Control Laws for Autonomous Landing on Pitching Decks

by

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Abstract

Title: Autonomous Landing of a Quadcopter onto a Pitching Deck

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This thesis addresses the problem of landing a quadcopter onto the pitching deck of a ship, as a partial simulation of a shipboard landing during high seas. The deck is modeled to pitch with a sinusoidal motion. While shipboard landing consists of many stages, only the final landing is simulated in this research. Other ship motions, such as roll and heave, are likewise not considered. The goal is to land the quadcopter in the center of the deck with minimal position and pitch error, where position error measures how far fore or aft of the center of the landing pad the quadcopter lands, and pitch error measures the difference between the pitch of the quadcopter and the pitch of the deck at touchdown. Multiple landing strategies are tested in numerical simulations using Proportional Derivative (PD) and Zero Effort Miss/Zero Effort Velocity (ZEM/ZEV) control techniques. The PD controller is designed to match the pitch of the quadcopter with the pitch of the deck just before landing. The PD controller is shown to be able to reduce the pitch error significantly, but at the cost of position error. The ZEM/ZEV controller is designed to target landing on the ship when the pitch of the deck crosses zero. The ZEM/ZEV controller is shown to be able to land with high accuracy in position and pitch as long as the timing of when the deck would be horizontal was known. A mixture of ZEM/ZEV and PD control is able to land the quadcopter with low pitch and position error even during the presence of timing errors. The simulation results serve to define test conditions for a future experimental validation of the control laws in the FIT ORION Lab.
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Chapter 1
Introduction

Long Island Sound, May 6th, 1943: Captain Frank Gregory lands his Sikorsky R-4 helicopter onto the deck of the tanker SS Bunker Hill. The event made history as the first helicopter to land on a moving ship, and ushered in new possibilities for naval aviation [1].

![First shipboard helicopter landing](image)

**Figure 1**: First shipboard helicopter landing [1].

The helicopter was quickly found to excel in search and rescue, especially at sea. In 1947, while demonstrating the flight capabilities of the Sikorsky S-51 for the U.S. Navy, chief test pilot Jimmy Viner had a fortuitous moment. During the demonstration, the carrier USS *Franklin D. Roosevelt* was recovering aircraft when one of the planes had an engine failure and crashed into the ocean. Viner saw the impending crash, swiftly maneuvered his helicopter above the downed aircraft, and rescued the pilot and radioman from the ocean. Viner had performed the first air-sea rescue using a helicopter, and the U.S. Navy had
witnessed it [2]. Helicopters would go on to play important roles in troop transport, anti-submarine warfare, and search and rescue for the U.S. Navy as well as the Coast Guard. Table 1 shows that in the period 2012-2016, the Coast Guard has dealt with over 88,000 search and rescue cases and saved over 21,000 lives [3]. Helicopters serve a key role in search and rescue operations, as they can search larger areas more quickly than boats.

Table 1: U.S. Coast Guard Search and Rescue Statistics

<table>
<thead>
<tr>
<th>Year</th>
<th>Cases</th>
<th>Lives Saved</th>
<th>Lives Lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>19,856</td>
<td>4,102</td>
<td>655</td>
</tr>
<tr>
<td>2013</td>
<td>17,826</td>
<td>3,773</td>
<td>608</td>
</tr>
<tr>
<td>2014</td>
<td>17,555</td>
<td>3,900</td>
<td>597</td>
</tr>
<tr>
<td>2015</td>
<td>17,302</td>
<td>3,930</td>
<td>605</td>
</tr>
<tr>
<td>2016</td>
<td>16,298</td>
<td>5,450</td>
<td>626</td>
</tr>
</tbody>
</table>

The activities of navies and coast guards around the world are not limited to fair weather operations. Therefore, helicopters have to be able to fulfil their roles in all weather conditions and all sea states. Shipboard landing during hazardous sea states presents a serious challenge for the pilot. Not only is the pilot concerned with landing on a translating surface, but the flight deck is also pitching and rolling. Conditions can be so severe that shipboard helicopter operations are impossible or present a significant risk for the flight crew. For commanders of search and rescue operations, this can mean a difficult choice: put the lives of the helicopter and deck crews in danger, or give up the search. One solution to this problem is to use unmanned aerial vehicles (UAVs) instead of crewed helicopters. Although UAVs allow search and rescue operations to continue without putting a flight crew at risk, the aircraft still faces the challenging task of landing during turbulent seas.

The main difficulties in shipboard landings are landing at the center of the deck, and at a point when the deck is horizontal. The focus of this research is the development and numerical simulation of control laws for autonomous landing of a quadrotor UAV (quadcopter) onto the pitching deck of a ship. First, a mathematical model of the motion
dynamics of quadcopter is created in MATLAB Simulink. Using this model, a guidance and control algorithm is developed and tested in numerical simulations for landing on a pitching deck.

1.1 Piloted Shipboard Landing Procedures

In order to develop an autonomous landing system, the current piloted methods for shipboard landing must be studied [4] [5]. The U.S. military has published extensive documentation on the subject. As a helicopter approaches for a landing, the pilot is informed by the ship of the wind speed and direction over the deck, pitch and roll of the ship, and the heading of the ship [5]. From this point on, the ship will not change heading unless absolutely necessary, and the helicopter will continue to approach the ship until it enters the range of the glideslope indicator.

The glideslope indicator, seen in Figure 2, is a device that projects a tricolor beam of light towards the pilot. Each color will only be visible to the pilot at distinct altitudes and distances. If the pilot flies too high, the light will turn green, and if the pilot descends too low, the light will turn red. The goal of the pilot is to stay within the amber band, as this will
provide a 3° glideslope that will guide the aircraft towards the deck of the ship. The helicopter will intercept the glideslope indicator at approximately 350 ft. altitude and 1 nautical mile distance and then the glideslope indicator guides the pilot to a position 50 ft. above the stern of the ship. Once hovering above the landing deck, the pilot lowers the helicopter to touchdown. The pilot times touchdown with a moment of minimal deck motion, either at the top or bottom of a swell. Targeting the top of the swell is advised, as it avoids hard landings [4].

From this overview, several boundary conditions and requirements can be established for the development of automatic landing algorithms:

1. The deck of the ship can be assumed to not be yawing, as the ship will not be changing its heading during final approach and landing.
2. Approximate values of the roll and pitch of the deck will be known to the landing system.
3. Targeting the deck when it is at the top of a wave will be an initial design requirement.

### 1.2 Helicopter Landing Aids

Several devices have been designed to aid a flight crew during challenging landing conditions. One device is Curtiss-Wright’s Recovery Assist, Secure and Traverse (RAST) system [6]. A RAST landing involves the helicopter lowering a cable down to the flight deck, which the crew secures to the RAST system. The RAST system then winches the helicopter down to the flight deck. Once on the deck, the helicopter has a secure connection and is at lower risk of moving on the flight deck.

The U.S. Coast Guard uses a hydraulic gripping system called TALON on their H-65 helicopters to secure them to the deck on landing [4]. The TALON system is comprised of a hydraulic piston and a metal landing grid. The piston is mounted on the underside of a helicopter and has a gripper at the end. The landing grid is a circle located at the center of
the landing deck. When the helicopter lands, the piston extends and grips the landing grid, providing a connection between the helicopter and the deck. This system reduces the risk to crew members who would otherwise have to manually secure the helicopter to the deck and allows the helicopter to land when the deck is at a greater pitch or roll.

### 1.3 Autonomous Landing of Single Rotor Helicopters

Research into the automatic landing of UH-60 helicopters is currently ongoing at Pennsylvania State University [7]. The researchers at Penn State found that an attitude mismatch between the landing deck can be a serious concern. To solve the problem of a mismatch between the attitude of the aircraft and the landing deck, a “level-out” maneuver was proposed [7]. This maneuver has the aircraft matching the attitude of the landing deck immediately before touchdown. The helicopter then quickly reduces lift to land. This ensures that all of the landing gear contact the deck in approximately the same instance. During the level-out maneuver, the task of the control system is thus to match attitude instead of position. Due to helicopter dynamics, this often results in the helicopter drifting from its target location at the center of the landing pad. Because this maneuver introduces a drift in position, it must be conducted as late as possible. The research at Penn State showed that setting the level-out maneuver to occur 1 s before landing resulted in position drifts under 3 ft [7].

### 1.4 Vision Based Autonomous Landing

In order to make an autonomous landing, the UAV must know the state of the flight deck. A vision-based system can be used to acquire the necessary state information. The vision system will process an image, search for a target pattern, and using the known target pattern, determine the position and orientation of the UAV relative to the flight deck. Bai and Wang used this system in conjunction with Kalman filters to provide relative navigation data for their UAV [8]. This navigation data is then used in a trajectory planner to plot a course that brings the UAV directly above the landing deck. Bai and Wang utilize a PID controller to keep the UAV on this desired path. Once above the deck, the drone decreases its altitude and
lands. Bai and Wang were able to land an AR.Drone quadcopter on a 60 x 60 cm deck at a maximum pitch angle of 10°. With distance errors from the center of the platform averaging 7.9 cm, the experiment demonstrated that a simple PID controller was sufficient for autonomous landing on a pitching deck.

1.5 MQ-8C Fire Scout

Northrop Grumman currently has a UAV capable of shipboard landings: the MQ-8C Fire Scout. Based off the Bell 407, the Fire Scout provides reconnaissance, surveillance and target acquisition services. The Fire Scout has successfully performed autonomous landings on Navy vessels experiencing ± 2° of pitch and ± 5° of roll [9]. However, Northrop Grumman has demonstrated landing on slopes with up to ± 5° of pitch during land based tests.

![MQ-8 Fire Scout](image)

**Figure 3: MQ-8 Fire Scout [8]**

1.6 Mission Profile

A mission profile for this research can be established by combining elements of shipboard landings conducted by the U.S. military, as well as the work of Penn State and Bain and
Wong. The quadcopter will begin the approach 4 m behind and above the target flight deck. Next, the quadcopter will maneuver to match the position of the flight deck and hover above the deck at a 1 m altitude. Once a stable hover has been achieved, the quadcopter will begin to descend towards the flight deck, attempting to stay centered above it. When the quadcopter is close to touchdown, it will begin tracking the attitude of the flight deck. Once the attitude has been matched, the quadcopter will then quickly land. Figure 4 Shows the general mission profile.

Figure 4: Mission Overview
2.1 Quadcopters

Quadcopters are a type of rotorcraft that uses four rotors to produce thrust and control. Typically, quadcopters consist of two perpendicular crossmembers intersecting at a central hub. At the end of each crossmember is a propeller connected to a motor. The propellers on the first crossmember spin opposite to the propellers on the second crossmember. This balances out the reaction moment produced by the propellers and eliminates the need for a tail rotor. The microcontroller controlling these motors is located at the hub, along with the power source for the vehicle.

2.1.1 Quadcopter Configurations

There are two main flight configurations for quadcopters: the cross, and the “X” configurations. The cross configuration, as seen in Figure 5, aligns the crossmembers of the quadcopter with the body fixed axes $X_B$ and $Y_B$. Movement in the $Z_B$ direction is controlled by the sum of the thrust of the four propellers. Pitch about the $Y_B$ axes is controlled by varying the speeds between the front and rear propellers. Increasing the speed of the front propeller while decreasing the speed of the rear propellers will cause the aircraft to pitch upwards (positive pitch rate), while doing the opposite will cause the aircraft to pitch downwards (negative pitch rate). Roll about the $X_B$ axis is controlled by varying the speed between the right and left propellers. Increasing the speed of the left propeller, while decreasing the speed of the right propeller will result in a right-propeller-down roll angle (positive roll rate). Yawing moment is controlled by the reaction moments on propellers. The reaction moments depend on the speed of the propellers and the associated drag that must be overcome in order to keep the propellers rotating. Normally the speed of the propellers is balanced such that the moments produced by the clockwise pair (1 and 3) and the counterclockwise pair (2 and 4).
cancel. Creating an imbalance in the reaction moments allows the quadcopter to rotate about the $Z_B$ axis. This will be defined as yaw and is given the symbol $\psi$. Pitch and roll will be denoted by $\theta$ and $\phi$.

![Figure 5: Cross Configuration](image)

The second flight configuration is the “X” configuration seen in Figure 6. Instead of the cross members of the quadcopter being aligned with the body axes, they are instead offset by 45°. Controlling yaw about $Z_B$ works as in the cross configuration. Pitching motion is controlled by varying the speed of the front propeller pair (1 and 2) with respect to the rear propeller pair (3 and 4). Similarly, roll is controlled by varying speed between the left pair of propellers (1 and 4) and the right pair of propellers (2 and 3).
Translational motion for both configurations is achieved by pitching or rolling the aircraft in the desired direction. It is important to note that translation of the quadcopter depends on the attitude of the quadcopter.

2.1.2 AR Drone 2.0
The quadcopter modeled and simulated in this thesis is the AR Drone 2.0 by Parrot. The AR Drone was chosen because it is currently available in the FIT ORION Lab and will be used in future validation experiments. The AR Drone flies in the “X” configuration and has propellers that spin as shown in Figure 6. Each propeller is driven by motors with 15 W power. The four flat landing gear are located below the motors. The AR Drone has a forward-facing camera, as well as two ultrasound sensors to measure altitude. As seen in Figure 6, the $X_B$ axis will be aligned with the forward-facing camera. The drone contains a 3-axis inertial measurement unit to measure acceleration. The AR Drone receives commands via wifi, which makes it easy to interface with an external control computer.
According to the manufacturer’s specifications, the gear ratio between the propellers and the motors is 0.1179. Using the gear ratio and the minimum and maximum speeds of the motors, the minimum and maximum propeller speeds can be found and are listed in Table 2.

**Table 2: Motor and Propeller Angular Velocity Limits**

<table>
<thead>
<tr>
<th>Component</th>
<th>Minimum Angular Velocity</th>
<th>Maximum Angular Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor</td>
<td>1,084 (rad/s)</td>
<td>4,335 (rad/s)</td>
</tr>
<tr>
<td>Propeller</td>
<td>130 (rad/s)</td>
<td>500 (rad/s)</td>
</tr>
</tbody>
</table>

The remaining physical properties for the AR Drone are based on the work of Li and can be seen in Table 3 [10]. The thrust constant and torque constant are experimentally derived values that relate the angular velocity of the propellers to the thrust and moment produced.
Table 3: AR Drone 2.0 Physical Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>$m$</td>
<td>0.429 kg</td>
</tr>
<tr>
<td>Length</td>
<td>$l$</td>
<td>0.1785 m</td>
</tr>
<tr>
<td>Moment of Inertia About $X_B$ Axis</td>
<td>$I_{xx}$</td>
<td>$2.237 \cdot 10^{-3}$ kg m$^2$</td>
</tr>
<tr>
<td>Moment of Inertia About $Y_B$ Axis</td>
<td>$I_{yy}$</td>
<td>$2.985 \cdot 10^{-3}$ kg m$^2$</td>
</tr>
<tr>
<td>Moment of Inertia About $Z_B$ Axis</td>
<td>$I_{zz}$</td>
<td>$4.803 \cdot 10^{-3}$ kg m$^2$</td>
</tr>
<tr>
<td>Thrust Constant</td>
<td>$k$</td>
<td>$8.048 \cdot 10^{-6}$ N/(rad/s)$^2$</td>
</tr>
<tr>
<td>Moment Constant</td>
<td>$C_M$</td>
<td>$2.423 \cdot 10^{-7}$ N/(rad/s)$^2$</td>
</tr>
<tr>
<td>Propeller Moment of Inertia</td>
<td>$I_r$</td>
<td>$2.029 \cdot 10^{-5}$ kg m$^2$</td>
</tr>
</tbody>
</table>
2.3 Proportional Derivative Control

Proportional derivative control, also known as PD control, is one of the fundamental control methods. PD control uses an error term in order to determine the required control input. The error term is the desired value of a system state less the present value of each system state value. The PD controller then calculates a control value that is proportional to the sum of the error and the derivative of the error. A PD controller thus takes the form of Equation 1.

\[ u = k_P e + k_D \dot{e} \]  

(1)

In Equation 1, \( u \) represents the control effort, while \( e \) and \( \dot{e} \) represent the error and the derivative of the error respectively. The proportional and derivative gains are denoted by \( k_P \) and \( k_D \). Gains are proportional constants that indicate how heavily the error and the derivative of the error contribute to the control effort. The proportional control will act similarly to a spring. The farther the spring is stretched, in this case denoted by the error, and the stronger the spring, denoted by the proportional gain, the greater the force produced. The derivative control will act as a damper. As the error decreases over time due to the proportional control driving the system towards the desired state, \( \dot{e} \) will become negative. This results in a damping effect, where \( k_D \) will define how strong the damping is.

Figure 8: PD control block diagram.
2.3 ZEM/ZEV Control

The zero-effort-miss/zero-effort-velocity (ZEM/ZEV) control method was developed by Ebrahimi et al. [11] after expanding upon Bryon and Ho’s work on optimal control for a rendezvous [12]. The control method uses a combination of a zero-effort-miss (ZEM) term and zero-effort-velocity (ZEV) term in order to guide a vehicle to a target with a desired final velocity. The ZEM term represents the position error and the ZEV term represents the velocity error at the final time if no further control effort is applied. In Equations 2 and 3, \( r_d \) and \( r \) are the desired and current position vectors, \( v_d \) and \( v \) are the desired and current velocity vectors and \( t_{go} \) is the time to go until rendezvous.

\[
ZEM = r_d - r - v \cdot t_{go}
\]  
\[
ZEV = v_d - v
\]

According to the work of Bong Wie, Equation 4 shows the optimal acceleration vector for rendezvous, \( a \), calculated by the ZEM/ZEV controller [13].

\[
a = \frac{6ZEM}{t_{go}^2} - \frac{2ZEV}{t_{go}}
\]

2.4 Ship Dynamics

Ship dynamics share many basic similarities to aircraft dynamics. Ship motion occurs with six degrees of freedom, as seen in Figure 9 [14]. The translation components are surge, sway and heave, and the rotation components are roll, pitch and yaw. Ship motion is heavily influenced by wave action. The resulting dynamics due to a ship traveling through ocean waves are highly complex and include factors including, but not limited to: wave velocity, wave frequency, wave direction, wave height, ship geometry, and water depth.
Waves can be modeled as regular or irregular waves. Regular waves are composed of a single harmonic and can be characterized by a single sine or cosine wave, dependent on their height and period [15]. Irregular waves are used to more accurately model the randomness that is present in actual ocean conditions. Irregular waves can be modeled as a sum of components. These wave components can have various amplitudes, frequencies, and phase angles. Spectral formulas are a common way to model irregular ocean waves. Many models take the form of Equation 5.

\[
S(f) = \frac{A}{f^2} \exp(-B/f^4)
\]  

(5)
In Equation 5, $S$ is the wave height, $f$ is the wave frequency, and $A$ and $B$ are constants. The constants depend on the model; several definitions can be seen in [15].

Ocean conditions can be described by sea states. Sea states are used to refer to the relative turbulence of the ocean and are characterized by significant wave height and sustained wind speed. Significant wave height is defined as the mean wave height of the highest third of the waves. Table 4 shows the different conditions and the probabilities of encountering each sea state [16].

**Table 4: Sea States** [16]

<table>
<thead>
<tr>
<th>Sea state no.</th>
<th>Significant wave height (m)</th>
<th>Sustained wind speed (knots)*</th>
<th>North Atlantic</th>
<th>North Pacific</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Percentage probability of sea state</td>
<td>Modal wave period (s)</td>
<td>Most probable</td>
<td>Percentage probability of sea state</td>
</tr>
<tr>
<td>0–1</td>
<td>0–0.1</td>
<td>0.05</td>
<td>0–6</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>0.1–0.5</td>
<td>0.3</td>
<td>7–10</td>
<td>8.5</td>
</tr>
<tr>
<td>3</td>
<td>0.5–1.25</td>
<td>0.88</td>
<td>11–16</td>
<td>13.5</td>
</tr>
<tr>
<td>4</td>
<td>1.25–2.5</td>
<td>1.88</td>
<td>17–21</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>2.5–4</td>
<td>3.25</td>
<td>22–27</td>
<td>24.5</td>
</tr>
<tr>
<td>6</td>
<td>4–6</td>
<td>5</td>
<td>28–47</td>
<td>37.5</td>
</tr>
<tr>
<td>7</td>
<td>6–9</td>
<td>7.5</td>
<td>48–55</td>
<td>51.5</td>
</tr>
<tr>
<td>8</td>
<td>9–14</td>
<td>11.5</td>
<td>56–63</td>
<td>59.5</td>
</tr>
<tr>
<td>&gt;8</td>
<td>&gt;14</td>
<td>&gt;14</td>
<td>&gt;63</td>
<td>&gt;63</td>
</tr>
</tbody>
</table>

*Note: *knots* is a unit of speed equal to one nautical mile per hour.
Chapter 3  
Mathematical Modeling of Quadcopter Dynamics

To develop the autonomous landing system, a mathematical model of the UAV motion dynamics is first required. The UAV modeled will be the Parrot AR Drone 2.0. This model will allow different guidance and control strategies to be tested quickly, in preparation for future experimental testing. The following assumptions were made when modeling the system:

- The UAV consists of two symmetric rigid beams crossing at the center of mass of the vehicle, with point masses at each of the beams representing the motors.
- The UAV will fly in the “X” configuration.
- The thrust produced by the propellers is proportional to the square of their angular velocity.
- Ground effects are not considered.
- External disturbances such as wind gusts are not considered.

3.1 Coordinate Systems

Three reference frames will be needed for the model: the UAV body frame, the flight deck frame, and the inertial reference frame. The inertial reference frame is a standard right-hand coordinate system, and has its origin at the center of mass of the ship. As the model neglects heaving motion of the ship, this reference frame can be considered as inertial for the duration of the approach maneuvers. The subscript \( I \) denotes the inertial frame. The flight deck frame has its origin at the center of the flight deck. For the simulations run as part of this thesis, it is assumed that center of mass of the ship coincides with the centroid of the flight deck. Therefore, the \( Y_D \) axis of the deck frame and the \( Y_I \) axis of the inertial frame coincide, and the deck frame is free to rotate about the \( Y_D/Y_I \) axis. The angle of the rotation is the pitch of the deck, \( \theta_D \). The UAV body frame is a standard aircraft coordinate system with origin in the
center of gravity of the UAV. The body frame is identified by subscript $B$. The three coordinate systems can be seen in Figure 10.

To transform from the body fixed frame to the inertial frame, a rotation matrix is required. The yaw-pitch-roll rotation matrix $R$ performs this transformation [17]:

$$
R = \begin{bmatrix}
    \cos \theta \cos \psi & \sin \theta \sin \psi \cos \phi - \cos \theta \sin \phi & \sin \theta \sin \phi + \cos \theta \cos \phi \\
    \sin \theta \cos \psi & -\cos \theta \sin \psi \cos \phi + \sin \theta \sin \phi & -\cos \theta \sin \phi - \sin \theta \cos \phi \\
    -\sin \phi & \cos \phi & 0
\end{bmatrix}
$$

(6)

where $s$, $c$, and $t$ correspond to sine, cosine, and tangent respectively, and $\phi$, $\theta$, and, $\psi$ correspond to the Euler angles roll, pitch, and yaw. Rotational rates about the body frame axes $X_B$, $Y_B$ and $Z_B$ will be denoted by $p$, $q$ and $r$ respectively. To convert these rotational rates to Euler rates, a second transformation matrix $E$ is required:

$$
\begin{bmatrix}
    \dot{\phi} \\
    \dot{\theta} \\
    \dot{\psi}
\end{bmatrix} = E \begin{bmatrix}
    p \\
    q \\
    r
\end{bmatrix}
$$

(7)

$$
E = \begin{bmatrix}
    1 & s\phi t\theta & c\phi t\theta \\
    0 & c\phi & -s\phi \\
    0 & s\phi \frac{c\theta}{c\phi} & c\phi \frac{c\theta}{c\phi}
\end{bmatrix}
$$

(8)

Figure 10: Inertial frame, deck frame, and body fixed frame.
A final transformation matrix, $D$ is required to transform from the flight deck coordinates to the inertial frame.

$$
D = \begin{bmatrix}
c\theta_D & 0 & -s\theta_D \\
0 & 1 & 0 \\
s\theta_D & 0 & c\theta_D
\end{bmatrix}
$$

(9)

### 3.2 Newton-Euler Dynamics of the Quadcopter

Thrust produced by the rotors, and the acceleration due to gravity are the main external forces acting on the quadcopter. The moments acting on the quadcopter occur due to the thrust of the rotors being offset from the center of mass, as well as due to the countertorque created from the spinning propellers. If the quadcopter is rotating, it will also experience a moment due to gyroscopic precession.

With the forces and moments identified, the Newton-Euler equations can now be used to model the dynamics of the quadcopter. The Newton-Euler equations take the form of Equation (10).

$$
\begin{bmatrix}
I & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{V} \\
\omega \times mV
\end{bmatrix}
+ \begin{bmatrix}
\omega \times I\omega \\
F
\end{bmatrix}
= \begin{bmatrix}
F \\
\tau
\end{bmatrix}
$$

(10)

Where $I$ is an identity matrix, $m$ is the mass of the quadcopter, $V$ is the velocity, $\omega$ represents the angular velocity, $I$ denotes the 3x3 inertia matrix, and $F$ and $\tau$ denote the force and torque vectors acting on the quadcopter. The equations of motion with respect to the inertial frame can be seen in Equations (11)-(16).

$$
\ddot{X}_i = (c\phi s\psi + c\phi s\theta c\psi)\frac{U_1}{m}
$$

(11)

$$
\ddot{Y}_i = (c\phi s\theta s\psi - s\phi c\psi)\frac{U_1}{m}
$$

(12)

$$
\ddot{Z}_i = g - (c\phi c\theta)\frac{U_1}{m}
$$

(13)

$$
\dot{\phi} = \dot{\theta} \psi \left(\frac{I_y - I_z}{I_x}\right) + \dot{\theta} \frac{I_y}{I_x} U_4 + U_2
$$

(14)
\[
\ddot{\theta} = \dot{\phi} \dot{\psi} \left( \frac{l_z - l_y}{l_y} \right) - \dot{\phi} \frac{l_z}{l_y} U_4 + U_3
\]
\[\text{(15)}\]
\[
\ddot{\psi} = \dot{\phi} \dot{\theta} \left( \frac{l_z - l_y}{l_x} \right) - \frac{u_4}{l_x}
\]
\[\text{(16)}\]

The Euler rates are \(\dot{\phi}, \dot{\theta}, \text{ and } \dot{\psi}\). The moments of inertia in the inertial coordinate system are \(I_x, I_y, \text{ and } I_z\) and \(g\) is the acceleration due to gravity. The propeller inertia is \(I_r\). The control terms \(U_1-U_4\) represent the total thrust, roll, pitch, and yaw control respectively and are expressed in Equations (17) – (20) [18].

\[
U_1 = k \sum_{i=1}^{4} \Omega_i^2
\]
\[\text{(17)}\]
\[
U_2 = kl \frac{\sqrt{2}}{2} (\Omega_1^2 - \Omega_2^2 - \Omega_3^2 + \Omega_4^2)
\]
\[\text{(18)}\]
\[
U_3 = kl \frac{\sqrt{2}}{2} (\Omega_1^2 + \Omega_2^2 - \Omega_3^2 - \Omega_4^2)
\]
\[\text{(19)}\]
\[
U_4 = C_M (\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2)
\]
\[\text{(20)}\]

The distance from the center of gravity to the center of the rotor is \(l\). The angular velocity of propeller \(i\) is denoted by \(\Omega_i\). The angular velocity of the propellers will be the actual control input that the drone receives. It is important to note that the quadcopter has six degrees of freedom but only four control inputs. This results in an underactuated system. Thus, in order for the quadcopter to translate in the \(X_I - Y_I\) plane, the quadcopter must have a nonzero pitch or roll value.

For simulation and control purposes, it is useful to establish the equations of motion in the body fixed frame. Applying the Newton-Euler Equations (10) with respect to the body fixed frame yields the following Equations (21) – (26).

\[
\ddot{X}_B = -g \sin \theta + rv - qw
\]
\[\text{(21)}\]
\[
\ddot{Y}_B = g \sin \phi \cos \theta - ru + pw
\]
\[\text{(22)}\]
\[ \ddot{Z}_B = g \cos \phi \cos \theta + qu - pv - \frac{u_1}{m} \tag{23} \]

\[ \dot{p} = \frac{u_2}{I_{xx}} - l_r q (-\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4) - q r \frac{l_{xx} - l_{yy}}{l_{xx}} \tag{24} \]

\[ \dot{q} = \frac{u_3}{I_{yy}} - l_r p (\Omega_1 - \Omega_2 + \Omega_3 - \Omega_4) - p r \frac{l_{xx} - l_{yy}}{l_{yy}} \tag{25} \]

\[ \dot{r} = pq \frac{l_{xx} - l_{yy}}{l_{xx}} - \frac{u_4}{l_{zz}} \tag{26} \]

The body fixed velocity is denoted by \( \mathbf{V}_B = [u \ v \ w]^T \). The quadcopter has no control input in the \( X_B \) or \( Y_B \) directions in the body frame. However, the quadcopter can move in the \( X_I \) and \( Y_I \) directions of the inertial frame if the \( Z_B \) axis has nonzero components in the \( X_I \) or \( Y_I \) directions, and \( U_1 \) is nonzero. This means that for the quadcopter to translate in the \( X_I-Y_I \) plane, it must have some amount of pitch or roll. This also implies that the quadcopter cannot remain stationary if it has a nonzero pitch or roll.

### 3.3 Ship Dynamics

Due to the complexity of ship motion in irregular waves, regular waves were modelled for this study and the ship dynamics were limited to pitch only. With the assistance of Dr. Sahoo of Florida Tech’s Ocean Engineering Department, a simulation was run in MAXSURF, a commercial software simulating the response of a ship in irregular waves. The model used in MAXSURF approximated a hull design for current Coast Guard cutters, and was scaled to match the dimensions of the Coast Guards newest Legend class cutter. The Legend class cutter has a length of 120 m, a beam of 16 m and a draft of 7 m. The modal period used in the simulation was determined using the dispersion relationship for deep water waves. Solving the dispersion relationship for the wave period yielded Equation (27).

\[ T = \frac{\sqrt{\lambda^2 \pi}}{g} \tag{27} \]
The modal period is $T$, in seconds, $\lambda$ is the wavelength and $g$ is the acceleration due to gravity. Assuming a wavelength equal to the length of the ship, $\lambda = 120$ m, the period calculated was $T = 8.8$ s. Using these values and assuming that the ship moves perpendicular to the waves, the simulation was run for significant wave heights of 1-8 m. According to Table 4, the probability for experiencing significant wave heights higher than 8 m is less than 2%. Figure 11 shows the pitch results from the MAXSURF simulations. MAXSURF also showed that the average wave period was approximately 6.6 s. These results were used as a starting point when running landing simulations. The deck motion was modeled to pitch as a sine wave, with a maximum period of 8.8 s and a minimum amplitude of 4°. A greater emphasis was put on lower period waves because they will provide a more challenging landing than short period waves.

Figure 11: Significant Wave Height vs. Pitch
Chapter 4
Quadcopter Control

In order to successfully complete a landing, the quadcopter needs to control all six degrees of freedom. Altitude will be controlled by varying the total thrust produced by the four propellers. Roll and pitch will both be controlled by creating an imbalance in thrust between two propellers: roll between the right and left propellers, and pitch between the front and rear propellers. Yaw control will vary the propeller speeds such that the countertorque is not balanced between the four propellers, resulting in a torque about the $Z_B$ axis. Translation in the $X_I$-$Y_I$ plane requires a more involved process for control. To translate in the $X_I$ or $Y_I$ directions, the quadcopter will first have to orient its thrust vector towards the desired direction. This means that to move in the $X_I$ direction the quadcopter will first need to pitch, and for movement in the $Y_I$ direction the quadcopter will first need to roll. Thus, a cascaded control design overcomes the underactuated system.

4.1 Model Simplifications for Controller Implementation

In order for PD control to be applied effectively, the Newton-Euler model must be simplified. Following the work of Bouabdallah, gyroscopic effects were ignored, and a small angle assumption for yaw angle error was made [18]. Yaw was assumed small because the controller will always be driving the Euler yaw angle $\psi$ to zero. This does limit the quadcopter to a constant forward heading. Therefore, the equations of motion for the quadcopter in the inertial frame are simplified into Equations (28)-(33).

\[
\ddot{X}_I = -\theta \frac{U_1}{m} \quad \text{(28)}
\]

\[
\ddot{Y}_I = \phi \frac{U_1}{m} \quad \text{(29)}
\]

\[
\ddot{Z}_I = g - \frac{U_1}{m} \quad \text{(30)}
\]
The cascaded control design will use a series of PD controllers to sequentially control the six degrees of freedom of the quadcopter, given desired values. First, the altitude controller calculates a thrust $U_i$ to achieve the desired altitude. Next the position controller will calculate pitch and/or roll angles, which will orient the thrust vector of the quadcopter towards the desired $X_I$ and $Y_I$ positions. Lastly, the attitude controller will drive the roll and pitch angles to their desired values which were calculated from the position controller, as well as the yaw angle which will be desired zero always. An overview of this control scheme can be seen in Figure 12.

4.2.1 Altitude Control
The altitude controller calculates $U_1$, which represents the total thrust required from the propellers. Let $\tilde{Z}$ represent the altitude error, $Z$ the actual altitude, and $Z_d$ the desired altitude.

$$Z = Z_d - Z$$  \hfill (34)
Using Equation (30), a PD control law can be written, where \( K_{pl1} \) and \( K_{dl1} \) represent the proportional and derivative gains.

\[
U_1 = m \left[ g - (K_{pl1}\ddot{Z} + K_{dl1}\dot{Z}) \right] \tag{35}
\]

For altitude control using ZEM/ZEV the control law would instead be shown by Equation (36), where \( v_{dz} \) and \( v_Z \) represent the desired and current velocity in the \( Z \) direction.

\[
U_1 = m \left[ g - \left( \frac{2-z_x t_g}{t_g^2} - \frac{v_{dz}-v_Z}{t_g} \right) \right] \tag{36}
\]

These control laws drive the quadcopter to the desired altitude and keeps it there. Setting the error terms to zero verifies that the control law demands a thrust that exactly counteracts the gravity acceleration.

### 4.2.2 Position Control

The position controller will take as inputs the desired \( X_I \) and \( Y_I \) positions, \( X_d \) and \( X_{d} \), the current positions \( X \) and \( Y \), and the total required thrust \( U_1 \). Let \( \bar{X} \) and \( \bar{Y} \) be the position errors in \( X \) and \( Y \) respectively. Starting with the \( X \) control, a new virtual control \( A_X \) is defined in Equation (37). The virtual control is the acceleration in the inertial frame that the quadcopter requires to achieve position tracking. Substituting \( A_X \) into Equation (28) yields Equation (38).

\[
A_X = K_{px}\bar{X} + K_{dx}\dot{X} \tag{37}
\]

\[
A_X = -\theta_d \frac{u_1}{m} \tag{38}
\]

The pitch angle required to achieve the virtual control \( A_X \), \( \theta_d \), can be found by rearranging Equation (38).

\[
\theta_d = -\frac{mA_X}{u_z} \tag{39}
\]
Equation (39) enables the calculation of $\theta_d$, the desired pitch angle in radians which will achieve the virtual control $A_x$ using the total thrust $U_1$. The same process can be applied to $y$, resulting in a desired roll $\phi_d$.

$$\phi_d = \frac{mA_y}{U_1}$$  \hfill (40)

The desired roll and pitch angles, $\phi_d$ and $\theta_d$ are then input into the attitude controller.

### 4.2.3 Attitude Control

The attitude controller is the most important controller for the quadcopter. The attitude controller tracks $\phi_d$, $\theta_d$, and $\psi_d$.

$$\ddot{\phi} = \phi_d - \phi$$  \hfill (41)

$$\ddot{\theta} = \theta_d - \theta$$  \hfill (42)

$$\ddot{\psi} = \psi_d - \psi$$  \hfill (43)

A PD controller is used to determine the roll control $U_2$. Physically, $U_2$ represents an imbalance of thrust between the right-side propellers and the left-side propellers. This imbalance of thrust, along with the moment arm caused by the propellers not being located at the center of mass of the quadcopter, will result in a rolling moment.

$$U_2 = K_{p2}\dot{\phi} + K_{d2}\ddot{\phi}$$  \hfill (44)

Another PD controller can be used for the pitch control, $U_3$. The pitch control represents another imbalance of thrusts, this time an imbalance between the front and rear propellers.

$$U_3 = K_{p3}\dot{\theta} + K_{d3}\ddot{\theta}$$  \hfill (45)

A final PD controller will be used for the yaw control $U_4$. 

\[ U_4 = K_p \dot{\psi} + K_d \ddot{\psi} \]  

(46)

### 4.2.4 Converting Control to Actuation

The thrust values \( U_1-U_4 \) calculated by the controllers need to be realized by the actuators of the system, in this case the motors. This will mean converting thrust values into propeller angular velocities. As demonstrated by Li, the thrust is produced by the propellers is proportional to the square of the angular velocities [10]. Using Equations (17)-(20), a mapping can be computed between the control values and the propeller angular velocities.

\[
\begin{bmatrix}
U_1 \\
U_2 \\
U_3 \\
U_4
\end{bmatrix} = \begin{bmatrix}
k & k & k & k \\
v & -v & -v & v \\
v & v & -v & -v \\
-C_M & C_M & -C_M & C_M
\end{bmatrix} \begin{bmatrix}
\Omega_1^2 \\
\Omega_2^2 \\
\Omega_3^2 \\
\Omega_4^2
\end{bmatrix}
\]  

(47)

\[
v = \frac{kt\sqrt{2}}{2}
\]  

(48)

Taking the inverse of Equation (47) will solve for the square of the propeller angular velocities given the control values.

\[
\begin{bmatrix}
\Omega_1^2 \\
\Omega_2^2 \\
\Omega_3^2 \\
\Omega_4^2
\end{bmatrix} = \begin{bmatrix}
k & k & k & k \\
v & -v & -v & v \\
v & v & -v & -v \\
-C_M & C_M & -C_M & C_M
\end{bmatrix}^{-1} \begin{bmatrix}
U_1 \\
U_2 \\
U_3 \\
U_4
\end{bmatrix}
\]  

(49)

Equation (49) presents two problems: first, the values obtained are still the square of the angular velocities, and second these values can be negative. Since the propellers of the quadcopter are only designed to spin in one direction, a negative angular velocity cannot be realized. Instead, a saturation function will be employed, setting all negative values for Equation (49) equal to zero. The square root can be applied elementwise to the left-hand side of Equation (49), yielding the angular velocities of the rotors required to achieve the calculated control.
Chapter 5
Mission Simulation

Using the motion dynamics model described in Chapter 3, and the control approach from Chapter 4, the mission was simulated in Mathwork’s Simulink. The simulation consists of five main subsystems: ship dynamics, guidance, control, actuation and quadcopter dynamics. The guidance subsystem calculates the desired position and attitude for the quadcopter throughout the simulation. These desired values are input into the control subsystem. In the control subsystem, the cascaded control outlined in Chapter 4 determines the proper control values $U_1-U_4$ to achieve the desired position and attitude. These control values are then converted into propeller angular velocities by the actuation subsystem. The propeller velocities are finally input into the quadcopter dynamics. The simulation was ran at a step size of 0.001 s. A visualization of the model can be seen in Figure 13.

![Mission Simulation Diagram]

**Figure 13: Mission simulation**

The simulated mission consists of an approach phase and a landing phase. The maneuvers during the landing phase change depending on the landing strategy. The three landing strategies tested are: PD control with pitch matching, ZEM/ZEV control, and ZEM/ZEV control with pitch matching. The quadcopter begins every simulation 4 m behind the flight
deck and 3 m above it. During the approach phase, the quadcopter matches X and Y coordinates with the deck and establishes a stable hover 1 m above the deck. Once a stable hover is established, the landing phase begins. In all landing strategies, the goal of the landing phase is to land the quadcopter onto the center of the deck with a vertical landing velocity below 0.5 m/s and with minimal attitude and position error. Pitch error, position error and landing velocity will be referred to as landing qualities and are investigated for each landing strategy. For the simulation, a landing is said to occur when the altitude of at least one of the landing gear turns negative. At this point the simulation stops and results are recorded.

5.6 PD Control with Pitch Matching

The first landing strategy tested was the PD control with pitch matching. For this landing, the quadcopter begins the initial descent toward the flight deck using PD control to control the altitude. During this initial descent, the position control also maintains the X and Y position to stay centered above the flight deck. Once the quadcopter reaches an altitude of 0.25 m above the deck, the attitude control system begins tracking the pitch of the deck. At the same time, the attitude controller changes its gains in order to quickly land on the deck. The goal of this strategy is to perform an aggressive pitch matching maneuver (PMM) in order to match the pitch of the quadcopter with the pitch of the deck and then to land quickly before the deck can change its pitch significantly. Table 5 lists the gains used in this landing strategy.
Table 5: Gains used for Pitch Matching Maneuver of PD Controller

<table>
<thead>
<tr>
<th>Gains</th>
<th>Proportional</th>
<th>Derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude (Approach)</td>
<td>$K_{p1} = 0.8$</td>
<td>$K_{d1} = 3.5$</td>
</tr>
<tr>
<td>Altitude (Landing)</td>
<td>$K_{p2} = 2$</td>
<td>$K_{d2} = 0.9$</td>
</tr>
<tr>
<td>$X$ Position</td>
<td>$K_{px} = 1.5$</td>
<td>$K_{dx} = 3.5$</td>
</tr>
<tr>
<td>$Y$ Position</td>
<td>$K_{py} = 1.5$</td>
<td>$K_{dy} = 3.5$</td>
</tr>
<tr>
<td>Roll</td>
<td>$K_{p\phi} = 0.3$</td>
<td>$K_{d\phi} = 0.8$</td>
</tr>
<tr>
<td>Pitch (Approach)</td>
<td>$K_{p\theta1} = 0.3$</td>
<td>$K_{d\theta1} = 0.8$</td>
</tr>
<tr>
<td>Pitch (Pitch Matching)</td>
<td>$K_{p\theta2} = 1.2$</td>
<td>$K_{d\theta2} = 0.2$</td>
</tr>
<tr>
<td>Yaw</td>
<td>$K_{p\psi} = 0.3$</td>
<td>$K_{d\psi} = 0.3$</td>
</tr>
</tbody>
</table>

5.7 ZEM/ZEV Control

Another approach to landing on a pitching deck is to use ZEM/ZEV control. The ZEM/ZEV control is limited to controlling the altitude of the quadcopter during the landing phase. During the approach phase, a PD controller remains in control of altitude. Figure 14 shows
the new control architecture during the landing phase, which is very similar to the original system seen in Figure 12.

![Diagram of control architecture](image)

**Figure 14: Cascaded control during landing phase using ZEM/ZEV.**

In order to implement a ZEM/ZEV controller, a desired $z$ position, $z$ velocity, and time to go must be determined. Time to go, $t_{go}$, indicates the amount of time the aircraft has until it reaches its target. The desired position, $z_d$, will be zero. The desired $z$ velocity will be set to $0.5$ m/s. This is to ensure a firm landing of the quadcopter. It is preferable to have a harder landing, as opposed to a softer landing during shipboard operations to ensure that the helicopter can remain on the deck after initial contact has been made with the landing gear [4]. Assuming the deck pitches according to a sine wave, the time to go is set such that the helicopter lands at a moment when the deck pitch is flat. This reduces the guidance problem to a matter of timing as well as reduces the need for aggressive maneuvering in the final moments of landing.

There are two points on the pitch sign wave that the quadcopter can target for a flat deck landing: a peak and a trough. Simulations are run for both in order to determine if one is preferable to the other. For both peak and trough landings the period of the wave is varied from 1.1 s to 8.8 s for amplitudes of $4^\circ$, $8^\circ$, and $12^\circ$. Similar to the PD controlled landing, the landing qualities to be investigated are pitch error, position error, and landing velocity.
5.8 ZEM/ZEV with Pitch Matching

ZEM/ZEV with pitching matching will be a combination of the two previous landing strategies. The descent of the quadcopter will be controlled by a ZEM/ZEV controller, however when $t_{go} = 0.25\, s$ the quadcopter will begin tracking the pitch of the deck. By incorporating the pitch matching maneuver with the ZEM/ZEV controller, the quadcopter will be able to handle landings that do not occur when the deck is horizontal.

5.9 Results

5.9.1 PD Control with Pitch Matching

An overview of the position history of the quadcopter can be seen in Figure 16. The quadcopter is seen to begin behind and above the deck of the ship. The quadcopter begins approaching the ship until its X and Y position match the center of the deck. During this time, the quadcopter is also descending and successfully reaches an altitude of 1 m. Once the quadcopter is centered above the deck at 1m, the controller waits until the quadcopter has matched the horizontal velocity of the ship. In Figure 16 the ship is stationary, so the quadcopter merely has to hover above the deck with zero velocity. Once the velocity has been matched, the quadcopter enters the landing phase and begins its initial descent.
Figure 16: Position history

Figure 15: Position of the quadcopter during the landing phase.
In order to perform this landing, the altitude of the level out maneuver needs to be determined. The altitude of the level out maneuver was varied from 0.5 m to 0.1 m in 0.05 m increments, while the deck remained at a static 4°, 8°, and 12° pitch. This was done to determine how the altitude of the pitch matching maneuver effected the pitch and position errors. The deck pitch was chosen to be static in order to allow a better comparison of the effects that the altitude of the level out maneuver had on the landing. Figure 17 shows the relation between altitude of the PMM and pitch error at landing. Increasing the altitude of the PMM is seen to decrease the pitch error. This is expected because the higher the quadcopter is when it begins the PMM, the more time it has to match the pitch of the deck. This would lead to the assumption that a higher altitude for the pitch matching is desired. However, Figure 18 shows that increasing the altitude of the PMM results in higher error in X position at landing. This is a result of the dynamics of the quadcopter. As the quadcopter pitches up to match the pitch of the deck, the thrust vector of the quadcopter will also pitch back. This results in the quadcopter drifting backwards when it pitches up to match the pitch of the deck. The last result from this series of tests was the landing velocity. Figure 19 shows the relation between landing velocity and altitude of the pitch matching maneuver. Landing velocity increases with increasing altitude of the pitch matching maneuver and deck pitch. In order to strike a balance between pitch error, position error, and landing velocity, 0.25 m was chosen as the altitude that pitch matching should begin.
Figure 17: Altitude of PMM vs Pitch Error.

Figure 18: Altitude of PMM vs X Error
With the altitude for the PMM established, the landing strategy was tested in a more dynamic environment. The static deck was replaced with a pitching deck. The deck pitched according to a sine wave with period 6.6 s. This period was chosen since it was the average wave period from the MAXSURF simulations. Once again, the amplitude was varied from 4° to 12° in increments of 4°.

From Figures 17 – 19 the amplitude of the deck pitch can be seen to have a significant effect on the landing qualities. Therefore, the pitch of the deck at landing must also be varied. This can be done by landing on the wave at different times across its period. To do this, a targeting algorithm was developed. First, the time needed to land on a static flat deck was found. Next, a check function was used to determine when a new wave had just begun. After the new wave starts, the quadcopter waits a certain amount of time \( t_{\text{wait}} \) before landing. By varying \( t_{\text{wait}} \), the quadcopter can land at different points of the wave and thus at different pitch angles of the flight deck. In summary, the checks in order to initiate landing are as follows:
• Check that position is centered at 1 m above landing deck.
• Check that velocity of quadcopter matches ship.
• Wait for the start of a new wave.
• Wait designated amount of time.
• Initiate landing.

For the simulation the start of a new wave is detected by comparing the value of the deck pitch at the current time, \( \theta_1 \), to the value of the deck pitch at the previous sample time, \( \theta_2 \). If \( \theta_1 \geq 0 \) and \( \theta_2 < 0 \), then a new wave was considered to have begun. This simple check was designed with the thought that the quadcopter only requires the sign of the deck pitch via sensors in order to determine if a new wave has started. This reduces the required accuracy of the sensors.
Figure 20 shows how the pitch error at landing varies with the targeted phase. The minimum pitch error calculated occurs just after the deck pitch reaches zero. This occurs at 0.167 rad and 1.167 rad. What is unexpected is that the minimum does not occur at a time when the deck pitch is zero. Maximum pitch error similarly occurs slightly after the maximum pitch occurs at 0.67 rad and 1.67 rad. The results are fairly symmetrical whether landing on a rising (pitching up) or falling (pitching down) section of the wave.

**Figure 20: Pitch error vs. targeted phase**
Figure 21 shows the resulting position error as the targeted phase is varied. Minima and maxima occur at points corresponding to the minimum and maximum deck pitch at landing. This is consistent with previous results from Figure 20. Once again, the results are symmetric, implying that a peak landing has similar results as a trough landing. From the results in Figure 20 and Figure 21, targeting near a zero deck pitch appears to give the best results when trying to minimize pitch and position error.
The last landing quality to consider is landing velocity. Figure 22 shows the quadcopter’s vertical velocity at the moment of landing. This velocity is taken as the vertical velocity of the center of gravity of the quadcopter when one of its landing gear touches the deck. Minimum landing velocity occurs at 0, π and 2π. Wave amplitude has minimal effect on the minimum landing velocity. Wave amplitude does have an increasing effect on landing velocity as the deck pitch at landing increases. As seen in Figure 22, the maximum landing velocity occurs when the quadcopter attempts to land at a point of maximum pitch. The landing velocities fell under the desired 0.5 m/s.

![Landing Velocity vs. Targeted Phase](chart.png)

Figure 22: Landing velocity for targeted landing.
The minimum pitch and position errors were found to occur at the same phase for all three amplitudes tested and can be seen in Table 6. The minimum errors do not occur at a point where the deck is horizontal, but instead at a point just after. At these points the landing velocity was roughly the same across all amplitudes and was well below 0.5 m/s.

### Table 6: Landing Qualities at Minimum Pitch and Position Error.

<table>
<thead>
<tr>
<th>Pitch Amplitude (°)</th>
<th>Pitch Error (°)</th>
<th>Position Error (cm)</th>
<th>Phase</th>
<th>Deck Angle (°)</th>
<th>Landing Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>±0.011</td>
<td>±1.9</td>
<td>$\pi \frac{7\pi}{6}$</td>
<td>±1.9</td>
<td>0.28</td>
</tr>
<tr>
<td>8</td>
<td>±0.021</td>
<td>±3.9</td>
<td>$\pi \frac{7\pi}{6}$</td>
<td>±3.9</td>
<td>0.29</td>
</tr>
<tr>
<td>12</td>
<td>±0.03</td>
<td>±6.1</td>
<td>$\pi \frac{7\pi}{6}$</td>
<td>±5.9</td>
<td>0.29</td>
</tr>
</tbody>
</table>

An interesting result is that targeting a point just after a horizontal deck yields significantly less error than targeting a point just before a horizontal deck. This effect is exacerbated at higher pitch amplitudes. Table 7 shows that landing at $\frac{5\pi}{6}$ yields approximately triple the position error of landing at $\frac{\pi}{6}$.

### Table 7: Comparison Between Landing Before and After a Flat Deck

<table>
<thead>
<tr>
<th>Phase Targeted (°)</th>
<th>Pitch Error (°)</th>
<th>Position Error (cm)</th>
<th>Deck Angle (°)</th>
<th>Landing Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\pi}{6}$</td>
<td>−.03</td>
<td>6.1</td>
<td>5.9</td>
<td>0.28</td>
</tr>
<tr>
<td>$\frac{5\pi}{6}$</td>
<td>0.186</td>
<td>18.8</td>
<td>6.049</td>
<td>0.33</td>
</tr>
</tbody>
</table>
5.9.2 ZEM/ZEV Landing
The first simulation targeted a peak landing. In order to target the peak of a wave, a new series of checks needed to be made before initiating the landing phase. The checks in order to initiate a peak landing are now:

- Check that position is centered at 1 m above landing deck.
- Check that velocity of quadcopter matches ship.
- Wait for the start of a new wave. Record time $t_1$.
- Wait for pitch to be a maximum. Record time $t_2$.
- Calculate $t_{go}$ using $t_1$ and $t_2$
- Initiate landing.

The time at maximum pitch can be found in a way similar to determining the start of a new wave. However, instead of using the pitch of the deck, the pitch rate $\dot{\theta}$ is used. Denoting the pitch rate at the current time and previous time by $\dot{\theta}_1$ and $\dot{\theta}_2$ respectively, the maximum pitch can be found if $\dot{\theta}_1 \leq 0$ and $\dot{\theta}_2 \geq 0$. Next, $t_{go}$ can be calculated by Equation (50).

$$t_{go} = t_2 - t_1$$  \hspace{1cm} (50)

Having calculated $t_{go}$, the ZEM/ZEV controller now has everything it requires to control the descent of the quadcopter.

Figure 23 shows the pitch error at landing as a function of wave period for 4°, 8°, and 12° amplitude waves. It is immediately apparent that for periods greater than 3.3 s, the ZEM/ZEV controller successfully targets the peak and lands when the deck is flat, resulting in almost no attitude mismatch at landing. However, for periods below 3.3 s, there is significant error, especially for 1.1 s period waves. Increasing wave amplitude further increases the pitch error for periods below 3.3 s. Figure 24 shows that period and amplitude have almost no effect on
position error. Figure 25 shows the same trend as Figure 23: the controller can successfully achieve a landing velocity of 0.5 m/s at wave periods of 3.3 s or greater but cannot at lower periods. Unlike the pitch error, the landing velocity is not affected by the wave amplitude.

Figure 23: Pitch error for a peak landing using ZEM/ZEV altitude control
Figure 24: Position error vs. period for ZEM/ZEV landing.

Figure 25: Landing velocity vs. period for ZEM/ZEV landing.
The cause of the poor landings at short wave periods was investigated. The high landing velocity suggests that the controls may be saturated. In order to determine whether this was the case, the propeller rates were plotted for the landing phase. Figure 26 shows the propeller rotation rates when landing on a wave with 1.1 s period. The region where the propeller rate is a steady 360 rad/s denotes the quadcopter hovering over the deck. The propeller rate dropping indicates the switch to the landing phase. The graph shows the propeller rates during landing are at the minimum possible for the AR Drone's motors. This implies that the AR Drone is simply unable to achieve the commanded thrusts demanded by the ZEM/ZEV controller for waves with period 1.1 s or less. This simulated behavior must be confirmed in future experiments.

Figure 26: Propeller rates during landing on 1.1s wave.
From Figure 23 and Figure 25, it is seen that the critical wave period above which the ZEM/ZEV controller can successfully land is between 2.2 and 3.3 s. In order to find the minimum wave period that the controller could land with the targeted landing velocity of 0.5 m/s, the simulation was rerun and the wave period varied between 2.2 and 3.3 seconds. Noting from Figure 25 that the landing velocity was not a function of amplitude, the simulation was only conducted using a wave amplitude of 12°. Figure 28 shows that at period of approximately 2.7 s, the ZEM/ZEV controller can successfully land at the desired velocity of 0.5 m/s. Figure 27 shows that the pitch error reduces as the wave period approaches 2.7 s and remains around 0 as the period increases beyond 2.7 s. This shows that landing the AR Drone with a ZEM/ZEV controller will require targeting waves that have a period of at least 2.7 s. For this approach to be successful the drone must be able to gather information about the wave period, either from onboard sensors or by other means. Using this data, the drone would wait for a suitable wave to land.

![Critical Period vs. Pitch Error](image_url)
Simulations for identifying peaks were reordered to identify troughs. The checks for trough landing are:

- Check that position is centered at 1m above landing deck.
- Check that velocity of quadcopter matches ship.
- Wait for pitch to be a maximum. Record time $t_1$.
- Wait for pitch to be minimum. Record time $t_2$.
- Calculate $t_{go}$ using $t_1$ and $t_2$
- Initiate landing.

Figure 28: Landing velocity in short period region.
Figure 29 shows the pitch error as a function of period for the trough landing. This trend appears equal and opposite to the pitch errors plotted in Figure 23.

Figure 29: Period vs. pitch error for trough landing.
Figure 30: Period vs. position error for trough landing.

Figure 31: Period vs. landing velocity for trough landing.
Figure 30 and Figure 31 closely resemble the corresponding graphs for the peak landing. This shows that landing velocity and position error are similar when performing a peak landing as compared to a trough landing. Once again, the ZEM/ZEV controller had difficulties at wave periods below 3.3 s. Shorter wave periods were again examined to determine the critical period for a landing that achieved the desired final velocity. Figure 32 shows that the critical period for trough landings is approximately 2.7 s which is the same for the peak landing as seen in Figure 28. The pitch error, as seen in Figure 33, is once again equal and opposite compared to the peak landing. At 3.3 s and above, the pitch error is seen to stabilize around 0° error.

![Critical Period for Trough Landing](image)

**Figure 32:** Critical period for trough landing.
The main discrepancy between the peak and trough landing lies in the pitch error at landing. Figure 34 shows the comparison. Only periods from 1.1 s to 3.3 s were graphed, as this was where differences were evident. Above wave periods of 3.3 s, the peak and trough landing converged to the same values. The absolute difference between the pitch errors for peak and trough landings was taken according to Equation (51).

\[ \Delta_{abs} = \left\| \bar{\theta}_p \right\| - \left\| \bar{\theta}_T \right\| \]  

(51)

The pitch errors for peak and trough landings are \( \bar{\theta}_p \) and \( \bar{\theta}_T \) respectively. Table 8 confirms what Figure 34 shows graphically. When landing on a peak vs. a trough, the pitch errors at wave periods below 2.7 s will be opposite in sign and nearly equal in value.
Table 8: Comparison of Pitch Error Between Peak and Trough Landing

<table>
<thead>
<tr>
<th>Pitch Amplitude (°)</th>
<th>Max Absolute Difference (°)</th>
<th>Wave Period at Max Difference (s)</th>
<th>Average Absolute Difference (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.018</td>
<td>2.2</td>
<td>0.0059</td>
</tr>
<tr>
<td>8</td>
<td>0.031</td>
<td>3.3</td>
<td>0.0069</td>
</tr>
<tr>
<td>12</td>
<td>0.046</td>
<td>3.3</td>
<td>0.012</td>
</tr>
</tbody>
</table>

Figure 34: Comparison of pitch error between peak and trough landings.
5.9.3 ZEM/ZEV Landing with Timing Error

Simulations were conducted to investigate how the controller handles errors in the sensors that detect pitch. Due to errors in the sensors that measure the deck pitch, the quadcopter may calculate an incorrect $t_{go}$. This will be simulated by introducing an error in $t_{go}$ when running the simulation. Adding a timing error into the simulation will help outline how accurate the calculation of $t_{go}$ needs to be in order to have a successful landing. Equation (52) will be the new equation for $t_{go}$.

$$t_{go} = t_2 - t_1 + e_{go}$$  \hspace{1cm} (52)

Where $e_{go}$ will be the timing error in $t_{go}$. The timing error will be varied from -1 s to 1 s in increments of 0.2 s. A ZEM/ZEV landing will be conducted for a wave with period 6.6 s and pitch amplitudes of 4°, 8° and 12°.
Figure 35: Timing error vs pitch error for peak landing.

Figure 36: Timing error vs pitch error for trough landing.
Figure 37: Timing error vs landing velocity for peak landing.

Figure 38: Timing error vs landing velocity for trough landing.
Figure 39: Timing error vs position error for peak landing.

Figure 40: Timing error vs position error for trough landing.
Figure 35 and Figure 36 show that varying the timing error results in a sinusoidal trend in the pitch error. This is merely a result of landing at a different phase of the wave. Figure 39 and Figure 40 show, as expected, that the position error is not affected by the timing error and remains insignificant. Figure 37 and Figure 38, which plotted the landing velocity, yield more interesting results. The peak and trough plots are nearly identical. The landing velocity exponentially decreased towards 0.5 m/s as the timing error went from -1 s to 0 and remained constant afterward. What is of note is that the landing velocity remains significantly above the target velocity even when the calculated $t_{go}$, after adding the timing error, is above the critical $t_{go}$. At first this was thought to have been caused by the quadcopter hitting the deck sooner than anticipated due to the attitude mismatch at landing. However, it was seen that when the timing error was positive, and the attitude mismatch was now equal and opposite, the controller achieved the targeted 0.5 m/s landing velocity.

5.9.4 ZEM/ZEV Landing with PMM

Having run the ZEM/ZEV landing with timing error, it was apparent that timing error can pose a significant problem when landing. In order to mitigate this potential problem, the pitch matching maneuver developed for the PD landing strategy was added to the ZEM/ZEV control. The modified landing algorithm first attempts to target a landing when the deck is horizontal. Once the quadcopter is close to landing, the quadcopter begins matching its pitch to the pitch of the deck. In this way, even if there is a nonzero timing error the attitude mismatch is corrected before landing. Targeting the deck when it is horizontal will minimize the amount of control effort required and allow a better attitude matching at landing. Since Figure 37 and Figure 38 show that the sign of the timing error had a strong effect on landing velocity, simulations were conducted assuming timing errors of -1 s and 1 s.

Instead of using altitude, $t_{go}$ was used to determine when to perform the PMM. The simulation was run varying the timing of the PMM from $t_{go} = 0.1$ s to $t_{go} = 0.5$ s in 0.5 s increments and the landing qualities were plotted. Both peak landings and trough landings were conducted for completeness.
The pitch error, seen in Figure 41 and Figure 42, decreases when the PMM is performed at a higher values of $t_{go}$. However, the position error increases when the PMM is performed at a higher value of $t_{go}$. Landing velocity is also closer to the targeted 0.5 m/s when the PMM is performed at a low value of $t_{go}$. The landing velocity for timing error values of -1 s is much higher than when the timing error is 1 s. This was expected from previous results. However, the landing velocity when $e_{go} = -1$ s appears to reach a minimum when $t_{go}$ is 0.25 s or 0.3 s. In order to strike a balance between the three landing qualities, $t_{go} = 0.25$ s is chosen to be the time of the PMM. At 0.25 s the pitch error is reduced to 2°. Furthermore, increasing the $t_{go}$ value results in diminishing position error as well as landing velocity.
Figure 41: Time to go vs. pitch error for peak

Figure 42: Time to go vs pitch error for trough
Figure 43: Time to go vs position error for peak landing.

Figure 44: Time to go vs position error for trough landing.
Figure 45: Time to go vs landing velocity for peak landing.

Figure 46: Time to go vs landing velocity for trough landing.
With the timing of the PMM established, the simulation was conducted again for more pitch amplitudes. A peak and trough landing were simulated for pitch amplitudes of $4^\circ$, $8^\circ$ and $12^\circ$ with a 6.6 s period. Once again, the timing error was varied from -1 s to 1 s in 0.2 s increments. Pitch error, position error, and landing velocity were plotted for each landing against the timing error. For both the peak and trough landings the position error, pitch error, and landing velocity were affected more by negative timing error than by positive timing error. Adding the level out maneuver kept the pitch error below $3^\circ$ for all of the simulations.

Figure 47: Timing error vs pitch error for peak landing.
Figure 48: Timing error vs position error for peak landing.

Figure 49: Timing error vs landing velocity for peak landing.
Figure 50: Timing error vs pitch error for trough landing.

Figure 51: Timing error vs position error for trough landing.
The three landing strategies simulated showed different trends in landing qualities. The PD controller was able to best match the pitch of the quadcopter with that of the deck. Figure 20 shows that, even at a pitch amplitude of 12° and targeting the deck at max amplitude, the pitch error remained under 0.2°. The PD controller was able to land with a very low position error when targeting a flat deck. However, position error increased dramatically when the deck was pitched at landing. When the deck pitched with an amplitude of 12° the maximum position error was over 0.3 m. The PD controller was able to keep the landing velocity below 0.5 m/s for all landings. However, the landing velocity was a result of tuning the controller. If a different landing velocity was desired the gains would have to be tuned again, possibly

Figure 52: Timing error vs landing velocity for trough landing.

5.10 Discussion

The three landing strategies simulated showed different trends in landing qualities. The PD controller was able to best match the pitch of the quadcopter with that of the deck. Figure 20 shows that, even at a pitch amplitude of 12° and targeting the deck at max amplitude, the pitch error remained under 0.2°. The PD controller was able to land with a very low position error when targeting a flat deck. However, position error increased dramatically when the deck was pitched at landing. When the deck pitched with an amplitude of 12° the maximum position error was over 0.3 m. The PD controller was able to keep the landing velocity below 0.5 m/s for all landings. However, the landing velocity was a result of tuning the controller. If a different landing velocity was desired the gains would have to be tuned again, possibly
resulting in changes to position and pitch error. Another advantage of the PD controller was that it required no prediction of the pitching motion of the ship. The controller simply attempted to track the current deck pitch as it made its final descent. This made the landing qualities highly dependent on where the ship was on the wave during landing. For example, if the quadcopter happened to land when the ship was at a high pitch angle, the resulting position error at landing would also be high due to the drift caused by the level out maneuver.

The ZEM/ZEV landing produced excellent landing qualities as long as the period of the wave was above 2.7s. If no timing error was present, assuming the sensors are perfect, the ZEM/ZEV controller had practically no pitch and position error. However, the ZEM/ZEV landing relies heavily on proper timing in order to target a flat deck. When timing error is present, the ZEM/ZEV controller experiences high pitch errors without an attitude matching maneuver. Figure 53 shows that adding the PMM reduced the pitch error significantly when the ZEM/ZEV landing experienced timing errors. It should be noted that adding the PMM did increase the pitch error when the timing error was zero. Thus the PMM causes some pitch error when otherwise there would be none. This small increase in pitch error is greatly outweighed by the reduction in pitch error at nonzero timing error values. The level out maneuver did add position error to the landing, as the quadcopter drifted when matching the pitch of the deck. Figure 54 shows that for a 12° peak landing the position error almost reached 2.5cm. Considering the pitch error reduced by the level out maneuver, this added position error was minor.
Figure 53: Pitch error with and without pitch matching maneuver.

Figure 54: Position error with and without pitch matching maneuver.
The landing velocity was seen to be closer to the target velocity when the timing error was positive, as seen in Figure 49 and Figure 52. Interestingly, the pitch error was also slightly lower when the timing error was positive as opposed to negative. Thus, if possible the landing strategy should gravitate towards landing after a horizontal deck as opposed to before. Since the trough and peak landings tended to be equal and opposite in their errors, this means that both are viable landing targets. Therefore, other factors should be taken into account when deciding where to target the landing. One factor could be the direction of the position error. If a trough landing is targeted and the landing strategy tends toward landing after the trough as opposed to before the trough, then the position error will be positive. This means that the quadcopter will land more towards the stern of the ship. This is desirable for destroyers and cutters. Both destroyers and cutters have superstructure fore of the flight deck, but do have a clear region aft of the flight deck. This means that if an aircraft lands aft of the center of the flight deck it will run no risk of a collision with the superstructure. Furthermore, if the quadcopter is attempting to land on a positively pitched deck, then the thrust vector of the quadcopter will be oriented correctly for a wave off. In the event of a wave off, the aircraft would maneuver away from the ship and perform another approach and attempt another landing. In this situation, the quadcopter would merely need to increase thrust in order to move away from the ship. If the quadcopter had been pitched forward, it would instead need to first pitch back and then increase thrust in order to move away from the deck of the ship.

Table 9 shows the general comparison between the PD and ZEM/ZEV landing strategies. While the PD controller provided excellent pitch matching between the quadcopter and the deck, it suffered from high position error, especially when attempting to land on a steep deck. The pure ZEM/ZEV controller was able to land with almost no position error and no pitch error if the period of the pitching motion was above 2.7s. However, the ZEM/ZEV controller did not handle timing errors well. The ZEM/ZEV with pitch matching was able to handle timing errors up to ±1 s. Therefore, the best landing strategy to use will be governed by how precisely the aircraft can time a landing on a horizontal deck. If a landing can be timed very well, then a pure ZEM/ZEV controller will provide the best results. If a landing can be
conducted within 1 s of a horizontal deck, then ZEM/ZEV with pitch matching should be employed. If no timing system is available, then PD control can be used, however it may provide poor position error.

Table 9: Comparison Between PD and ZEM/ZEV Landings

<table>
<thead>
<tr>
<th></th>
<th>Pitch Error</th>
<th>Position Error</th>
<th>Landing Velocity</th>
<th>Timing Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD</td>
<td>Very Low</td>
<td>Highly Variable</td>
<td>Indirectly Controlled</td>
<td>No</td>
</tr>
<tr>
<td>ZEM/ZEV</td>
<td>Dependent on timing error</td>
<td>Very Low</td>
<td>Directly Controlled</td>
<td>Yes</td>
</tr>
<tr>
<td>ZEM/ZEV with pitch matching</td>
<td>Low</td>
<td>Low</td>
<td>Directly Controlled</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Chapter 6
Conclusion

The work presented in this paper proposed different landing strategies for landing a quadcopter onto a pitching deck of a ship. While shipboard landings involve much more complicated ship dynamics than just pitch motion, the results shown can be used as a starting point for further research into this topic. The three landing strategies simulated were PD control with pitch matching maneuver, ZEM/ZEV control, and ZEM/ZEV control with pitch matching maneuver. Of the three, ZEM/ZEV with pitch matching performed the best, if a landing could be timed close to when the deck is horizontal.

The PD controller was able to land the quadcopter with minimal pitch error and required no prediction of the ship motion. However, if the pitch matching maneuver was conducted when the ship was significantly pitched, then the PD controller resulted in a high position error. The landing velocity resulting from the PD controller was not directly controlled, but could be indirectly controlled through gain tuning. The ZEM/ZEV controller was able to land with almost no pitch or position error as long as the period of the pitching motion was greater than approximately 2.7s. The ZEM/ZEV controller was also able to land with a specified velocity. This is beneficial for shipboard landings because helicopters need to land softly enough in order not to damage the aircraft, but solidly enough as to avoid multiple contacts with the deck. The ZEM/ZEV controller required the pitching motion of the ship to be predicted for a half period. For a real sea state, the pitching motion of the ship would not follow a regular sine wave. If the ZEM/ZEV controller experienced timing errors when trying to target a flat deck, then the pitch error could be significant, and the landing velocity could be much higher than desired. Thus, a combination of ZEM/ZEV and PD control was tested. This approach reduced the pitch error seen in the ZEM/ZEV controller when timing errors were present, while adding minor position error. Targeting a landing when the deck was at a small angle yielded the lowest pitch and position errors. However, when the pitch
matching maneuver was used, the lowest pitch and position errors occurred when the quadcopter landed just after the deck was horizontal.

6.1 Future Work

Further work must be conducted to account for roll, heave, and other ship motions when performing an autonomous landing. Disturbances such as wind gusts could also be modeled in order to make the simulation more realistic.

Experimental validation of the simulation results is another important next step in this research. The simulations were designed to use the AR Drone 2.0 which is currently available for research use at FIT’s ORION Lab.

One key assumption that was made was that the deck of the ship pitched with a sinusoidal motion. Real ocean conditions will not present such a regular and easily predicted ship motion such as a sine wave. One way to deal with this limitation is to develop a way to predict the ship motion accurately. The ZEM/ZEV landing strategy only takes a half period to complete, which means that the ship motion will only need to be predicted for a half period of the wave at a time. Another approach would be to still assume that the wave will pitch with a sinusoidal motion, but to also measure the error between the predicted pitch and the actual measured pitch of the deck. This error term between the predicted and actual wave could be used in a controller to make corrections to the landing.
References


