Quantum theory of the linewidth of a laser with a saturable absorber: phase diffusion

Michael Sokol*
University of Dayton
Department of Electrical Engineering
300 College Park
Dayton, Ohio 45469-0226

Abstract. A quantum theory of the spectral width of a laser with a saturable absorber is presented. We perform a density matrix calculation yielding the off-diagonal element $\rho_{n,n+1}(t)$ which is proportional to the average value of the electric field $\langle E(t)\rangle$ associated with the laser field having a large photon number n. The general linewidth theory is applied to any general laser which has saturable absorber features for a portion of the building of the laser field. We have found that the saturable absorber action contribution to the linewidth of such a laser can be substantial.
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1 Introduction

In the Scully-Lamb theory, the diagonal elements $\rho_{n,n}(t)$ of the density operator come to nonzero, steady-state values. Such is not the case for the off-diagonal element $\rho_{n,n+1}(t)$ which decays to zero. This element determines the ensemble average of the electric field. A decay of the off-diagonal elements implies a simultaneous decay of the average electric field which can be interpreted as being due to the laser phase diffusing from its nominal monochromatic value, thus producing a finite laser linewidth. As is well known from all sustained oscillators, the field amplitude is constrained to fluctuate about its steadystate value, yet the phase can fluctuate freely. Thus, a decay of the ensemble average of the electric field is due to a diffusion in phase denoted by a diffusion coefficient D, resulting in a diminishing vector sum. An alternate statement is that the field becomes uncorrelated with its value at an earlier time.[1] In the standard quantum theory of the laser *approximate* values for this phase diffusion rate can be calculated using a Master Equation derivation that gives the general density matrix elements $\rho_{n,m}(t)$, we let $m=n+1$ and obtain[2] the off-diagonal elements $\rho_{n,n+1}(t)$. We have reported the results of a similar calculation dealing with the linewidth floor in low-threshold lasers.[3]

2 Model of the Laser System

We consider a model of a general absorber-type laser as a four-level atomic gain medium and a resonant two-level saturable absorber medium. The two media are taken to be of similar atomic species but acting independent of one another, making them obey linearity and hence superposition. After introducing a four-level atomic medium as shown in Fig. 1 and the saturable absorber medium in Fig. 2, the two independent systems we take as comprising the general absorber-type laser.

2.1 *Four-Level Atom Model*

Much previous work on these systems is based on a rate equation analysis of a four-level system, shown in Fig. 1, where level refers to energy level.

The model has a ground state 0 (later we model the decay from the lasing levels as occurring to different intermediate levels c and d which in turn decay to the ground state), an excited level 1, an upper laser level a, and a lower laser level b. This lower level of the lasing transition, level b, is assumed to decay very fast, thus rapidly depopulating this level. The pump excites atoms into level 1 and an assumption is made that the population of level 1 rapidly decays to the upper lasing level a. The rapid nonradiative decay from the upper pump level 1 effectively results in direct pumping of the upper lasing level a at the rate P. Any changes in the population of level 0 due to the pump P is assumed small. A further assumption of this quantum model is the presence of phase-destroying collisions that occur at a much larger rate than the spontaneous emission rate of level a. In this limit, the polarization of the atoms, described in part by the off-diagonal density matrix elements damps out to zero. Thus we need only be concerned with tracking the populations of the atomic levels in such a limit. In writing these equations, we assume that the decay rate from the lower laser level b is much larger than that of the upper laser level a. Yokoyama and Brorson have written the rate equations for the population inversion N and photon number n in a semiclassical model of the four-level atomic laser system.[4] A majority of the previous work is based on semiclassical rate equations. A rate equation analysis gives us only average values of the photon number $\langle n\rangle$ or average values of intensity $\langle I\rangle$ in the cavity.

*Current affiliation: Florida Institute of Technology, Dept. of Electrical Engineering, 150 W. University Blvd., Melbourne, FL 32901.

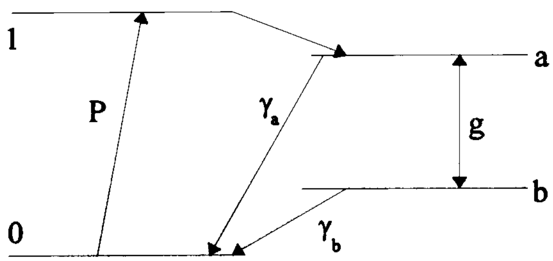

Fig. 1 Four-level atom model. The lasing action occurs over the *a* to *b* energy levels.

2.2 *Two-Level Absorber Model*

A fundamental difference between the operation of a general absorber-type laser and a four-level atomic laser is that an absorber-type laser gain medium is typically absorbing until a number of electrons is pumped into the gain region. The pumping events are supplied by the injection current which injects electrons into the gain region. Until there are more electrons in the upper lasing level a than there are in the lower laser level b, the lasing material will act as a net absorber at the lasing wavelength.

The primary mission in this paper has been the development of a quantum model of an absorber-type laser. Previous work using a four-level atom model of a laser has been done by Rice and Carmichael,[5] Bjork and Yamamoto,[6] and by Jin et al.[7] Rice and Carmichael have given relevant numerical solutions to a master equation similar to the evolution equation presented in this section. They also extend their qualitative results to a bad cavity regime. The model for our gain media for the general absorber-type system is that we model the problem as a four-level atom system and a resonant two-level atom system. Our desire was a theory sophisticated enough to model the available absorber-type laser experimental data. The theory we present in this section has that ability. We have modeled the absorption present in the absorber-type lasers by injecting two-level atoms from their ground state into the laser cavity at the rate L, as in Fig. 2. Such constructed ''loss'' atoms are taken to be exactly resonant with the laser field and act as absorbers of the laser radiation. Our model includes the feature of net absorption below a certain pump rate. We believe our theory sufficiently describes the absorber-type lasers, avoiding the complex many-body effects inherent in such laser systems. Our theory is pictorially represented as

Absorber laser

= four-level atoms injected to the upper laser level

+ two-level atoms injected to the lower laser level.

We shall derive the master equation for the general absorber-type laser and use it to find the phase-diffusion coefficient D, and hence the linewidth of these lasers in the following section.

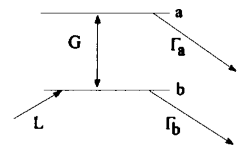

Fig. 2 Absorber atom level structure.

3 Derivation of the Master Equation for Saturable Absorber-Type Lasers

3.1 *Introduction*

The super-diagonal density matrix element $\rho_{n,n+1}(t)$ is the function expanded over in constructing the ensemble average of the electric field as

$$\langle E(t)\rangle = \frac{1}{2}E_0\sum_n \rho_{n,n+1}(t)\sqrt{n+1}\,\exp(-i\nu t)+\text{c.c.} \qquad (1)$$

It is the decay of the off-diagonal element $\rho_{n,n+1}(t)$ that implies a simultaneous decay of the electric field $E(t)$, thus producing the laser linewidth according to the Heisenberg uncertainty relationship which limits the precision with which we can measure the energy. Such a relationship states that the spontaneously emitted radiation is not totally monochromatic, but has a frequency spectrum of a width inversely proportional to a time t approximated by the cavity lifetime τ.

3.2 *The Master Equation for the Four-Level Atom System*

We assume in the usual Weisskopf-Wigner approximation that the decay of the lasing levels is independent of the stimulated coupling of the laser levels. The equation of motion of the density matrix for the field is superposed to the standard equation of motion of the density matrix representing cavity loss yielding the density matrix equation in the number representation. We have derived this equation in Reference 2 and have reported the standard result in Reference 3 to be

$$\begin{aligned}\dot{\rho}_{n,m}(t) &= \dot{\rho}_{n,m}(t)_{\text{gain}}+\dot{\rho}_{n,m}(t)_{\text{loss}}\\ &= -\frac{Pg^2\gamma_b\gamma_+(n+1+m+1)+g^2(n-m)^2}{\gamma_a\gamma_b\gamma_+^2+2\gamma_+^2g^2(n+1+m+1)+g^4(n-m)^2}\\ &\quad *\rho_{n,m}(t)+\frac{2Prg^2\gamma_b\gamma_+\sqrt{nm}}{\gamma_a\gamma_b\gamma_+^2+2\gamma_+^2g^2(n+m)+g^4(n-m)^2}\\ &\quad *\rho_{n-1,m-1}(t)-\frac{C}{2}(n+m)*\rho_{n,m}(t)\\ &\quad +C\sqrt{(n+1)(m+1)}\\ &\quad *\rho_{n+1,m+1}(t), \end{aligned} \qquad (2)$$

where P is the pumping rate of atoms from the ground state to the upper lasing level, g is the coupling rate of the four-level atoms to the laser field, the γ's are the decay rates of their subscripted energy level, C is the rate of photon removal from the laser cavity, and $\gamma_+ = (\gamma_a + \gamma_b)/2 + \gamma_{ph}$, with γ_{ph} the rate of phase destroying collisions. The diagonal element $\rho_{n,n}(t)$ corresponds to a probability of having n photons in the cavity, and is found by substituting $n = m$ into this last evolution Eq. (2), giving

$$\dot{\rho}_{n,n}(t) = \frac{-2Pg^2\gamma_b\gamma_+(n+1)}{\gamma_a\gamma_b\gamma_+^2 + 4\gamma_+^2 g^2(n+1)} * \rho_{n,n}(t) + \frac{2Pg^2\gamma_b\gamma_+(n)}{\gamma_a\gamma_b\gamma_+^2 + 4\gamma_+^2 g^2(n)} * \rho_{n-1,n-1}(t) - C(n) * \rho_{n,n}(t) + C(n+1) * \rho_{n+1,n+1}(t). \tag{3}$$

We can further manipulate the atomic coefficients into dimensionless low-threshold parameters ϵ and σ by introducing

$$\epsilon = \frac{\gamma_a}{\gamma_b}, \quad \sigma = \frac{g^2}{\gamma_+\gamma_b}, \quad \frac{\epsilon}{2\sigma} = \frac{1-\beta}{\beta}, \tag{4}$$

where the thresholdless parameter β is the ratio of spontaneous emission into the lasing mode in terms of atomic parameters as

$$\beta = \frac{\dfrac{g^2}{\gamma_+\gamma_b}}{\dfrac{g^2}{\gamma_+\gamma_b} + \dfrac{1}{2}\dfrac{\gamma_a}{\gamma_b}}. \tag{5}$$

The two low-threshold parameters ϵ and σ are standard in the quantum theory of the laser. The derived ratio $(\epsilon/2\sigma)$ is a ratio of the spontaneous emission into nonlasing modes 1-β to the spontaneous emission into a lasing mode β. Manipulating the steady-state form of Eq. (3) into a form that includes Eqs. (4) and (5) gives a recursion relation for the density matrix elements of a four-level atom as

$$\rho_{n+1,n+1}(t) = \frac{P/C}{\dfrac{1-\beta}{\beta} + (1+\epsilon)(n+1)} \rho_{n,n}(t). \tag{6}$$

This result is the basis for the work presented in Chapter III (A) of Reference 2 and is also reported in Reference 3.

The β values are obtained by fitting the light output curve to the injection pump power. The values of β for gas, conventional semiconductor, and microdisk lasers are on the order of 10^{-8}, 10^{-5}, and 10^{-1}. The β has always existed and has not been considerable until device dimensions on the order of a wavelength were realized. These microlasers have linear dimensions on the order of a micron where a significant fraction of the spontaneous emission can be coupled back into the cavity. Using these β values allows us to estimate that the ratio of ϵ to σ for gas, semiconductor, and microdisk lasers is 10^8, 10^5, and 10^1. The inverse of this ratio gives the fraction of spontaneous emission into the lasing mode. In low-threshold laser theory we are interested in this ratio where the ϵ and σ are in terms of β, as the third part of Eq. (4) shows.

3.3 *The Master Equation for the Saturable Absorber System*

The absorber action is mimicked by the injection of atoms in the lower of a pair of levels resonant with the cavity laser field. These atoms then serve as saturable absorbers. Repeating the development reported in the last section with initial conditions appropriate for injection in the lower laser level b rather than the upper laser level a, leads to a contribution to the density matrix equation of motion of

$$\dot{\rho}_{n,m}(t) = -L\left[1 - \frac{\Gamma_b\Gamma_+ G^2(n+m) + \Gamma_a\Gamma_b\Gamma_+^2}{\Gamma_a\Gamma_b\Gamma_+^2 + G^4(n-m)^2 + 2G^2\Gamma_+^2(n+m)}\right] \times \rho_{n,m}(t) + L\left[\frac{\Gamma_a\Gamma_+ 2G^2\sqrt{n+1}\sqrt{m+1}}{\Gamma_a\Gamma_b\Gamma_+^2 + G^4(n-m)^2 + 2G^2\Gamma^{+2}(n+1+m+1)}\right] \times \rho_{n+1,m+1}(t). \tag{7}$$

Here, the Γ's are the decay rates for the absorber atom levels resonant with the laser field and G is the coupling of these levels to that field. So, including the terms for the cavity loss and the injection of the four-level gain atoms as Eq. (2), superposed to the injection of the loss atoms as Eq. (7) leads to the general master equation for saturable absorber type-lasers as

$$\dot{\rho}_{n,m}(t) = -Pg^2\frac{[\gamma_b\gamma_+(n+1+m+1) + g^2(n-m)^2]}{\gamma_a\gamma_b\gamma_+^2 + 2\gamma_+^2 g^2(n+1+m+1) + g^4(n-m)^2} * \rho_{n,m}(t) + \frac{2Pg^2\gamma_b\gamma_+\sqrt{(}nm}{\gamma_a\gamma_b\gamma_+^2 + 2\gamma_+^2 g^2(n+m) + g^4(n-m)^2} * \rho_{n-1,m-1}(t) - L\left[1 - \frac{\Gamma_b\Gamma_+ G^2(n+m) + \Gamma_a\Gamma_b\Gamma_+^2}{\Gamma_a\Gamma_b\Gamma_+^2 + G^4(n-m)^2 + 2G^2\Gamma_+^2(n+m)}\right] * \rho_{n,m}(t) + L\left[\frac{\Gamma_a\Gamma_+ 2G^2\sqrt{n+1}\sqrt{m+1}}{\Gamma_a\Gamma_b\Gamma_+^2 + G^4(n-m)^2 + 2G^2\Gamma^{+2}(n+1+m+1)}\right] * \rho_{n+1,m+1}(t) - \frac{C}{2}(n+m) * \rho_{n,m}(t) + C\sqrt{(n+1)(m+1)} * \rho_{n+1,m+1}(t). \tag{8}$$

This is the central theoretical result of this work and is a new result. Recall that with $n = m$ gives the probability of n intracavity photons. These can be found using the above relation and detailed balance. With the parameter definitions of Eqs. (4) and (5) and taking $\Gamma_b \equiv 0$ we find the recursion relation for a saturable absorber-type laser as

$$\rho_{n+1,n+1}(t) = \frac{P/C}{\left[\frac{1-\beta}{\beta}+(1+\epsilon)(n+1)\right]\left[1+\frac{L/C}{n+1}\right]}\rho_{n,n}(t). \tag{9}$$

This result constitutes a new recursion relation for a lasing media which contains absorptive action during a portion of the building of the lasing field. This has been described by the injection of atoms in the lower level of the two-level atom absorber media that is resonant with the lower laser level of the gain media that undergoes injection of atoms in the upper laser level. We believe that the theory is applicable to other four-level lasers such as Nd:YAG and dye lasers[9] that have absorption prior to the onset of lasing.

4 Derivation of the Linewidth using a Phase-Diffusion Coefficient

We can obtain the evolution equation for the off-diagonal elements in a saturable absorber-type laser by substituting $m=n+1$ into Eq. (8) giving

$$\begin{aligned}\dot{\rho}_{n,n+1}(t) = &-\frac{Pg^2[\gamma_b\gamma_+(2n+3)+g^2]}{\gamma_a\gamma_b\gamma_+^2+2\gamma_+^2 g^2(2n+3)+g^4}*\rho_{n,n+1}(t)\\ &+\frac{2Pg^2\gamma_b\gamma_+\sqrt{n(n+1)}}{\gamma_a\gamma_b\gamma_+^2+2\gamma_+^2 g^2(2n+1)+g^4}*\rho_{n-1,n}(t)\\ &-\frac{LG^2[\Gamma_a\Gamma_+(2n+1)+G^2]}{\Gamma_a\Gamma_b\Gamma_+^2+2\Gamma_+^2 G^2(2n+1)+G^4}*\rho_{n,n+1}(t)\\ &+\frac{2LG^2\Gamma_a\Gamma_+\sqrt{(n+1)(n+2)}}{\Gamma_a\Gamma_b\Gamma_+^2+2\Gamma_+^2 G^2(2n+3)+G^4}\\ &*\rho_{n+1,n+2}(t)-\left(\frac{C}{2}\right)(2n+1)*\rho_{n,n+1}(t)\\ &+C\sqrt{(n+1)(n+2)}*\rho_{n+1,n+2}(t).\end{aligned} \tag{10}$$

The terms proportional to L are due to the injection of the absorber atoms. The Γ's are the decay rates and G is the atomic coupling parameter for the absorber atoms as indicated in Fig. 2. We assume that the reduced density matrix elements are initially in a pure-state which allows one to use the factorization ansatz method in Scully-Lamb theory, i.e.,

$$\rho_{n,n+1}(t) = \sqrt{\rho_{n,n}\rho_{n+1,n+1}}\exp(-Dt/2), \tag{11}$$

where D is a time-independent phase diffusion coefficient. With this, the ensemble average of the electric field associated with the laser field as in Eq. (1) becomes

$$\langle E(t)\rangle = \langle E(0)\rangle\cos(\nu t)\exp(-Dt/2). \tag{12}$$

Substitution of the ansatz Eq. (11) into the evolution Eq. (10) gives

$$\begin{aligned}\frac{-D}{2} = &-\frac{Pg^2[\gamma_b\gamma_+(2n+3)+g^2]}{\gamma_a\gamma_b\gamma_+^2+2\gamma_+^2 g^2(2n+3)+g^4}\\ &+\frac{2Pg^2\gamma_b\gamma_+\sqrt{n(n+1)}}{\gamma_a\gamma_b\gamma_+^2+2\gamma_+^2 g^2(2n+1)+g^4}*\frac{\rho_{n-1,n}(t)}{\rho_{n,n+1}(t)}\\ &-\frac{LG^2[\Gamma_a\Gamma_+(2n+1)+G^2]}{\Gamma_a\Gamma_b\Gamma_+^2+2\Gamma_+^2 G^2(2n+1)+G^4}\\ &+\frac{2LG^2\Gamma_a\Gamma_+\sqrt{(n+1)(n+2)}}{\Gamma_a\Gamma_b\Gamma_+^2+2\Gamma_+^2 G^2(2n+3)+G^4}*\frac{\rho_{n+1,n+2}(t)}{\rho_{n,n+1}(t)}\\ &-\left(\frac{C}{2}\right)(2n+1)+C\sqrt{(n+1)(m+1)}\\ &*\frac{\rho_{n+1,n+2}(t)}{\rho_{n,n+1}(t)}.\end{aligned} \tag{13}$$

The two unique ratios of participating density matrix elements are

$$\begin{aligned}\frac{\rho_{n-1,n}(t)}{\rho_{n,n+1}(t)} &= \sqrt{\frac{\rho_{n-1,n-1}(t)}{\rho_{n+1,n+1}(t)}}\\ \frac{\rho_{n+1,n+2}(t)}{\rho_{n,n+1}(t)} &= \sqrt{\frac{\rho_{n+2,n+2}(t)}{\rho_{n,n}(t)}}.\end{aligned} \tag{14}$$

Substitution Eq. (14) using Eq. (13) gives the participating ratio of the elements as

$$\sqrt{\frac{\rho_{n-1,n-1}(t)}{\rho_{n+1,n+1}(t)}} = \left(\frac{1}{P/C}\right)\sqrt{\left[\frac{1-\beta}{\beta}+(1+\epsilon)(n+1)\right]\left[1+\frac{L/C}{n+1}\right]\left[\frac{1-\beta}{\beta}+(1+\epsilon)(n)\right]\left[1+\frac{L/C}{n}\right]}$$

$$\sqrt{\frac{\rho_{n+2,n+2}(t)}{\rho_{n,n}(t)}} = (P/C)\sqrt{\frac{1}{\left[\frac{1-\beta}{\beta}+(1+\epsilon)(n+2)\right]\left[1+\frac{L/C}{n+2}\right]\left[\frac{1-\beta}{\beta}+(1+\epsilon)(n+1)\right]\left[1+\frac{L/C}{n+1}\right]}}. \tag{15}$$

Substituting these ratios of the matrix elements in Eq. (15) into Eq. (13) and specializing to the typical case in which the ratio of the upper and lower lasing level decay rate, $\gamma_a/\gamma_b=\epsilon$ becomes much less than unity, and for average photon number $\overline{n}$ much larger than unity, we find a diffusion coefficient for saturable absorber type lasers

$$D=\frac{C}{2\overline{n}}+L\frac{\sigma}{\overline{n}}. \tag{16}$$

The linewidth D thus depends on the total losses $(C/2)+L\sigma$. This result was obtained analytically and checked using MAPLE and has some features new to laser linewidth theory. We can observe that the linewidth depends on the two losses. The $(C/2\overline{n})$ term is the linewidth contribution due to the spontaneous emission and is always present in any laser system. The $(L\sigma/\overline{n})$ term is the contribution due to the loss introduced by the saturable absorber. We can take the rate $L\sigma$ to be the effective loss rate due to the presence of an absorptive mechanism in the gain media of a laser. The $(C/2)+L\sigma$ plays a role similar to the results of a Langevin treatment of the laser. In such analysis, the diffusion coefficient relates the fluctuation and dissipation constants through a fluctuation-dissipation theorem, implying proportional fluctuation power. The lasers that have absorption are dye, solid-state, and semiconductor gain media. We find that the first term in Eq. (16) reproduces Schawlow-Townes result in agreement with the geometrical vector-kick model given in Reference 8. The second term is due to the presence of the saturable absorber. The $L\sigma$ term represents the inherited noise due to dissipation of energy of a lasing medium.

Some estimation of the magnitudes of these terms is desirable at this point. The term dependent on the injection of the saturable absorbers becomes important when the rate of injection L becomes larger than the cavity loss rate C. For the quantum theory to be comparable with the rate equation prediction for the steady-state photon number above threshold we constrain Eq. (16) to have

$$L=\gamma_a N(1-\beta). \tag{17}$$

For example, in the microcavity semiconductor lasers $N=10^5$ is the carrier number at transparency and β is on the order of 0.1 and $\gamma_a=10^8$. That is, L is on the order of 10^9. The cavity loss rate C for these devices is taken to be on the order of 10^{-11}. As discussed in Reference 3, the σ linewidth floor might be as large as 10^{-4}. For lasers modeled as a four-level system, it may be true for where the losses due to the absorption L play an important role in determining the laser linewidth. In other laser systems in which the saturable absorber plays a larger role, this may be even more important. It may be that the L rate in the second term is always smaller than the Schawlow-Townes term for any known laser system and cannot be comparable. We note that there is strong absorption action in Nd:YAG, dye, and semiconductor gain media.

5 Conclusion

The key results in this theoretical paper have been developed and reported in section 4 where a viable quantum mechanical theory of the linewidth, via a diffusion coefficient D, for absorber-type lasers was formed. The Scully-Lamb theory has been extended to this class of lasers that include a saturable absorber. We have obtained a master equation of the off-diagonal density matrix elements which are proportional to the average value of the electric field operator associated with a large photon number laser field. We have obtained approximate linewidth results that are new to laser theory for a gain medium having saturable absorber features for a portion of the laser beam evolution. This model has been derived for saturable absorber-type lasers, though the results are generally applicable to any four-level laser system involving saturable losses and models some aspects of dye lasers and their bleaching action as well as microcavity semiconductor devices. The model could yield a set of design relations used by device developers incorporating lasers with absorbing-type gain media.

Acknowledgments

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Michael Sokol holds the BSME, BSEE, MSEE, and MS in optical physics all from the University of Bridgeport, Connecticut. He also holds an MS in applied math, MSEO, and PhD in electrical engineering all from the University of Dayton, Ohio. He is presently at the Florida Institute of Technology where his research interests are in mathematical physics, laser theory, photonic devices, complex analysis, and nonlinear/quantum optics.