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Samuel Peter Kozaitis
Harold Gregory Andrews
Wesley E. Foor

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S. P. Kozaitis

**Florida Institute of Technology,
Department of Electrical and Computer Engineering
Melbourne, FL 32901-6988**

H. G. Andrews, W. E. Foor

**Rome Laboratory
Photonics Center
Griffiss AFB, NY 13441**

ABSTRACT

Using an optical technique, we classified images of natural terrain based on their fractal dimension. We calculated the fractal dimension from an optically generated power spectrum obtained with a magneto-optic spatial light modulator (SLM). By using the fractal dimension to classify images of natural terrain, our post processing was simpler than when a ring-wedge detector was used.

1.0 INTRODUCTION

In many pattern recognition applications, characteristic features provide critical information of a scene. For example, the power spectrum of an image may be sampled with a ring-wedge detector to obtain rotation and scale invariant features for product inspection or classification of terrain imagery.^{1,2} These features may form a 1-dimensional feature vector that is compared to a database of feature vectors using pattern recognition algorithms. If the power spectrum is generated optically, a substantial speed advantage may exist when compared to an electronic system. Although such a system may possess a speed advantage in the computation of the power spectrum, the comparison of a large feature vector to a large database in an attempt to find the best match may prove time consuming. To simplify this classification step, we examined the use of the fractal dimension to model natural terrain imagery.

Although several algorithms have been used to estimate the fractal dimension³, we used a technique based on an image's power spectrum.⁴ By examining the power spectrum as a function of spatial frequency, rotation invariance of a scene was obtained. Using the self-scaling characteristic of fractals, we obtained scale invariance. Our technique resulted in a number, the fractal dimension, that represented a terrain's "roughness". The fractal dimension was easily compared to dimensions of other images for classification. The technique was simpler than comparing imperfect matches of a feature vector with a large database as is often the case when a wedge-ring detector is used. In the next section we briefly discuss the fractal dimension followed by a description of our optical experiment. Then, we describe results and discuss our conclusions.

2.0 FRACTAL DIMENSION

Although there are a variety of practical issues associated with making an estimation of fractal dimension from finite data, it has been shown that fractals may be well-suited to some signal processing applications. For example, the estimation of fractal dimension has been used to perform texture segmentation and classification in images of natural terrain.⁵

The idea of a fractal dimension arises from the notion that the dimension required to measure an entity may not be an integer value. The Hausdorff-Besicovitch dimension is the dimension that, when used to measure an object, will yield a finite non-zero result. For instance, the dimensionality of a line segment is one, the tool for its measure being length. The two dimensional measure (area) of a line segment yields a zero measure. Conversely, using length to measure a two dimensional entity will yield a result of infinity, since an infinitely long line may be packed into an arbitrarily small planar region.

The Sierpinski Gasket (an approximation of which is shown in Fig. 1) is generated by removing the middle portion of a square, and then recursively doing the same to each of the eight remaining sub-squares. An infinite number of recursions are required for generating the Sierpinski Gasket. The measure of the dust left over will be infinite when using length, and zero when using area. This is said to have a fractional or fractal dimensionality.^{6,7}

Imagery of natural terrain (and computer generated terrain) vary in their fractal dimension, and can be thought of as a type of Brownian Surface.⁷ These differences can be used for image segmentation and for detecting regions of interest.

In one topological dimension, a Fractal Brownian line function $B_H(t)$ with a Fourier power spectrum $F_H(f)$ may be described as⁵

$$F_H(f) \propto f^{-\beta} \quad (1)$$

and H is related to β by

$$\beta = 2H + 1 \quad (2)$$

and to the fractal dimension D of $B_H(t)$ by

$$D = 2 - H. \quad (3)$$

Here $H \in (0,1)$ and is used to describe the fractional portion of the curve's dimensionality.

In two topological dimensions, a Fractal Brownian surface function $B_H(x,y)$ with a Fourier power spectrum $F_H(f, \theta)$ may be described as^{5,8}

$$F_H(f, \theta) \propto f^{-\beta} \quad (4)$$

where

$$B = 2H + 2. \quad (5)$$

Furthermore, under certain assumptions, a Fractal surface with power spectrum described as in Eq. (4) produces an image with power spectrum⁸

$$F_I(f, \theta) \propto f^{2-\beta} \quad (6)$$

From the power spectrum of an image, we calculated D from the slope of a line produced by a least-squares fit to a log-log plot of power to frequency.

In our study, we considered 256 x 256 binary images. We initially used the Sierpinski Gasket which has D in the limit of 1.89, then calculated D digitally using the power spectrum method for comparison. We summed the energy in the Fourier plane using rings 4 pixels wide with 32-bit floating point data. Using the power spectrum method, we digitally computed D to be 1.96. Error was introduced through the digital approximation of the image, the approximation of rings with square pixels, and quantization. Using the optical system described the next section, we calculated D to be 1.99. Additional error in the optical estimation primarily resulted from the shape of the laser beam, finite pixel size of the spatial light modulator (SLM) and optical aberrations. In this work, we were mainly interested in the relative difference of dimension between different classes of images rather than their actual value of D .

3.0 OPTICAL SYSTEM

We used a Fourier optical system like that shown in Fig. 2 to obtain the power spectrum of an image. The SLM was placed in a converging beam so we capture the power spectrum with 256 x 256 pixels of the CCD camera. Collimated light from a He-Ne laser with $\lambda = 633\text{nm}$ illuminated the system that included a Semetex 256 x 256 magneto-optic SLM. The location of a particular spatial frequency u is given by

$$u = \frac{x}{\lambda f} \quad (7)$$

where f is the focal length of the lens, λ is the wavelength of the light being used and x is the location in the Fourier plane. We used an achromatic lens with $f = 1000$ mm and the SLM had square pixels of $56\mu\text{m}$ on a side with $76\mu\text{m}$ center-to-center spacing. The power spectrum was recorded by a camera whose signal was passed through a logarithmic amplifier before being digitized to 8-bits by a computer. The image recorded by the camera was 256×256 pixels and was divided into 29 rings each 4 pixels wide. The SLM was operated in the maximum contrast mode and the intensity of the power spectrum was adjusted by neutral density filters. The computer calculated the fractal dimension from the slope of a line obtained with a least-squares fit on a log - log plot of power to spatial frequency.

4.0 EXPERIMENT

To evaluate the feasibility of our approach, we attempted to discriminate between three different types of terrain. We used images obtained from photographs of mountains, rivers and clouds that were taken from directly overhead at a scale of 1:16,000. The images were thresholded for display on our SLM and shown in Fig. 3.

With a digital system, many orders of magnitude of values can be represented in floating point form. We used a camera that is typical of a machine vision system and could distinguish only 8 bits. When the power spectrum was normalized, the intensity in the vicinity of the DC was often much greater than at other frequencies. Therefore, the value of intensity at frequencies other than in the vicinity of the DC would be measured to be near 0. Because the region around the DC dominated the spectrum, we let this portion saturate and blocked a circle of radius 10 pixels when digitizing the power spectrum. In this way, energy in higher spatial frequencies could be recorded. Furthermore, we passed the video signal from the camera through a logarithmic amplifier. The power spectrum of the images in Fig. 3 were recorded by our system and shown in Fig. 4.

We compared the results where the power spectrum generated optically to digital calculations. In addition, we compared results when the logarithmic amplifier was and was not used. When the logarithmic amplifier was not used, we decreased the density of the neutral density filter to allow more light through so that energy in the higher spatial frequencies could be recorded. A comparison of the results is shown in Table 1.

TABLE 1. Estimates of Fractal Dimension

Scene	Computed digitally	8-bit camera	8-bit camera with log amplifier
Mountains	1.44	1.49	1.35
Rivers	1.40	1.43	1.27
Clouds	1.30	1.40	1.22

The results showed that the values obtained optically generally agree with those computed digitally in a relative sense. For example, the value of D for the mountains was always higher than that of the riv-

ers and clouds. The estimates of D obtained with the logarithmic amplifier were generally lower, and the results obtained with a linear camera were generally higher than that obtained that computed digitally.

At high spatial frequencies, the recorded signal was dominated by noise. Therefore, we eliminated some of the values from the calculation of the slope of the line of energy to frequency because the noisy data points could change the value of the slope and therefore D . We eliminated values associated with the outermost rings until the slope remained constant. A graph of energy vs. frequency measured optically for images in Figs. 3a and 3c are shown in Fig. 5. It can be seen that for Fig. 3c, noise dominates at lower frequencies than for Fig. 3a. Finally, when an image was displayed on the SLM, often a few rows or columns of the device would yield constant dark or bright values. This characteristic could have changed our estimates of D by changing the amount of energy along the axis in the Fourier plane.

5.0 SUMMARY AND CONCLUSIONS

We calculated the fractal dimension of an image from an optically generated power spectrum using a magneto-optic SLM. Our results showed that values obtained optically generally agree with those computed digitally in a relative sense. Using this method, we expect a substantial speed advantage when compared to an electronic system or optical system using a ring-wedge detector for classifying terrain. Our technique resulted in a single number, the fractal dimension to characterize a terrain. The fractal dimension was easily compared to dimensions of other images for classification. Because the classification was based on a single value, it may not be able to discriminate as well as other methods.

6.0 ACKNOWLEDGMENT

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7.0 REFERENCES

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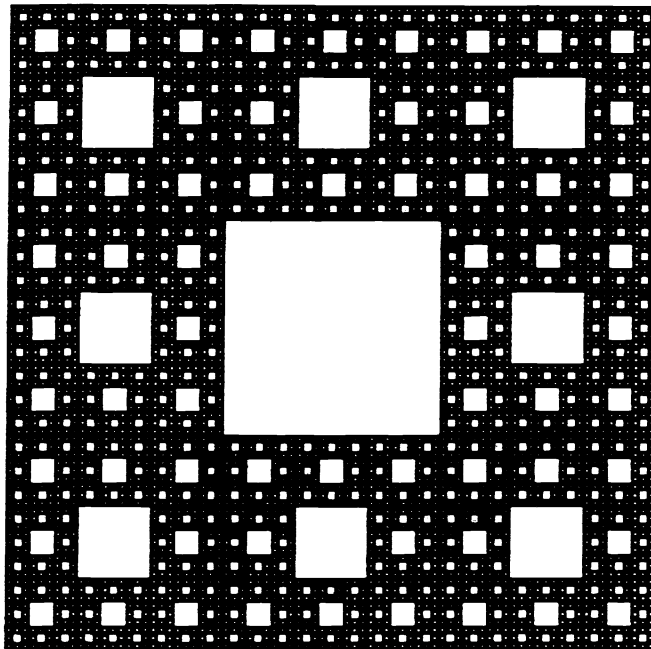


FIGURE 1. Approximation of Sierpinski Gasket

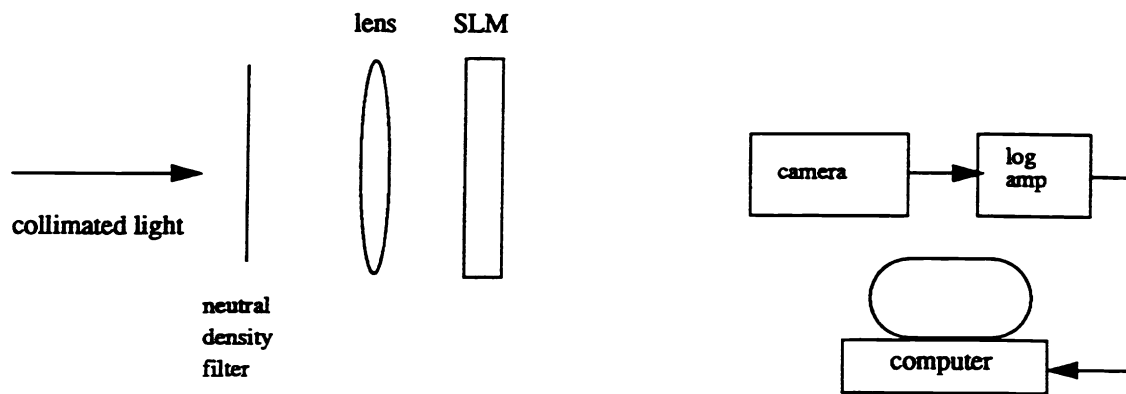
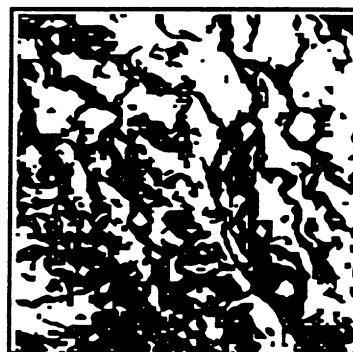


FIGURE 2. Schematic diagram of optical system



(a)



(b)



(c)

FIGURE 3. Images of natural terrain

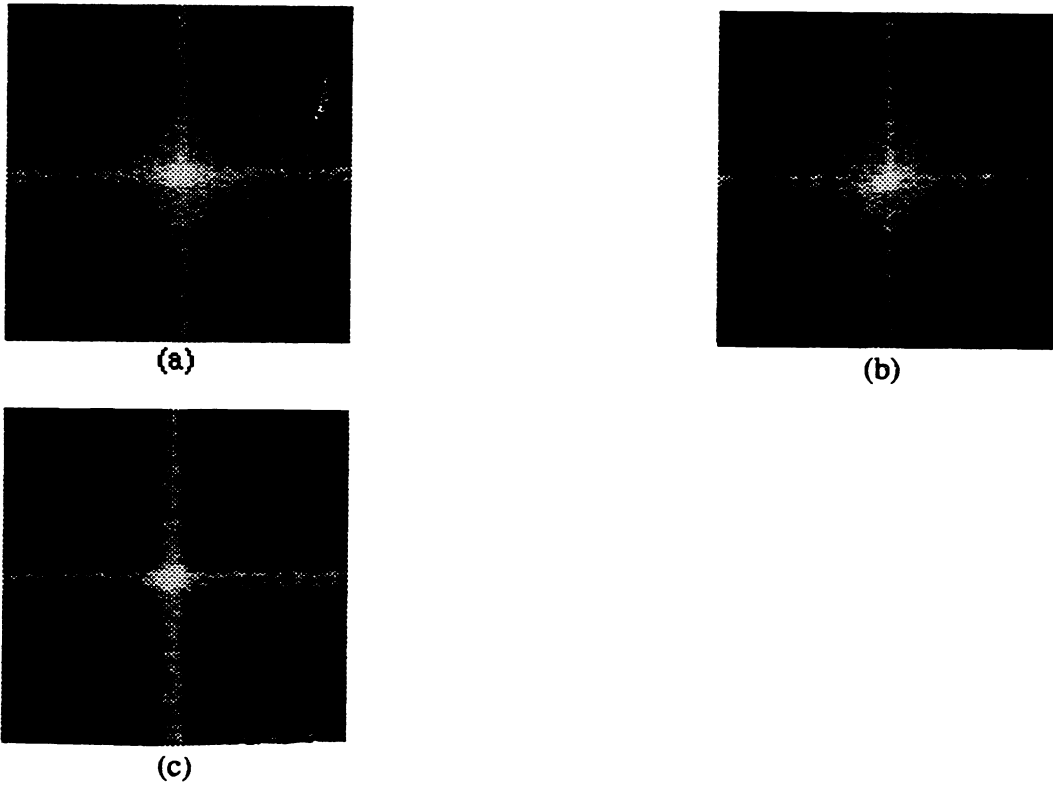


FIGURE 4. Power spectra of images of natural terrain generated optically

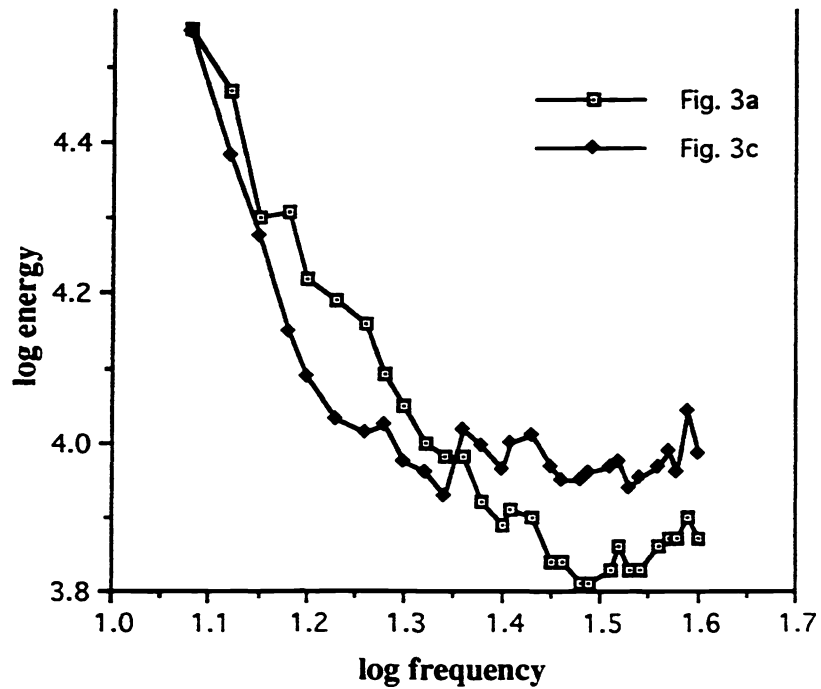


FIGURE 5. Graph of log energy vs. log frequency of Figs. 3a and 3c.