Obscured object detection via Bayesian target modeling techniques

Rufus H. Cofer

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R. H Cofer
Dept. of Electrical/Computer Engineering
Florida Institute of Technology
150 W. University Blvd.
Melbourne, FL 32901-6988
(407) 768-8000 x8818
rhcofer@ee.fit.edu

ABSTRACT

Underground objects are by nature often severely obscured although the general character of the intervening random media may be reasonably understood. The task of detecting these underground objects also implies that their exact location and or orientation is not known. To partially counter these difficulties, one may; however, be given a model of the target of interest, e.g. a particular tank type, a water pipe, etc.

To set up a quality framework for solution of the above problem, this paper utilizes the paradigm of Bayesian decision theory that promises minimum error detection given that certain probability density functions can be found. Within this framework, mathematical techniques are shown to handle the uncertainties of target location and orientation, many of the random obscuration problems, and how to make best use of the target model. The approach taken can also be applied to other synergistic cases such as seeing through obscuring vegetation.

1.0 INTRODUCTION

The problem of detecting objects underneath intervening layers of material is a difficult one which has attracted much attention.[1-5] Of the many ways which microwave radar can be applied to the problem, there is need of low-cost, compact and real time systems. The system of Figure 1. provides a potential answer. Here a

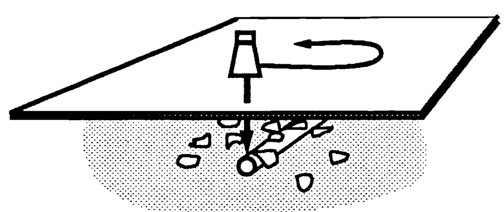

Figure 1. Imaging System

chirped or FM-CW system[1] is raster scanned over the region of interest to build up a volumetric image. The subsurface medium is typically modeled as homogeneous except for the object to be viewed.[4] However, Fig-

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ure 1. is more realistic. In fact, there are three major sources of interference: specular reflection from the surface, spurious paths between transmitter and receiver and diffuse surface scatter from the angular illumination of the beam. In addition, the subsurface medium is seldom homogeneous, although certain media such as snow can at times[1] be so considered. Due to the assortment of problems inevitably found in subsurface viewing, there is considerable need to move to more optimal techniques centered around better signal processing of the returns.

In Section 2.0, the essentials of optimal Bayesian detection/recognition are briefly described, in Section 3.0 and in Section 4.0, theory is advanced for the treatment of non-homogeneous media.

2.0 BAYESIAN DETECTION/RECOGNITION THEORY

The Bayesian recognizer is optimal in terms of probability of recognition and false alarm. It can be given[6] as

$$\text{If} \quad \Lambda > \eta \quad \text{then recognize } H_1 \quad \text{else recognize } H_0 \tag{1}$$

where Λ, the likelihood function, is

$$\Lambda = \frac{p_{R|H_1}(R|H_1)}{p_{R|H_0}(R|H_0)} \tag{2}$$

η, a decision threshold independent of the imaged data, R, is

$$\eta = \frac{(c_{10} - c_{00})\, P(H_0)}{(c_{01} - c_{11})\, P(H_1)} \tag{3}$$

and the c's are the relative costs of correct and erroneous recognitions of H_1 and H_0.

Letting H_1 and H_0 be the cases where there is or is not a subsurface target in the volumetric image, R, the Bayesian recognizer of Eq. (1) can be suited to the problem of detecting the presence or absence of the subsurface target. This is accomplished by properly evaluating Λ.

To evaluate Λ, one needs the probability density functions of $p_{R|H_1}(R|H_1)$ and $p_{R|H_0}(R|H_0)$. These are further developed below in Section 4.0. First; however, it is useful to further simplify the likelihood function, Λ.

3.0 SIMPLIFICATION OF THE LIKELIHOOD FUNCTION

Direct determination of the likelihood function is overly difficult. An approach toward simplification is to decompose it in terms of yet simpler functions. The process is begun by consideration of target indexing.

An indexing variable[7] is a target attribute such as position, orientation, articulation, size, etc. where every value is mutually exclusive. A target index, I_i, is an element of the Cartesian product of any such number of target indexing variables. The main use of indexing is in simplification of the calculation of $p_{\mathbf{R}|H_1}(R|H_1)$

$$p_{\mathbf{R}|H_1}(R|H_1) = \sum_i p_{\mathbf{R}|I_i,H_1}(R|I_i,H_1)\,p_{I_i|H_1}(I_i|H_1) \tag{4}$$

As further simplification, the volumetric subsurface image can generally be partitioned into various columns, r_j, with independent stochastic behaviors. This permits Eq. (4) to be expanded further to

$$p_{\mathbf{R}|H_1}(R|H_1) = \sum_i \prod_j p_{r_j|I_i,H_1}(r_j|I_i,H_1)\,p_{I_i|H_1}(I_i|H_1) \tag{5}$$

If no target is present, the development of $p_{\mathbf{R}|H_0}(R|H_0)$ is obviously independent of target indexing. Again partitioning the volumetric image into columns, r_j, with independent stochastic behaviors gives

$$p_{\mathbf{R}|H_0}(R|H_0) = \prod_j p_{r_j|H_0}(r_j|H_0) \tag{6}$$

Substituting Eq.'s (5) and (6) into Eq. (2) gives for the likelihood

$$\Lambda = \sum_i p_{I_i|H_1}(I_i|H_1) \prod_j \frac{p_{r_j|I_i,H_1}(r_j|I_i,H_1)}{p_{r_j|H_0}(r_j|H_0)} \tag{7}$$

Now for any columns, r_j, not affected by target presence

$$p_{r_l|I_i,H_1}(r_l|I_i,H_1) = p_{r_l|H_0}(r_l|H_0) \tag{8}$$

and so

$$\frac{p_{r_l|I_i,H_1}(r_l|I_i,H_1)}{p_{r_l|H_0}(r_l|H_0)} = 1 \tag{9}$$

Thus by the theorem of irrelevance[6], all regions, r_j, not affected by target presence need not enter into the $\prod$ calculation of Eq. (7). This greatly improves computational efficiency and retention of precision during the likelihood calculation.

Now considering the local likelihood within Eq. (7)

$$\Lambda_j \equiv \frac{p_{r_j|I_i,H_1}(r_j|I_i,H_1)}{p_{r_j|H_0}(r_j|H_0)} \tag{10}$$

the column of data, r_j, can be partitioned into observations, ϑ_k, based on depth of viewing in the column

$$\Lambda_j = \frac{p_{\vartheta_n, \vartheta_{n-1}, \dots, \vartheta_1 | I_i, H_1}(\vartheta_n, \vartheta_{n-1}, \dots, \vartheta_1 | I_i, H_1)}{p_{\vartheta_n, \vartheta_{n-1}, \dots, \vartheta_1 | H_0}(\vartheta_n, \vartheta_{n-1}, \dots, \vartheta_1 | H_0)} \tag{11}$$

or

$$\Lambda_j = \frac{p_{\vartheta_n | \vartheta_{n-1}, \dots, \vartheta_1, I_i, H_1}(\vartheta_n | \vartheta_{n-1}, \dots, \vartheta_1, I_i, H_1)}{p_{\vartheta_n | \vartheta_{n-1}, \dots, \vartheta_1, H_0}(\vartheta_n | \vartheta_{n-1}, \dots, \vartheta_1, H_0)}$$

$$\bullet \frac{p_{\vartheta_{n-1} | \vartheta_{n-2}, \dots, \vartheta_1, I_i, H_1}(\vartheta_{n-1} | \vartheta_{n-2}, \dots, \vartheta_1, I_i, H_1)}{p_{\vartheta_{n-1} | \vartheta_{n-2}, \dots, \vartheta_1, H_0}(\vartheta_{n-1} | \vartheta_{n-2}, \dots, \vartheta_1, H_0)}$$

$$\bullet \quad \bullet \quad \bullet$$

$$\bullet \frac{p_{\vartheta_1 | I_i, H_1}(\vartheta_1 | I_i, H_1)}{p_{\vartheta_1 | H_0}(\vartheta_1 | H_0)} \tag{12}$$

But since

$$p_{\vartheta_i | \vartheta_{i-1}, \dots, \vartheta_1, I_i, H_1}(\vartheta_i | \vartheta_{i-1}, \dots, \vartheta_1, I_i, H_1) = p_{\vartheta_i | \vartheta_{i-1}, \dots, \vartheta_1, H_0}(\vartheta_i | \vartheta_{i-1}, \dots, \vartheta_1, H_0) \tag{13}$$

for depths shallower than the target, then

$$\Lambda_j = \frac{p_{\vartheta_n | \vartheta_{n-1}, \dots, \vartheta_1, I_i, H_1}(\vartheta_n | \vartheta_{n-1}, \dots, \vartheta_1, I_i, H_1)}{p_{\vartheta_n | \vartheta_{n-1}, \dots, \vartheta_1, H_0}(\vartheta_n | \vartheta_{n-1}, \dots, \vartheta_1, H_0)} \tag{14}$$

providing further simplification of the overall calculation.

4.0 TREATMENT OF NON-HOMOGENEOUS MEDIA

As shown in Figure 1., subsurface media is typically non-homogeneous. The effects of this non-homogeneity is mitigated somewhat by the fact that the imaging process is essentially volumetric where there are separate returns at different depths. While 3-D viewing helps, it is no panacea. Intervening objects such as ore bearing rocks in sedentary subsurface media can reflect significant energy, which in turn affects the illumination of the actual target being sought at a deeper depth. Reduced target illumination causes the target to be more difficult to see in the clutter thereby reducing possibility of correct detection and recognition. What is needed is a means of adjusting the recognition process for the effects of intervening non-homogeneous media. To accomplish this, it is necessary to model the received signal chain, Figure 2., to determine relationships between the ϑ_k.

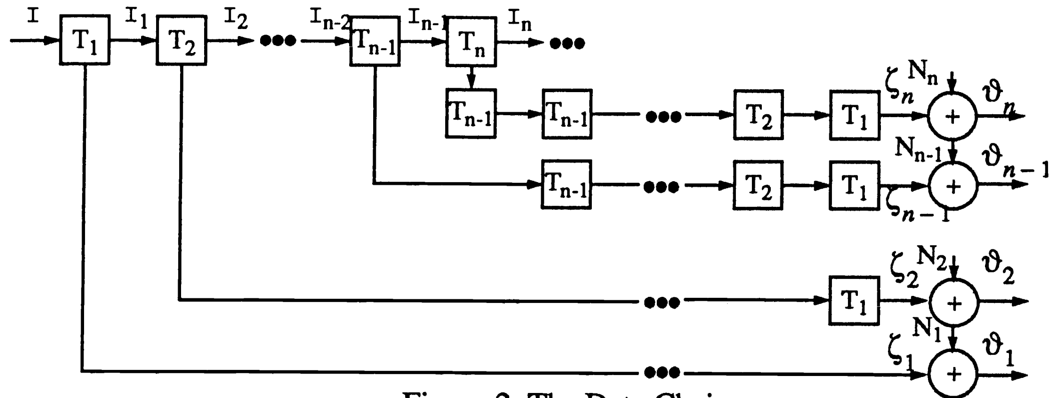

Figure 2. The Data Chain

The radar illumination at the ground surface is I. After it has gone through the first layer, it is I_1, the second layer, I_2, and so forth.The target index is such that the target resides at the I_n'th layer if it is present. The reflectance at each layer is α_i. Since the reflected signal must return through intervening media, the output signal, ϑ_k, for the k'th layer is

$$\vartheta_k = \alpha_k \prod_{l=1}^{k-1} (1-\alpha_l)^2 I + N_k \tag{15}$$

In general, the α_i's are independent random variables with probability density functions, $p(\alpha_i)$. In the case of the n'th layer, the probability can be conditioned on the presence or absence of the target as $p(\alpha_i|H_1)$ and $p(\alpha_i|H_0)$ respectively. All other $p(\alpha_i)$ at lesser depths do not depend upon the present or absence of the target. The N_k's of Eq. (15) represent the radar receiver noise and clutter for the various depths.

Through attention to the signal chain modeling, it is now possible to reduce Eq. (14) to

$$\begin{aligned}\Lambda_j = (\sum_{\alpha\text{'s}}(&p_{\vartheta_n|\alpha_{n-1},\ldots,\alpha_1,I_i,H_1}(\vartheta_n|\alpha_{n-1},\ldots,\alpha_1,I_i,H_1)\\ &\bullet p_{\alpha_{n-1},\ldots,\alpha_1|\vartheta_{n-1},\ldots,\vartheta_1}(\alpha_{n-1}|\vartheta_{n-1},\ldots,\vartheta_1))\\ &/(\sum_{\alpha\text{'s}}(p_{\vartheta_n|\alpha_{n-1},\ldots,\alpha_1,H_0}(\vartheta_n|\alpha_{n-1},\ldots,\alpha_1 p,H_0)\\ &\bullet p_{\alpha_{n-1},\ldots,\alpha_1|\vartheta_{n-1},\ldots,\vartheta_1}(\alpha_{n-1}|\vartheta_{n-1},\ldots,\vartheta_1)))\end{aligned} \tag{16}$$

While Eq. (16) is composed of more terms, each term is becoming more simpler in character.

Continuing

$$p_{\alpha_{n-1}, \ldots, \alpha_1 | \vartheta_{n-1}, \ldots, \vartheta}(\alpha_{n-1} | \vartheta_{n-1}, \ldots, \vartheta_1) = p_{\alpha_{n-1}, \ldots, \alpha_1 | \zeta_{n-1}, \ldots, \zeta_1}(\alpha_{n-1} | \zeta_{n-1}, \ldots, \zeta_1) \prod_{i=1}^{n-1} p_{\zeta_i | \vartheta_i}(\zeta_i | \vartheta_i) \tag{17}$$

Given knowledge of the receiver noise probability density function, the $p_{\zeta_i | \vartheta_i}(\zeta_i | \vartheta_i)$ are easily computed.

Computation of $p_{\alpha_{n-1}, \ldots, \alpha_1 | \zeta_{n-1}, \ldots, \zeta_1}(\alpha_{n-1} | \zeta_{n-1}, \ldots, \zeta_1)$ proceeds as follows. First, solve for the ζ_j's in terms of the α_i's as follows

$$\begin{aligned} \zeta_1 &= \alpha_1 \\ \zeta_2 &= \alpha_2 (1-\alpha_1)^2 \\ \zeta_3 &= \alpha_3 (1-\alpha_2)^2 (1-\alpha_1)^2 \\ &\vdots \end{aligned} \tag{18}$$

and compute the Jacobian, $J[\alpha_1, \ldots, \alpha_{n-1}]$.

Now, solve for the α_i's in terms of the ζ_j's as follows

$$\begin{aligned} \alpha_1 &= \zeta_1 \\ \alpha_2 &= \frac{\zeta_2}{(1-\zeta_1)^2} \\ \alpha_3 &= \frac{\zeta_3}{((1-\zeta_1)^2 - \zeta_2)^2} \\ &\vdots \end{aligned} \tag{19}$$

Noting that the probability density functions, $p_{\alpha_i}(\alpha_i)$, are typically independent due to the random nature of media results in

$$p_{\alpha_{n-1}, \ldots, \alpha_1 | \zeta_{n-1}, \ldots, \zeta_1}(\alpha_{n-1} | \zeta_{n-1}, \ldots, \zeta_1) = p_{\alpha_1}(\zeta_1) p_{\alpha_2}\left(\frac{\zeta_2}{(1-\zeta_1)^2}\right) p_{\alpha_3}\left(\frac{\zeta_3}{((1-\zeta_1)^2 - \zeta_2)^2}\right) \ldots J[\zeta_1, \ldots, \zeta_{n-1}] \tag{20}$$

which is easily computed.

The final probabilities to be expressed in terms of simpler quantities are $(p_{\vartheta_n|\alpha_{n-1},\dots,\alpha_1,I_i,H_1}(\vartheta_n|\alpha_{n-1},\dots,\alpha_1,I_i,H_1)$ and $(p_{\vartheta_n|\alpha_{n-1},\dots,\alpha_1,H_0}(\vartheta_n|\alpha_{n-1},\dots,\alpha_1,H_0)$. From the ϑ_n signal path of Figure 2., and Eq. (18)

$$\zeta_n = \alpha_n (1-\alpha_{n-1})^2 \dots (1-\alpha_1)^2 \tag{21}$$

At this point only α_n is a random variable since the other α's are know via conditioning. Thus the calculation of the probability density function ζ_n is straightforward given the probability density function of the radar reflectivity of the n'th layer. This probability is dependent upon whether the target or background is assumed to be present.

Given the probability density function of ζ_n, the probability density function of ϑ_n is straightforward - based on the probability density function for the receiver noise, N_i.

This completes the full development of the Bayesian likelihood ratio required for optimal decision in terms of simple constituent probabilities. Figure 3. shows the required interface structure of the resulting signal process module.

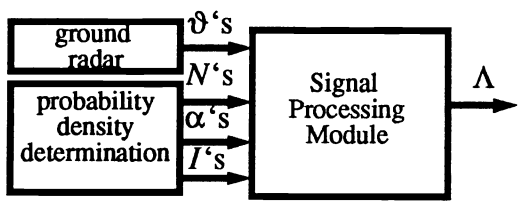

Figure 3. Signal Processing Module

5.0 CONCLUSIONS AND FUTURE WORK

This paper has shown the theoretical development required to more properly account for non-homogeneous media in underground radar. It has also shown many of the mathematical simplifications which are possible in practice.

Our future work is now centered in two areas. We have interest in probability density determination adaptively based upon the radar returns themselves to allow more automatic bootstraping of the process. We also are working to further simplify the calculations within the signal processing module. The technique of maximum current interest is heuristic pruning of low informational terms making up the likelihood ratio. One candidate means involves making early decision as to target indexing and then seeing if the decision is supported by high informational pathways through the reduced domain likelihood computation.

6.0 REFERENCES

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