JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 94, NO. B10, PAGES 13,673–13,679, OCTOBER 10, 1989

The ~60-Year Power Spectral Peak of the Magnetic Variations Around London and the Earth's Rotation Rate Fluctuations

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Maximum entropy spectral analysis has been applied to the annual mean magnetic data around London for the years 1839–1974. These data were compiled by Malin and Bullard. The existence of a ~60-year spectral line is clearly found. In addition, cross power spectral density analyses using a two-channel maximum entropy method were applied to both the variations of the annual mean declination around London from 1900 to 1974 and the fluctuations of the Earth's rotation rate from 1876 to 1974. The coherence is high (0.98), and the phase angle indicates that the changes of the magnetic declinations around London lag behind the fluctuations of the Earth's rotation rate by 20 ± 4 years.

1. Introduction

The secular variations of the Earth's magnetic field have been a subject of interest for the last 100 years. One method to study any time series is power spectral density analysis [*Blackman and Tukey*, 1959]. *Currie* [1973*a*, *b*] made a spectral density analysis of separate time series of the twentieth century magnetic data at 28 observatories. The method used was the maximum entropy spectral analysis (MESA). A spectral line of ~60 years was observed at most but not all of the 28 observatories. To take into account all the observatory data, *Jin and Thomas* [1977] analyzed the geomagnetic dipole field variations from 1900 to 1969. The dipole fields were derived from the spherical harmonic analysis of the annual mean geomagnetic data of all magnetic observatories. The spectral peak at 60 years was found in the dipole field variations. Recently, *Langel et al.* [1986] examined the geomagnetic data from 1903 to 1982. In particular, they performed a power spectral analysis of the secular variations of the Gauss-Schmidt coefficients, g_l^m and h_l^m, up to third order and third degree. They found the existence of the 60-year period in the secular variations of g_1^0, h_1^1, g_2^0, h_2^1, g_2^2, g_3^0, g_3^1, g_3^2, and g_3^3. Despite the evidence, the 60-year spectral line has been looked upon with some skepticism among researchers because the magnetic data used are often only about 70–80 years in length.

An excellent compilation of magnetic declinations and inclinations around London from 1570 to 1975 was published by *Malin and Bullard* [1982]. The data were corrected for their site differences, and tabulations were given with corrections made with respect to Greenwich. The long record of magnetic data around London provides an opportunity to reexamine the 60-year spectral line. *Malin and Bullard* [1982] reported the absence of the 60-year line in the power spectrum of the difference between the smooth and less smooth spline functions derived from the data. It prompted us to analyze the magnetic data around London using the maximum entropy method developed by *Burg* [1967, 1968, 1977]. The magnetic data used are from 1839 to 1974.

It is accepted that the origin of the main geomagnetic field lies with the current in the fluid core of the Earth. Basic physical principles imply that a close relationship must exist between the fluctuations of the rotation rate of the Earth and the secular variations of the geomagnetic field. The power spectral analysis of the geomagnetic dipole field and the length-of-day (l.o.d.) fluctuations by *Jin and Thomas* [1977] confirmed their spectral similarity and the existence of the common spectral peak at approximately 66 years. *Le Mouel and Courtillot* [1982] reported that the first derivative of declination at Paris leads the variation of the Earth's rotation rate by approximately 10 years. Since the variation of the declination can be related to the westward drift of the geomagnetic field, cross power spectral analyses using a two-channel maximum entropy method were also included in this research to find the phase relationship and the coherence between the fluctuations of the rotation rate of the Earth and the variations of the declinations at London. The magnetic data and the l.o.d. data used for the cross-correlation are from 1900 to 1974 and from 1876 to 1974, respectively.

2. Two-Channel Maximum Entropy Spectral Analysis

The method of maximum entropy spectral analysis of a single time series has been well documented in the book *Modern Spectrum Analysis*, edited by *Childers* [1978]. In particular, the article by *Ulrych and Bishop* [1975] gives detailed computer programs. MESA does not require either a zero or a periodic extension of the data. It has good resolution and is extremely useful if the periods one seeks are comparable to the length of the data. *Chen and Stegen* [1974] demonstrated that MESA is applicable even if the data length is only three fourths of the sinusoid's period.

Cross power spectral analysis is briefly discussed in the paper by *Ulrych and Jensen* [1974]. More detailed discussions on multichannel MESA are provided by *Strand* [1977*a*,

Paper number 89JB01241.
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b]. Following is a brief summary of the algorithms associated with the two-channel MESA:

If $y(1), y(2), \ldots, y(n)$ represent a single time series with time increment Δt, then MESA searches for a near-optimum filter such that

$$y(t) + A_{1m}y(t - \Delta t) + \cdots + A_{mm}y(t - m\Delta t) = e_m(t) \tag{1}$$

where $m < n$ is the number of lags, the A_{km} are the filter coefficients, and $e_m(t)$ is the output. The sequence $e_m(t)$ resembles white noise for the appropriate filter, and the power spectral density is given by

$$P(f) = \frac{P_m \Delta t}{\left|1 + \sum_{k=1}^{m} A_{km} \exp\left[-j2\pi k f \Delta t\right]\right|^2} \tag{2}$$

where P_m is the power of the output $e_m(t)$, and f is any frequency that lies between the positive and negative Nyquist frequency f_N, $1/(2\Delta t)$. For a two-channel MESA one forms a two-channel vector time series

$$Y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} \tag{3}$$

Both the forward filter and the backward filter are to be found such that

$$Y(t) + \sum_{k=1}^{m} F_{km}^{*} Y(t - k\Delta t) = E_m(t) \tag{4}$$

$$Y(t) + \sum_{k=1}^{m} B_{km}^{*} Y(t - k\Delta t) = E'_m(t) \tag{5}$$

where F_{km} and B_{km} are 2×2 matrices and asterisks denote the Hermitian conjugation, since the time series can be of complex form. The optimum filters satisfy the condition that the mean square values of $E_m(t)$ and $E'_m(t)$ are minima. The resulting F_{km} and B_{km} satisfy the equations

$$\begin{bmatrix} R_0 & R_1 & R_2 & \cdots & R_m \\ R_{-1} & R_0 & R_1 & \cdots & R_{m-1} \\ R_{-2} & R_{-1} & R_0 & \cdots & R_{m-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ R_{-m} & R_{-m+1} & R_{-m+2} & \vdots & R_0 \end{bmatrix} \begin{bmatrix} 1 \\ F_{1m} \\ F_{2m} \\ \vdots \\ \vdots \\ F_{mm} \end{bmatrix} = \begin{bmatrix} P_m \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} R_0 & R_{-1} & R_{-2} & \cdots & R_{-m} \\ R_1 & R_0 & R_{-1} & \cdots & R_{-m+1} \\ R_2 & R_1 & R_0 & \cdots & R_{-m+2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ R_m & R_{-1} & R_{-2} & \vdots & R_0 \end{bmatrix} \begin{bmatrix} 1 \\ B_{1m} \\ B_{2m} \\ \vdots \\ \vdots \\ B_{mm} \end{bmatrix} = \begin{bmatrix} P'_m \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \tag{7}$$

where

$$R_k = E[Y(t)Y^*(t - k\Delta t)] \qquad R_{-k} = R_k$$

$$P_m = E[E_m(t)E_m^*(t)]$$

$$P'_m = E[E'_m(t)E'_m(t)]$$

and $E[\ ]$ is the expected value. P_m and P'_m can be computed by the recursion formula

$$P_m = P_{m-1} - C_m^* P'_{m-1} C_m \tag{8}$$

$$P'_m = P_{m-1} - (C'_m)^* P_{m-1} C'_m \tag{9}$$

where $C_m = F_{mm}$ and $C'_m = B_{mm}$ are the forward and backward reflection coefficients. *Strand*'s [1977*b*] article contains detailed calculations of C_m and C'_m. The power spectral density can be calculated by

$$G(f) = \Delta t F^{-1}(1/z)^* P_n F^{-1}(1/z) \tag{10}$$

where $F(Z) = I + F_{1m}Z + \cdots + F_{mm}Z_n$ and $Z = \exp[-j2\pi f\Delta t]$.

$G(f)$ assume the form

$$\begin{pmatrix} G_{11}(f) & G_{12}(f) \\ G_{21}(f) & G_{22}(f) \end{pmatrix} \tag{11}$$

where G_{11} is the power spectral density of time series 1, G_{22} is the power spectral density of time series 2, and G_{12} is the power spectral density of the combined time series. The squared coherence and the phase angle Θ are given by

$$\text{coh}^2(f) = \frac{|G_{12}(f)|^2}{G_{11}(f)G_{22}(f)} \tag{12}$$

$$\Theta = \tan^{-1}\left[\frac{\text{Im } G_{12}(f)}{\text{Re } G_{12}(f)}\right] \tag{13}$$

3. Data Selection and Method of Analysis

The magnetic data used in the present study are taken from *Malin and Bullard* [1982, Tables A1 and A2]. Not all the data are used, for the following reasons: First, there are data for only 18 years from 1676 to 1776 and data for only 34 years from 1776 to 1838, with gaps varying from 2 to 7 years. We require yearly data for the power spectral density analysis. Second, since there is a switch in the general trend of the change of the declination from westward to eastward around 1830, we cannot justify removing the linear trend for a time span extended backward beyond 1830. The magnetic declination data chosen are from 1839 to 1974. Only data with dates listed at midyear, such as 1851.5, are used. These data exist every year without a gap for the chosen span of years. If more than one data point is given for a particular year, we calculated an average value. The maximum deviation of the value from the mean is about 18 min. The chosen magnetic inclination data are from 1843 to 1956. The same criteria were used with the exception of the point at 1950. Even though its date is listed as 1950.6 instead of 1950.5, it was used for the sake of extending the data from 1951 to 1956. The time sequences of the chosen data are given in Figures 1 and 2. Single-channel MESA was applied to these magnetic data. The main weaknesses of MESA are the lack of theoretical criteria for the optimal filter length m and the problem of spurious peaks. Akaike's criterion [*Akaike*, 1974] of using the minimum value of the final prediction error (FPE) is used by some researchers, but it does not always work. The FPE is defined as

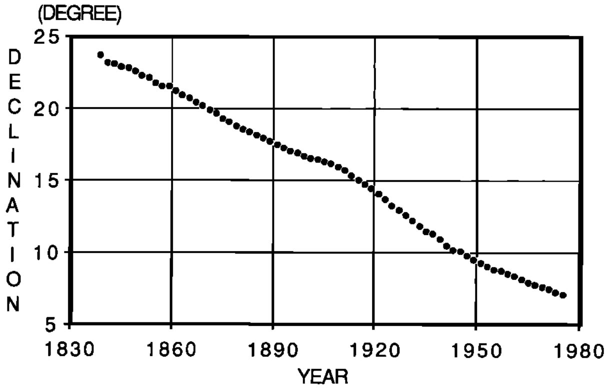

Fig. 1. The magnetic declinations at London from 1839 to 1974.

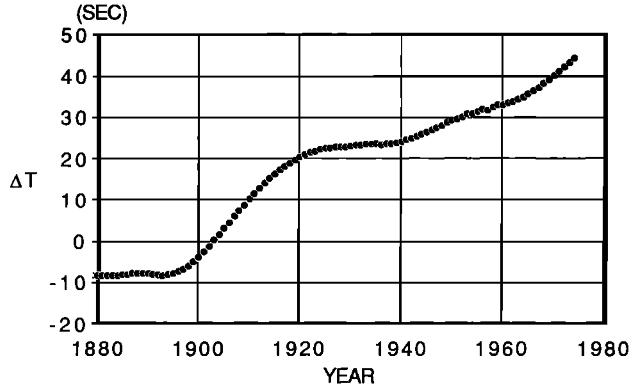

Fig. 3. Time discrepancy curve from 1881 to 1978. ΔT is ephemeris time minus universal time.

$$\mathrm{FPE}(m) = P_m\left(\frac{N + m + 1}{N - m - 1}\right) \tag{14}$$

where N is the number of the data point.

In a previous article by *Jin and Thomas* [1977] it was demonstrated with a simulated time series that a relatively large number of prediction error filter (PEF) coefficients is justified if the data have a linear trend superimposed with periodic variations. On the other hand, the detrended time series does change the data depending on the length of the time span and the variations. Hence a practical approach was adopted. Both the original time series and linearly detrended time series with varying numbers of PEF coefficients were analyzed. This is to be compared with the analysis of a simulated time series which resembles the real time series under consideration. In this case the simulated time series is made of a linear function resembling the linear trend of the declination curve at London and a sinusoidal function of 60-year period. The MESA of the simulated time series serves as a guideline to see whether the number of the PEF coefficients used in the actual data analyses is reasonable. In addition, the FPE are calculated. MESA is deemed to perform well if the FPE are rapidly reduced as the filter length is increased through the first few values of m. To avoid the problem of spurious peaks, only the spectral peaks that appear in both the original time series and the detrended time series are considered.

Because of the fluctuations of the Earth's rotation rate, the observed positions of the Sun, Moon, and planets, based on

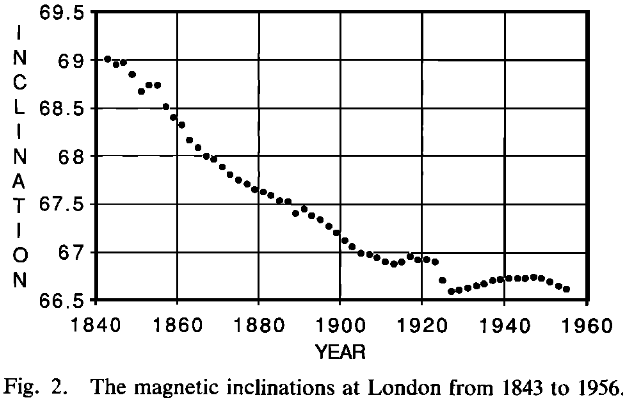

Fig. 2. The magnetic inclinations at London from 1843 to 1956.

a rotating Earth (universal time), are different from those calculated from Newtonian mechanics based on a uniform time (ephemeris time). Ephemeris time minus universal time, ΔT, is given annually in the American Ephemeris and Nautical Almanac. "Minus ΔT" from 1881 to 1978 is shown in Figure 3. The values of ΔT from 1881 to 1961 are the same values used in the single-channel MESA by *Jin and Thomas* [1977], and those from 1961 to 1978 are taken from the article by *Morrison* [1979]. We simply add Morrison's data to our existing data file. Since our interest is in the decade fluctuations, the data from the ephemeris are adequate [*Munk and MacDonald*, 1975]. The so-called length-of-day fluctuation is proportional to the time derivative of ΔT, i.e., l.o.d. fluctuation = $2.738 \times 10^{-3} \times d(\Delta T)/dt$ seconds (ephemeris time). Having identified the ~60-year peak, we applied the two-channel MESA to the linearly detrended magnetic declination data at London from 1900 to 1974 and the fluctuation of the rotation rate of the Earth from 1876 to 1978. The choice of the time spans will be explained in section 4. The objectives of the two-channel MESA were to determine the coherence and the phase angle of the 60-year spectral peak between the changes of the magnetic declination and the rotational rate of the Earth.

4. RESULTS AND CONCLUSIONS

A power spectral density analysis of the original magnetic declination data at London from 1839 to 1974 is given in Figure 4. The total number of PEF coefficients used is 85, and the number of spectral estimates, or Δf, is 300. The relative peaks occur at periods of 66.7, 40, 26, 19, 15, 12, and 11 years, but the amplitudes at 26, 19, 12, and 11 years are very small. The declination data with a linear trend removed is given in Figure 5. The detrended declination, ΔD, is given by $\Delta D = D(t) - D'(t)$, where $D'(t) = a + b\,(t - 1839)$, and a and b are determined from the data points in Figure 1. The power spectral density of the detrended declinations is shown in Figure 6. The number of PEF coefficients is 20, and the number of spectral estimates is 300. Again the main peak is at 66.7 years. There is also a broad peak of very small amplitude at 12.5 years. The number of PEF coefficients used is significantly reduced from 85 to 20 in the detrended declination time series because of the improvement in the signal to noise ratio. The power spectral density of the original magnetic inclination time series is given in Figure 7. The number of PEF coefficients and the number of spectral

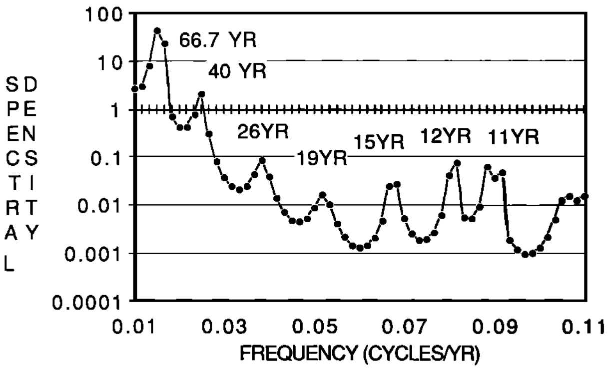

Fig. 4. Power spectral density of the declination data at London from 1839 to 1974. The number of PEF coefficients used is 85, and the number of spectral estimates is 300.

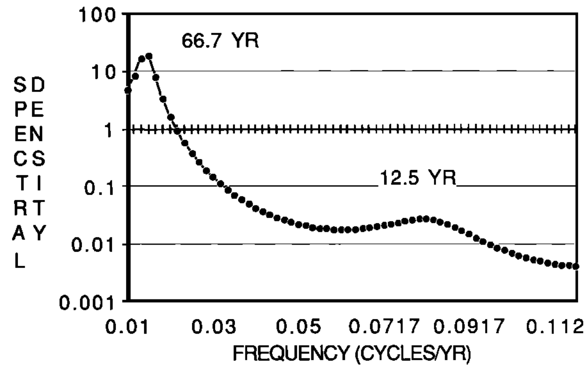

Fig. 6. Power spectral density of the detrended declination data at London from 1839 to 1974. The number of PEF coefficients used is 20, and the number of spectral estimates is 300.

estimates used are the same as those used in Figure 4. The relative peaks are at periods of 54.5, 31.6, 22.2, 16.7, 13.0, 10.9, and 9.7 years. The detrended magnetic inclination time series and its power spectral density are given in Figures 8 and 9, respectively. The number of PEF coefficients used in Figure 9 is 50. The peaks are at periods of 54.5, 31.6, 21.4, 12.8, and 9.8 years. The number of PEF coefficients used is also reduced in analyzing the detrended inclination data. The reduction is not as much as that in the case of detrended declination because the variations of the magnetic inclination are smaller than those of the magnetic declination. If one examines the detrended declination time series in Figure 5, it can be noted that significant change occurs around 1900. Thus a simulated time series is constructed with a line plus one period of a sinusoid starting at the seventieth point. Its equation is given as

$$y(t) = 24 - 0.125t \qquad t = 0, \cdots, 69$$

$$y(t) = 24 - 0.125t + 0.1 \sin\left(\frac{2\pi t}{60}\right) \qquad t = 70, \cdots, 136 \tag{15}$$

The analysis of the simulated time series indicates that the spectral peak at 60 years will appear only if the number of the PEF coefficients is greater than 75. This is similar to what was observed in the power spectral density analysis of the time series of the magnetic data without the linear trend removed. Power spectral density analyses of the magnetic declination and inclination data indicate only a common spectral peak around 60 years. The uncertainty in the location of the peak is somewhat difficult to determine. *Chen and Stegen* [1974] reported a 20% variation with respect to the initial phase in their experiments with MESA as applied to a single period of sinusoids. Using the average value of the results from both the declination and the inclination time series, we assign the location of the peak to be at 60 years with an uncertainty of 12 years.

Since a relatively large variation occurred in the magnetic declination and began around 1900, the two-channel MESA was applied to the time series of ΔD at London from 1900 to 1974 and the detrended minus ΔT from 1876 to 1978. Furthermore, in order to obtain a better estimate of the phase information between the fluctuations of the Earth's rotation rate and the variations of the magnetic field, different sets of ΔT with the beginning year varying from 1876 to 1904 are used. Each set of "minus ΔT" contains 75 points and is detrended separately. The detrended minus ΔT time series were then combined with the ΔD time series for the analysis. A total of 15 sets were analyzed. For all the two-channel analyses the ΔT time series is the first one, and the declination time series is the second one. A set of detrended minus ΔT time series with the beginning year 1890 together with the detrended declination data is shown in Figure 10. It should be noted that only the data points from 1890 to 1964 in Figure 3 are used for this particular detrended ΔT curve. Akaike's final prediction error criterion [*Strand*, 1977*a*] was used to

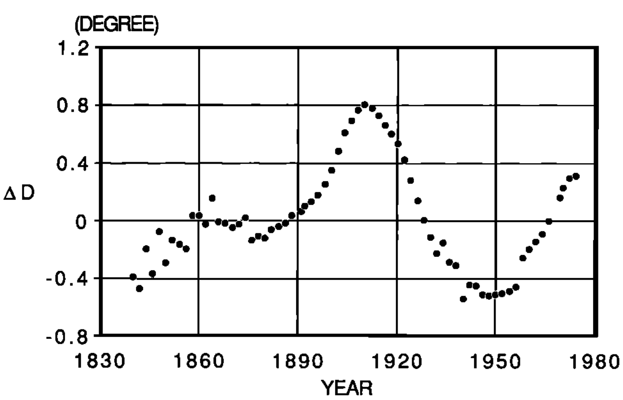

Fig. 5. Detrended declination data at London from 1839 to 1974.

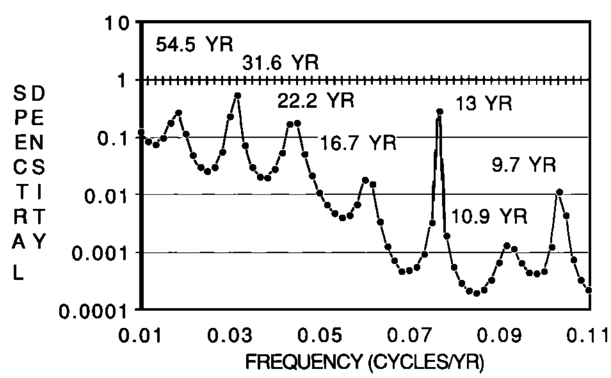

Fig. 7. Power spectral density of the inclination data at London from 1843 to 1946. The number of PEF coefficients used is 85, and the number of spectral estimate is 300.

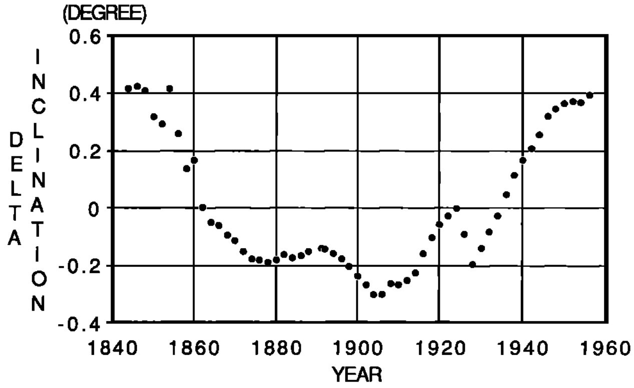

Fig. 8. Detrended inclination data at London from 1843 to 1956.

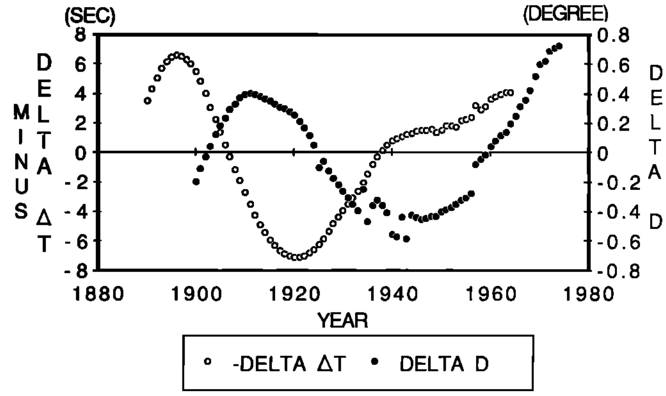

Fig. 10. Detrended "minus ΔT" time series from 1890 to 1964 and the detrended declination data at London from 1900 to 1974.

give a rough indication of the performance of the two-channel MESA. According to this criterion, MESA is performing well if det P_m is rapidly reducing as the filter length is increased through the first few values of m.

The results in the location of the spectral peak and the phase angle are about the same for three to 12 filter coefficients. The number of filter coefficients was thus chosen to be nine. For the set of the two time series depicted in Figure 10 the resulting absolute value of the cross power spectral density G_{12} and the corresponding phase Θ versus frequency are shown in Figure 11. The number of spectral estimates was chosen to be 256, separated by 1.953×10^{-3} cycle/yr. Only one spectral peak is found from the two-channel MESA. It is at a frequency of 0.01758 cycle/yr, which corresponds to a period of approximately 57 years. The squared coherence is 0.97, and the phase angle is 60° at that frequency. The apparent time lag is 9.5 years, which is calculated from $\Theta = 2\pi f \tau$. Since the beginning year of the declination time series is ahead of this particular "minus ΔT" time series by 10 years, the true time lag is 19.5 years. Similar two-channel MESA were carried out for the same ΔD time series combined with 15 different sets of "minus ΔT" time series. The results of the two-channel MESA of the 15 sets of l.o.d. and magnetic declination data are summarized in Table 1. It is seen that the spectral peaks occur around the frequency of 0.01758 cycle/yr, or a period of 57 years. The average squared coherence is 0.96. The plot of the phase angles versus the initial time difference is given in Figure 12. The initial time difference equals the beginning year of the magnetic data minus the beginning year of the ΔT data. It can be seen that the phase angle is approximately zero for an initial time difference of 20 years. Thus the variations of the declination at London lag behind the fluctuations of the Earth's rotation rate by 20 years. We assign an uncertainty of 4 years to the time lag, as suggested by the experiments with sinusoids.

In conclusion, the single-channel MESA of the relatively extensive magnetic data at London from 1839 to 1974 clearly identifies the existence of the ~60 year spectral peak. This disagrees with the results of *Malin and Bullard* [1982]. According to their paper, the least squares spline functions were first derived from 250-year magnetic data. Then the power spectra of the difference between the smooth and the less smooth spline functions, based on 5-year values and tapered with a triangular filter, were obtained. They commented that the absence of an 11-year cycle is due to the smoothing process and that such a variation was present in the original data as indicated by the residuals of the observatory data from the smooth curve. The smoothing process is probably also related to the absence of the 60-year peak, as we observed the peaks around 11 years in our analysis.

Our two-channel MESA of the magnetic declination data from 1900 to 1974 and the length-of-day data from 1876 to 1978 indicate that the variations of the declination at London lag behind the rotation rate change of the Earth by about 20 ± 4 years. This is in general agreement with the cross-

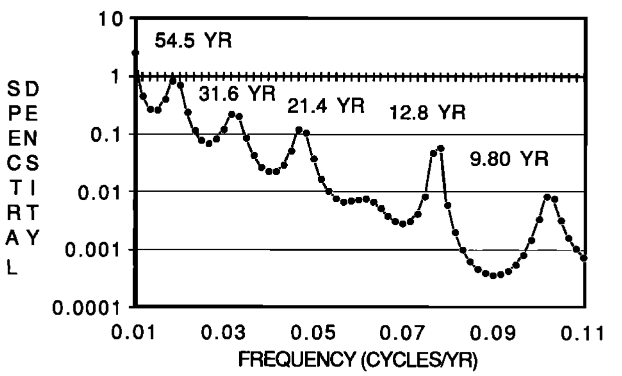

Fig. 9. Power spectral density of the detrended inclination data at London from 1843 to 1956. The number of PEF coefficients used is 50, and the number of spectral estimate is 300.

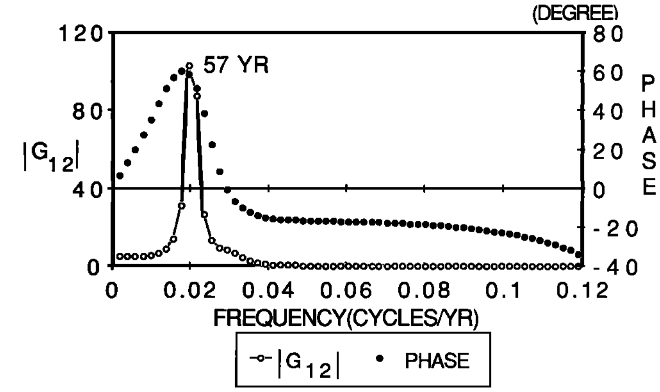

Fig. 11. Magnitude of the cross power spectral density of the detrended "minus ΔT" from 1890 to 1964 and the detrended declination data at London from 1900 to 1974. The number of PEF coefficients used is 9, and the number of spectral estimates is 256.

TABLE 1. Results of Two-Channel MESA

Initial Time Difference, years	Frequency, Cycle/yr	Squared Coherence	Apparent Phase, deg	Apparent Time Lag, years	Actual Time Lag, years
24	0.01758	0.98	−24.6	−3.9	20.1
22	0.01758	0.99	−10.3	−1.6	20.4
20	0.01953	0.99	3.2	0.5	20.5
18	0.01953	0.99	16.8	2.4	20.4
16	0.01953	0.99	27.2	3.9	20.0
14	0.01758	0.99	38.1	6.0	20.0
12	0.01953	0.98	46.9	6.7	18.7
10	0.01758	0.97	60.1	9.5	19.5
8	0.01758	0.97	66.6	10.5	18.5
6	0.01758	0.95	74.1	11.7	17.7
4	0.01758	0.93	77.6	12.3	16.3
2	0.01953	0.91	90.2	12.8	14.8
0	0.02148	0.94	104	13.5	13.5
−2	0.01953	0.91	120	17.0	15.0
−4	0.01953	0.97	141	20.1	16.1

correlation calculation by *Langel et al.* [1986] that the l.o.d. fluctuations from 1893 to 1972 lead the rate of change of the Gauss-Schmidt coefficient g_1^0 from 1903 to 1982 by approximately 13 years. It is in disagreement with the results of *Le Mouel et al.* [1981]. They reported that the first derivative of the declination at Paris leads the decade fluctuations of the Earth's rotation by approximately 10 years. Since no actual cross-correlation study was done by Le Mouel et al. except the visual inspection of the two sets of curves, the question on leading and lagging can be ambiguous. The 60-year spectral peak was observed in the variations of the dipole field of the Earth [*Jin and Thomas*, 1977]. It is also found in the secular variation of some of the Gauss-Schmidt coefficients other than the dipole terms [*Langel et al.*, 1986] and in the magnetic field data at some of the magnetic observatories [*Currie*, 1973*a*]. Thus the 60-year spectral peak is associated with the entire main geomagnetic field. Even though this study is primarily on the cross-correlation between the variations of the magnetic data around London and the fluctuations of the rotation of the Earth, it would be expected that the variations of the geomagnetic field at other locations are related to the l.o.d. fluctuations as well. The study of the variations of the amplitude and the phase angle of this 60-year spectral peak as a function of locations would improve our understanding of the core-mantle coupling and the convective motion in the core.

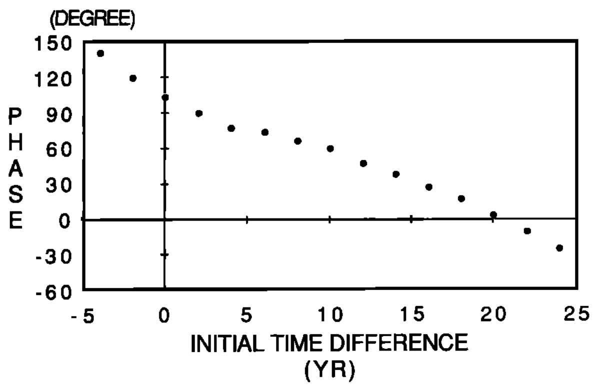

Fig. 12. Phase versus the initial time difference, where the initial time difference equals the beginning year of the magnetic data minus the beginning year of the ΔT data.

The question as to the true periodic nature of the 60-year peak cannot be answered with this limited data analysis; certainly there is no evidence that this can be traced back to the prehistoric past. Nevertheless, we have assumed that fluctuations of both the magnetic data and the l.o.d. data to be random with the possibility of the inclusion of periodic as well as transient variations. Hence the 60-year peak can be related to the hydromagnetic oscillation as suggested by *Braginsky* [1970] and *Hide* [1966]. We learned from *Braginsky* [1987] that both he and *Rivin* [1985] have shown that the 60-year oscillation of l.o.d. started near 1860.

Acknowledgments. The authors wish to thank Otto Neal Strand and Russell B. Chadwick of NOAA for providing the multichannel MESA programs and Florida Institute of Technology for the use of the VAX computer. We also want to thank S. I. Braginsky and the other anonymous reviewers whose comments have led to an improved paper.

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(Received October 31, 1988;
revised May 25, 1989;
accepted June 14, 1989.)