1)
% title( 'Au')
xlabel('Wavelength [' \text{\textmu m}]');
ylabel('Permittivity ');

plot(1e6 * Lmba1, real(Au), 'b', 1e6 * Lmba1, imag(Au), 'b--', 'LineWidth', 3); legend('Real Au', ' Imaginary Au')

Wp1 = 8.8128 \times 10^{15};
g1 = 3.8828 \times 10^{14}
wf = 1.1297
Edol = \text{wf} - \left( \frac{Wp1.2}{W1.2 + \text{i} \cdot W1 \cdot g_1} \right); C_r = \text{Edo1};

% figure(3)
% title( 'Cr')
% xlabel('W '); ylabel('Permittivity ');
% plot(1e6 * Lmba1, real(Edo1), 'c', 1e6 * Lmba1, imag(Edo1), 'c--', 'LineWidth', 3); legend('Real Cu', ' Imaginary Cu')

Wp3 = 2.0598 \times 10^{16}
g3 = 2.2876 \times 10^{14};
wf = 1.000;
\[ E_{do3} = \text{wf} - \left( \frac{Wp3.2}{W1.2 + \text{i} \cdot W1 \cdot g3} \right); A_l = E_{do3} \]

% figure(5)
% title( 'Al')
```matlab
% xlabel('W '); ylabel('Permittivity '); %
plot(1e6.*Lmba1,real(Edo1),'k',1e6.*Lmba1,imag(Edo1),'k--','LineWidth',3);legend('Real Al',' Imaginary Al')

Wp4=1.3354e16
g4=9.6875e13
wf=3.7325;
Edo4=wf-((Wp4.^2)./(W1.^2+1i.*W1.*g4));Ag=Edo4;

% figure(7)
% title('Pd')
% xlabel('W '); ylabel('Permittivity ''); %
plot(1e6.*Lmba1,real(Pd),'b',1e6.*Lmba1,imag(Pd),'b--','LineWidth',3);legend('Real Pd',' Imaginary Pd')

% EE= [Lmba1' real(Pd)'  imag(Pd)'];
figure(3)
plot(1e6.*Lmba1,real(Au),'m',1e6.*Lmba1,real(Cr),'c',1e6.*Lmba1,real(Al),'k',1e6.*Lmba1,real(Ag),'y','LineWidth',3)
xlabel('Wavelength [ \mum ]'); ylabel('Real Permittivity');legend('Real Au','Real Cr','Real Al','Real Ag');
figure(4)
plot(1e6.*Lmba1,imag(Au),'m-',1e6.*Lmba1,imag(Cr),'c--',1e6.*Lmba1,imag(Al),'k-',1e6.*Lmba1,imag(Ag),'y--','LineWidth',3);
xlabel('Wavelength [ \mum ]'); ylabel('Imaginary Permittivity');legend('Imaginary Au','Imaginary Cr','Imaginary Al','Imaginary Ag')

figure(5)
plot(1e6.*Lmba1,real(Au),'m',1e6.*Lmba1,real(Al),'k ',1e6.*Lmba1,real(Ag),'g','LineWidth',3)
```
xlabel('Wavelength [\mum ]'); ylabel('Real Permittivity'); legend('Real Au', 'Real Al', 'Real Ag');
figure(6)
plot(1e6.*Lmba1, imag(Au), 'm-', 1e6.*Lmba1, imag(Al), 'k--', 1e6.*Lmba1, imag(Ag), 'g-', 'LineWidth', 3);
xlabel('Wavelength [\mum ]'); ylabel('Imaginary Permittivity'); legend('Imaginary Au', 'Imaginary Al', 'Imaginary Ag')