Bit allocation considering mean absolute error for image compression

Hemen Goswami
Samuel Peter Kozaitis

Bit-allocation considering mean absolute error for image compression

H. Goswami, and S. P. Kozaitis
Florida Institute of Technology
Division of Electrical, and Computer Science and Engineering
150 West University Boulevard
Melbourne, FL 32901

1. INTRODUCTION

In lossy image compression schemes, often some distortion measure is minimized to arrive at a desired target bit rate. The distortion measure that has been most studied is the mean-squared-error (MSE). However, perceptual quality often does not agree with the notion of minimization of mean square error[1-3]. Since MSE can not guarantee the optimality of perceptual quality, others error measures have been investigated. Others have found strong mathematical and practical perspective to choose a different error measure other than MSE, especially for image compression[2]. In Ref. 2 it is argued that the mean absolute error (MAE) measure is a better error measure than MSE for image compression from a perceptual standpoint. In addition, the MSE measure fails when only a small proportion of extreme observations is present[3]. In this paper we develop a bit allocation algorithm to minimize the MAE rather than MSE.

2. MEAN ABSOLUTE ERROR FOR A UNIFORM QUANTIZER

The distortion measure in L^p sense is given by the following,

$$D = E[|\ x - \hat{x}|\ ^p] = \sum_i \int_{x_{i-1}}^{x_i} |\ x - y_i|\ ^p p_X(x)dx, \tag{1}$$

where $p_X(x)$ is the probability density function of the input signal, y_i is the quantizer output for a given input x within the quantizer levels of i-1 and i. The MAE and MSE are obtained for $p = 1$ and $p = 2$ respectively. The MAE for a uniform quantizer can be written as,

$$= E[|x - y_i|] \cong \sum_{i=1}^{N} |x - y_i| p_X(x)dx \tag{2}$$

$$\cong \sum_{i=1}^{N} p_X(y_i) \int_{y_i - \Delta/2}^{y_i + \Delta/2} |x - y_i| dx \tag{3}$$

$$= \frac{\Delta}{4} \sum_{i=1}^{N} p_X(y_i)\Delta \tag{4}$$

$$\cong \frac{\Delta}{4} \int_{y_1 - \Delta/2}^{y_N + \Delta/2} f(x)dx \cong \frac{\Delta}{4} \tag{5}$$

In *Visual Information Processing IX*, Stephen K. Park, Zia-ur Rahman, Editors,
Proceedings of SPIE Vol. 4041 (2000) • 0277-786X/00/$15.00

where Δ is the quantization step size. The above expression for MAE for a uniform quantizer is obtained with these approximations: there is no overload distortion, the input pdf is constant within a quantization step size, and the definition of a Riemann integral, $\Sigma f(y_i)\Delta$, is approximately equal to the integral of $f(y_i)$.

3. BIT-ALLOCATION USING MAE

In subband coding, the input signal is transformed into various subbands. The target bit rate R is then allocated to the subbands to minimize the distortion that is the MAE in this case. An iterative algorithm can be found to find the optimal bit allocation. Assuming $\Delta = (b-a)/N$, where N is the number of quantization levels for the input signal bounded between a and b, the distortion D can be expressed in terms of the input signal characteristics as,

$$D(R) = \frac{\Delta}{4} \cong \frac{(b-a)}{4N} \cong m_x 2^{-R} \tag{6}$$

where m_x is the MAE of the input to the quantizer. We assume here that minimization of MAE of the transformed coefficients is equivalent to the minimization of MAE of the input coefficients. Eq. (6) can be presented in a more general form taking into account the relative subband size, the quantizer performance index, and weighting factor for perceptual coding. Considering N subbands, the total distortion in the transform domain can be written as,

$$D = \sum_i D_i(R_i) = \sum_i \alpha_i \omega_i \varepsilon_i m_{xi} 2^{-R_i} \tag{7}$$

where α_i is the relative subband size, ε_i is the quantizer performance index, and ω_i is the weighting factor (for perceptual coding).

Now the problem of optimal bit allocation is reduced to minimization of Eq. (7) with respect to the following constraint,

$$\sum_i \alpha_i R_i = R_C \tag{8}$$

where R_c is the total target bit rate. The minimization of Eq. (7) with the constraint given by Eq. (8) can be carried out using the Lagrange multiplier λ,

$$\text{minimize}\{D(R) + \lambda R_c\} = \text{minimize}\{\sum_i \alpha_i \omega_i \varepsilon_i m_{xi} + \lambda R_i\}. \tag{9}$$

Assuming $\varepsilon = \varepsilon$, and $\dfrac{\lambda}{\varepsilon} = \overline{\lambda}$, we obtained the following closed form solutions,

$$R_i = \log_2\left(\frac{w_i m_{xi} \log 2}{\overline{\lambda}}\right) \tag{10}$$

and

$$\overline{\lambda} = 2^{\sum_i \alpha_i \log_2(\omega_i m_{xi} \log 2) - R_C} \tag{11}$$

From Eqs. (10) and (11), the bit rate for individual subband is calculated iteratively. The iterative algorithm can be expressed as:

1. Find bit rate for each subband and set negative bit rate to zero.
2. Adjust the relative subband size accordingly by reducing the number of subbands whose bit rate is set to zero.
3. Repeat step 1 until all bit rates are non-negative.

3. EXPERIMENTS

The following steps were used in image compression experiments. The target bit rate was allocated among various subbands to minimize both the MSE and MAE separately. We used 13 subbands (four levels of the wavelet transform) with the first subband corresponding to the low resolution version of the image. The next subband corresponded the vertical high-frequencies, then the next to vertical and horizontal high-frequencies, then horizontal high-frequencies, at the largest scale (low resolution). The numbering of the subbands continued at the next scale in this fashion, then the next, for all four scales. Using the Daubechies minimum phase wavelet with eight coefficients, some reconstruction results are shown in Fig. 1. Other results are shown in Table 1.

The results were for four compression ratios of 1.0, 0.5, 0.25, and 0.125. We considered two performance metrics, the peak signal-to-noise ratio (PSNR) = 10 $\log_{10}(255^2/\text{MSE})$, and the MAE.

Considering the MAE performance metric, the MAE optimization results were generally lower for the uniform weighting except at 0.125 bpp. Using the PSNR as the performance metric the MAE also gave improved results except for at 0.125 bpp. Figure 3 shows the reconstructed images at 0.5 bpp.

Table 1 Results of MSE and MAE bit-allocation

	MSE optimization		**MAE optimization**	
bpp	**PSNR**	**MAE**	**PSNR**	**MAE**
1.0	19.76	21.34	20.05	20.62
0.5	18.41	24.88	18.41	24.83
0.25	18.00	25.40	18.10	25.27
0.125	17.33	27.88	17.31	28.04

4. CONCLUSION

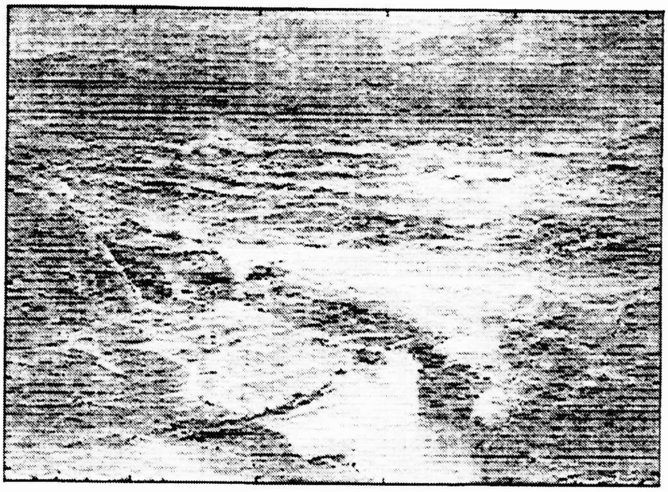
We showed that bit-allocation for wavelet transform coding could be optimized considering the MAE.

5. REFERENCES

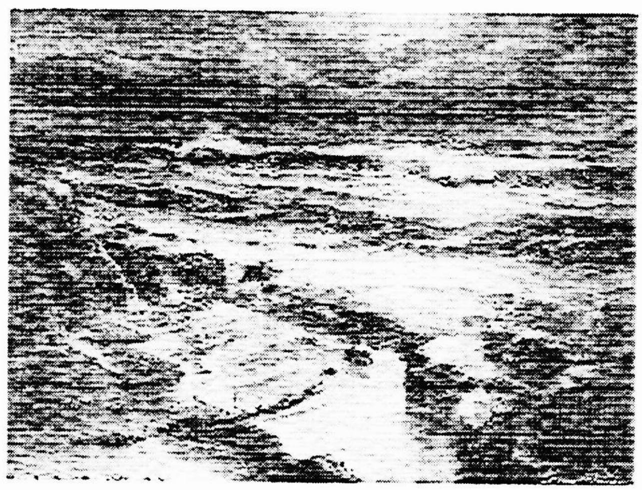
1. G. Strang, and T. Nguyen, "Wavelets and Filter Banks," Wellesley-Cambridge Press, (1996)

2. R.A. Devore, B. Jawaerth, and J. Lucier, "Image Compression through Wavelet Transform Coding," *IEEE Trans. on Information Theory*, Vol 38(2), 719-746 (1992)

3. M. Shaw, C.L. Nikias, "Signal processing with fractional lower order moments: Stable processes and their applications, " *Proceedings of the IEEE*, Vol 81(7), 986-1009 (1993)



(a) MSE optimization



(b) MAE optimization

Figure 1 Reconstruction results for 0.5 bpp.