Reduction of multiplicative noise using higher-order statistics

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ABSTRACT

We used a higher-order correlation-based method for signal denoising of images corrupted by multiplicative noise. Using the logarithm of an image, we applied a third-order correlation technique for identification of wavelet coefficients that contained mostly signal. In our approach, we examined wavelet coefficients in an environment where the contribution from the second-order moment of the noise had been reduced. Our results compared favorably and were less sensitive to threshold selection when compared to a second-order wavelet denoising method.

Keywords: denoising, wavelet transform, higher-order statistics

1. INTRODUCTION

Images generated by synthetic aperture radar (SAR) often contain a granular noise referred to as speckle. The speckle usually degrades the image by interfering with the image understanding problem. A SAR image is generally formed by coherently processing the returns of successive radar pulses. There are variations in intensity from pixel to pixel that often appears as a granular type of noise. The intensity of the speckle has been shown to be a multiplicative noise, and there have been several approaches to suppress the speckle [1-4]. In addition, by using the logarithm of the speckle image, the noise can be thought of as additive, and filters designed for additive noise can be used. There are a variety of nonlinear filters that have been shown to be useful for reducing speckle noise. Nonlinear filtering techniques have been introduced that are based on the theory of robust estimation[5]. In addition, methods based on the wavelet transform have been shown to be useful[6]. We also used a wavelet-based approach to remove multiplicative noise from signals. We retained or set to zero wavelet coefficients based on higher-order statistics. Higher-order correlations are an extension of the more familiar second-order cross-correlation function. In our approach, we applied a third-order correlation technique to identify wavelet coefficients that contained mostly signal. Using this approach us to improve the separation between signal and noise.

2 DENOISING

A straightforward approach to denoising involves setting wavelet coefficients that are mostly noise to zero. This typically involves scaling an image, taking its wavelet transform, thresholding the wavelet coefficients, then taking the inverse wavelet transform. When considering multiplicative noise, the logarithm of a signal may be used so that the noise appears additive, and methods developed for additive noise may be used. In the additive case, after calculating the wavelet transform of the scaled image, the noisy wavelet transform coefficients are usually subjected to a soft or hard threshold. A hard threshold indicates that wavelet transform coefficients are retained only if their absolute value is greater than or equal to a threshold t. A soft threshold is similar to a hard threshold, but the threshold is subtracted from the absolute value of coefficients greater than the threshold.

There are different ways to choose a threshold. One threshold that is easy to implement, has been shown to work well for sparse data, and produces a good visual quality of a denoised signal is described as[7]

$$t = \sigma(2\log(N))^{1/2}\ , \tag{1}$$

where N is the number of wavelet coefficients to be thresholded, and σ is the standard deviation of the noise. The noise variance is often estimated from the median value of the wavelet coefficients at the smallest scale. After thresholding, the inverse wavelet transform is performed.

3. HIGHER-ORDER CORRELATIONS

The correlation between two functions has been often used as a measure of their similarity. The conventional correlation function is a second-order correlation, and is a special case higher-order correlations.[8,9] The nth-order correlation of the signal $f(x)$ is defined as

$$f_n(\tau_1,\ \tau_2,\ .\ .\ .\ \tau_{n-1}) \equiv \sum_{k=0}^{N-1} f\ (x)\ f\ (\tau_1{+}x)\ f\ (\tau_2{+}x)\ .\ .\ .\ f\ (\tau_{n-1}{+}x) \tag{2}$$

where the nth-order correlation is a function of n - 1 independent variables. For n = 2, Eq. (2) becomes the second-order correlation of $f(x)$ which is the familiar autocorrelation function.

We considered the third-order or triple correlation because it is simpler and easier to calculate than other higher-order correlations. The third-order correlation, n = 3, of a one-dimensional function is a function of two variables. From Eq. (2) the third-order correlation of $f(x)$ is

$$f_3(\tau_1,\ \tau_2) = \sum_{k=0}^{N-1} f\ (x)\ f\ (\tau_1{+}x)\ f\ (\tau_2{+}x) \tag{3}$$

where $f_3(t_1,t_2)$ is symmetric with respect to its variables τ_1, and τ_2.

The third-order correlation coefficient $f_3(0,0)$, can be found by sampling the triple correlation $f_3(t_1,t_2)$, at zero displacement where $\tau_1 = \tau_2 = 0$. From Eq. (3), the third-order correlation coefficient becomes

$$f_3(0,\ 0) = \sum_{k=0}^{N-1} f^3(x) \tag{4}$$

which shows that the third-order correlation coefficient $f_3(0, 0)$, of $f(x)$, can be calculated directly as the sum of the cubes of $f(x)$ from $k = 0$ - N-1.

4. HIGHER-ORDER STATISTICS FOR SIGNAL DENOISING

In this section we described a method for denoising using third-order correlations. We considered a noisy signal that contained multiplicative noise of unknown spectral density. Our goal was to select wavelet coefficients that were due primarily to the signal. We thought of wavelet coefficients as correlations between an input signal and a wavelet at a particular scale and translation. The noisy input signal was described as $s(x) = f(x)n(x)$ where $f(x)$ represented the signal and $n(x)$ the noise .

We first took the logarithm of the signal so the noise term would be additive. Then, we calculated the cross-correlation function between the preprocessed input signal and each scaled and translated wavelet. The resulting correlation function was labeled as $b_{jk}(\tau)$. Then, we calculated the third-order autocorrelation coefficient of $b_{jk}(\tau)$. Using Eq. (4), the third-order autocorrelation coefficient of the second-order correlation result was described as

$$b3_{jk}(0,0) = \sum_{\tau=1}^{2m=1} \left(b_{jk}(\tau)\right)^3 , \tag{5}$$

where m is the length of the wavelet, and the summation is performed only on that portion of the signal. Because the preprocessed input signal could be considered as a signal plus noise, we considered the output of the second-order correlation as consisting of two parts. One part was the correlation between the input signal and a wavelet, and the second part was the correlation between the noise and the wavelet. The second-order correlation result was written as

$$b_{jk}(\tau) = fb_{jk}(\tau) + nb_{jk}(\tau) , \tag{6}$$

where $fb_{jk}(\tau) = log(f(x)) \circledast b_{jk}(\tau)$, and $nb_{jk}(\tau) = log(n(x)) \circledast b_{jk}(\tau)$, represented the signal-wavelet correlation, and the noise-wavelet correlation respectively. Substituting Eq. (6) into Eq. (5), and rearranging, the expression for the third-order autocorrelation coefficient can be written as[10]

$$b3_{jk}(0,0) = \sum_{\tau=1}^{2m-1} \left(fb_{jk}(\tau)\right)^3 + 3\sum_{\tau=1}^{2m-1} \left(fb_{jk}(\tau)\right)^2 nb_{jk}(\tau) + \sum_{\tau=1}^{2m-1} fb_{jk}(\tau)\left(nb_{jk}(\tau)\right)^2 + \sum_{\tau=1}^{2m-1} \left(nb_{jk}(\tau)\right)^3 . \tag{7}$$

The last term is the only all noise term and will have a contribution from the third-order moment of the noise. The second term will be proportional to the mean of the noise. The first term is only term that is strictly due to the signal.

The third term is related to the product of signal and the noise power. It can be minimized if the mean of the signal is set to zero.

5. RESULTS

We compared our method to a more conventional wavelet denoising approach described in section 2. In both cases we performed a logarithm on the signal before the wavelet transform, and the inverse logarithm after denoising. We multiplied signals by noise of a Rayleigh distribution for different values of β, where β is the square root of the variance, divided by the mean. Then we attempted to remove the noise using the denoising algorithms. In both methods we calculated the variance σ^2 by examining the wavelet transform at the finest scale. The threshold for the conventional denoising approach was that of Eq. (4). For the third-order approach we used $t = 3\sigma^2$. In all cases, we used three levels of the wavelet transform and used Daubechies minimum-phase orthogonal 4-tap wavelets with 2 vanishing moments. Figure 1 shows a sample 1-D signal. Figure 2 shows the mean-squared error (MSE) as a function. The results showed that the third-order method consistently showed improved results when compared to the conventional method.

We also performed experiments on the SAR image shown in Fig. 3 that had a Rayleigh-distributed histogram, and noise was contributed as before Due to computational burden of the third-order approach, we denoised each row separately, then denoised each column of the original noisy separately, then, averaged the two results. The results comparing the second- and third-order approach are shown in Fig. 4. The figure shows that the third-order method consistently showed improved results when compared to the conventional method. Although we showed only a few test signals, in many cases we found the third-order method provided improved MSE values when compared to the second-order approach.

6 CONCLUSION

We used a higher-order correlation-based method for signal denoising of multiplicative noise. In our approach, we determined if wavelet coefficients contained mostly noise, or signal, based on third-order statistics. In this domain it was easier to separate a signal from noise than in a second-order domain.

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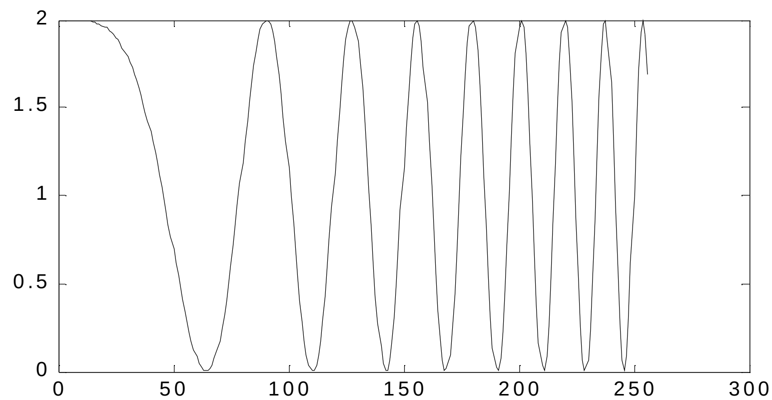

Figure 1 1–D signal used in denoising experiments.

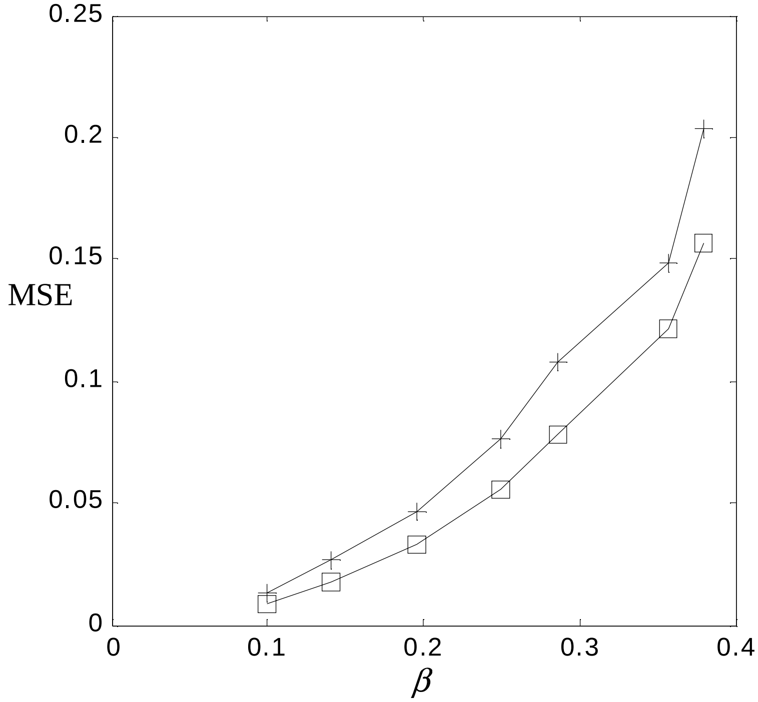

Figure 2 Comparison of 2nd (+), and 3rd ( ), -order denoising methods for the signal in Fig. 1.

Figure 3 SAR image used in denoising experiments.

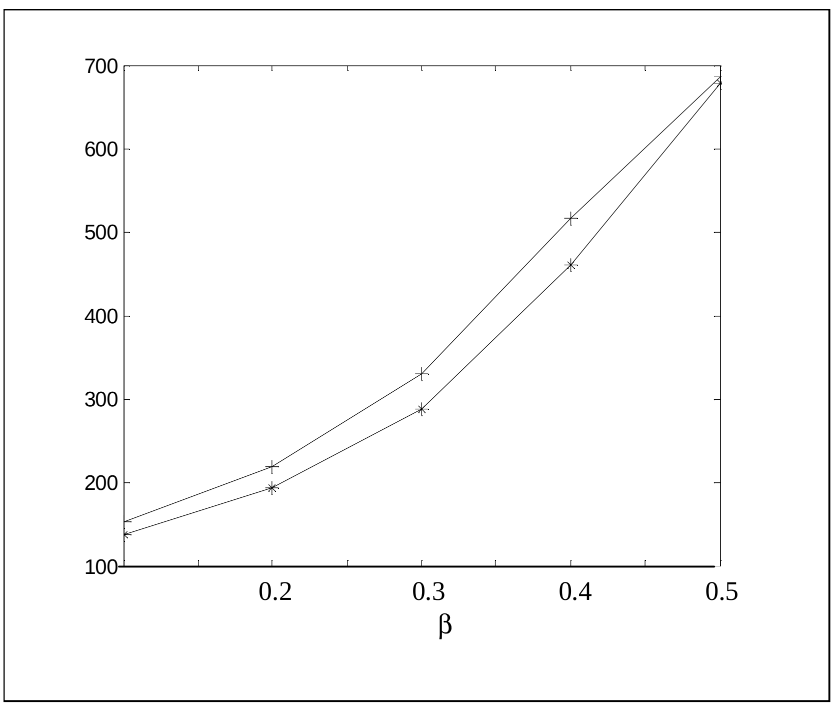

Figure 4 Comparison of 2nd (+), and 3rd (*), -order denoising methods for the image in Fig. 3.