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Robust Linear Quadratic Regulation using Neural Network

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ABSTRACT

Using an Artificial Neural Network(ANN) trained with the Least Mean Square(LMS) algorithm we have designed a robust linear quadratic regulator for a range of plant uncertainty. Since there is a trade-off between performance and robustness in the conventional design techniques, we propose a design technique to provide the best mix of robustness and performance. Our approach is to provide different control strategies for different levels of uncertainty. We describe how to measure these uncertainties. We will compare our multiple strategies results with those of conventional techniques e.g. H_∞ control theory. A Lyapunov equation is used to define stability in all cases.

1. INTRODUCTION

In modern control theory much time is spent searching for the robust system design technique e.g. H_∞ control theory, but there is always trade-off between the performance and the robustness. There also are limitation on the robustness bound. In this paper we suggest a possible design technique using an ANN to satisfy the performance, the robustness, and possibly extended the boundary of robustness. If we can design a control system with a number of compensators based on different levels of uncertainty and we can then chase the appropriate compensator based on the system response, then we have a robust control system. (see Fig. 1) Conventional technique for compensator can guarantee performance, but can not guarantee performance in the presence of uncertainties. We will show a design technique that can meet the robustness and performance. In this paper we restrict ourselves to the discrete and stochastic case.

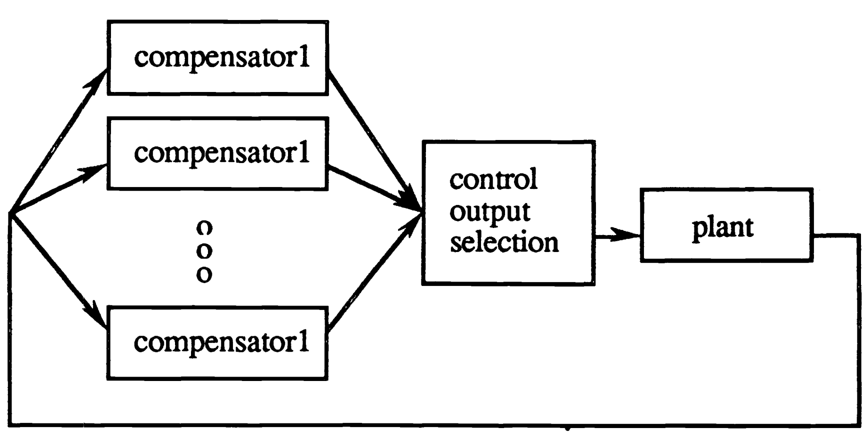

Fig. 1. Possible control system with multiple compensators

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2. LINEAR QUADRATIC REGULATION PROBLEM

In this section we introduce the discrete-time static output-feedback control problem. Given the nth-order plant

$$x(k+1) = Ax(k) + Bu(k) + Dw(k) \qquad k = 0, 1, 2, \ldots \tag{2.1}$$
$$y(k) = Cx(k) \tag{2.2}$$

determine a static output feedback law

$$u(k) = Ky(k) \tag{2.3}$$

that satisfies the following design criteria:

i) the closed-loop system (2.1) - (2.3) is asymptotically stable.

ii) the performance functional

$$J(k) = \lim_{k \to \infty} E\left[x^T(k)R_1x(k) + u^T(k)R_2u(k)\right] \tag{2.4}$$

is minimized.

Note that since we cannot apply this problem statement to the ANN-based controller, we need to modify the above statement which can be suitable to the ANN case.

ANN based controller is designed to minimize the cost function

$$J(u) = \sum_{i=1}^{N} x_i^2 \qquad \text{where N = number of state variables.} \tag{2.5}$$

Therefore desired output(control signal **u**) can be calculated by solving the equation (2.6) for the control signal **u**

$$\frac{\partial J}{\partial u} = 0 \tag{2.6}$$

Once an ANN is tranined with the desired control signal **u** obtained from equation (2.6), we have to check the stability for the closed-loop system. Note that we are going to use a Lyapunov stability theorem mentioned in the section 4.

3. CONTROL SYSTEM USING AN ANN

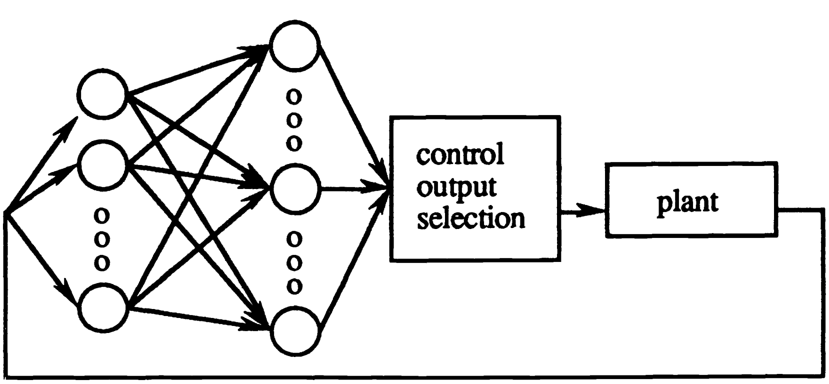

Fig. 2. suggested control system

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Architecture

The proposed ANN architecture is shown in Fig. 2. It has only input and output layers, and we use a linear activation function for the neurons. We have as many input neurons as we have state variables. The number of output neurons is based on the number of regions into which the range of uncertainty is devided. Control output selection is based on decision routines described in the section 5.

Training

Training sets consists of time series data generated by a several different initial value of the states. We use the LMS algorithm to train the NN. Desired value for any given input can be calculated by solving equation (2.6) for control signal u. Training was done using following steps.

1) Select several different initial values for the state variable x.
2) Apply plant output values to the ANN inputs and calculate the value of the output neurons.
3) Using the desired system response(equation 2.6) calculate the error and adjust the weights based on the error and using the LMS algorithm.
4) Do step 2) and 3) during transient response period T.
5) Repeat step 1) through 4) until the weights have converged for all initial value sets.
6) Increase the level of uncertainty until desired range of the uncertainty is reached.(when this level is achieved training is complete.) If you meet the required uncertainties, then you finish the training, otherwise continue the process.
7) Test stability and performance. If it meets requirements, then go to step 6, otherwise generate the new output neurons and go to step 1.

Testing

1) Select initial values for the state variable x.
2) Apply input values to the ANN inputs and calculate the output values.
3) If it is the very first iteration, use the norminal case output values for the control signal u. otherwise, choose the proper control signals calculated by step 4) based on the stability map.
4) Measure the state variables x and decide which region of uncertainty we are in.
5) Repeat step 2) through 4) as required.

we can construct the stability map using the Lyapunov stability theorem described in the following section.

4.STABILITY TEST

Lyapunov stability theorem: For a given linear time-invariant digital system described by the difference equation

$$x(k+1) = Ax(k) + Dw(k)$$

the system is asymptotically stable if and only if, given $V = DD^T$, there exists a positive-definite matrix Q such that

$$Q = AQA^T + V$$

Using Lyapunov stability theorem for a trained closed-loop system we can draw the stability region versus uncertainties.

In order to construct the stability map we apply this theorem with different levels of uncertainty. (example is given in following section .)

5. EXAMPLES

As an example we choose following dynamic system

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$$x(k+1) = \begin{bmatrix} (-3+\Delta_1) & 0 \\ 0 & (2+\Delta_2) \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k) + w(k)$$

where Δ_1 and Δ_2 = uncertainty

LQR with H_∞ - constraint

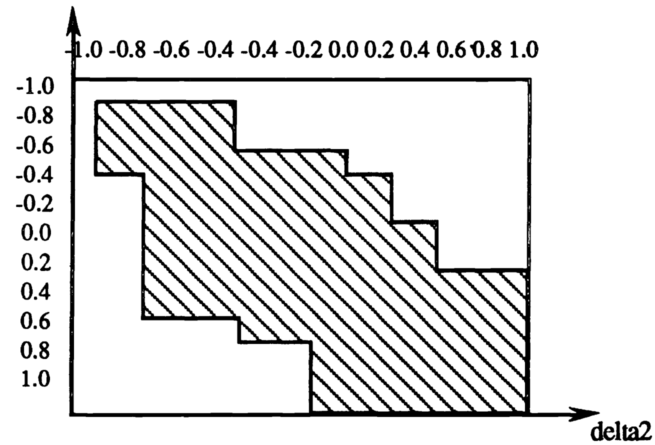

Fig.3. Stability Map for LQR control system

NN

Training pattern is simply chosen for four different initial values (x1,x2)={(0.5,0.5),(0.1,0.1),(-0.5,-0.5),(-0.1,-0.1)}. Desired control signal can be determined by equation (2.5) and (2.6) as follows

$$J = x_1^2(k+1) + x_2^2(k+1)$$
$$= [(-3+\Delta_1)x_1(k) + u(k)]^2 + [(2+\Delta_2)x_2(k) + u(k)]^2$$

Let $\frac{\partial J}{\partial u} = 0$

$$u(k) = -0.5[(-3+\Delta_1)x_1(k) + (2+\Delta_2)x_2(k)]$$

we train the closed-loop system with the desired control signal until we have small error and the requred performance.

In order to decide the uncertainties from the measured values we use the following technique. For a given close-loop system

$$x_1(k+1) = (-3+\Delta_1)x_1(k) + w_1x_1(k) + w_2x_2(k) + du(k)$$
$$x_2(k+1) = (2+\Delta_2)x_2(k) + w_1x_1(k) + w_2x_2(k) + du(k)$$
$$\Delta_1 = \frac{x_1(k+1)}{x_1(k)} + 3 - w_1 - w_2\frac{x_2(k)}{x_1(k)} - \frac{n(k)}{x_1(k)}$$
$$\Delta_2 = \frac{x_2(k+1)}{x_2(k)} - 2 - w_2 - w_1\frac{x_1(k)}{x_2(k)} - \frac{n(k)}{x_2(k)}$$

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$$E[\Delta_1] = \frac{x_1(k+1)}{x_1(k)} + 3 - w_1 - w_2\frac{x_2(k)}{x_1(k)}$$
$$E[\Delta_2] = \frac{x_2(k+1)}{x_2(k)} - 2 - w_2 - w_1\frac{x_1(k)}{x_2(k)}$$

we can then determine the proper control signal to allow us o obtain the expected uncertainty.

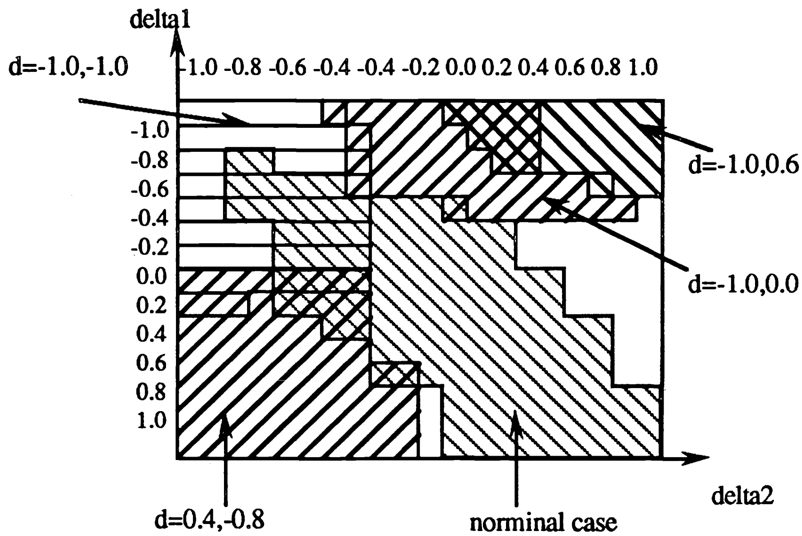

Fig.4. Stability Map for NN-based control system

Note that "d" in the fig.4. means uncertainties.

6. CONCLUSIONS

In this paper we investigated the robust control system of employing an ANN-based controller. We constructed the system and test it successfully with the assumption that the magnitude of noise was less than the one of uncertainty. We can increase the boundary of the uncertainty as much as we can until it does not destroy the controllability of the system. For future study of this idea we will develop the techniques to detect the level of uncertainty from the system response in the large scale of control system.

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