DESIGN OF A COMPACT HELICAL COUNTERFLOW HEAT EXCHANGER

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Abstract

Compact heat exchangers are desirable in many aerospace applications. New additive manufacturing approaches, such as 3D printing, have enabled the fabrication of heat exchange devices utilizing geometries that cannot be fabricated using traditional approaches. The new geometries enabled by 3D printing may result in higher heat transfer using smaller devices, however, constraints associated with the fabrication of these devices also impose potential performance degradations. This document presents the design and analysis of a novel, compact counter flow heat exchanger which uses helically shaped passages to enhance the effectiveness of the heat transfer. Although the helical passages increase the heat transfer and reduce the size of the device, 3D print build constraints mandate that the passages are constructed with a lean angle for structural support that also increases the overall pressure loss of the fluid. An analytical model is developed, that can be used to trade the size and mass of the device for required heat transfer performance and acceptable levels of fluid pressure loss. Various working fluids, including water and cryogens are considered and designs that meet specified heat transfer goals while minimizing the pressure loss and volume of the device are presented. These designs are compared against a straight channel counter flow heat exchanger which can be fabricated using traditional approaches. This work demonstrates that for the same working fluids and for a set of given geometric constraints a tradeoff between heat exchange, pressure loss and compactness is observed while designing an optimized model.
# Table of Contents

Abstract...........................................................................................................................................iii
Table of Contents................................................................................................................................iv
List of Figures......................................................................................................................................vi
List of Tables .......................................................................................................................................viii
List of Symbols ............................................................................................................................... ix
Acknowledgments ............................................................................................................................. xi

1. Introduction ...................................................................................................................................... 1
   1.1. Background .................................................................................................................................. 1
       1.1.1 Classification According to Flow Path ................................................................................... 1
       1.1.2 Classification According to Transfer Processes ...................................................................... 3
       1.1.3 Classification According to Construction ............................................................................... 3
       1.1.4 Classification According to Compactness ............................................................................. 6
   1.2. Motivation .................................................................................................................................... 7
   1.3. Objectives ..................................................................................................................................... 8
   1.4. Approach ..................................................................................................................................... 9
   1.5. Thesis Overview .......................................................................................................................... 9

2. Counterflow Heat Exchanger Analysis Overview ............................................................................. 11

3. Counterflow Heat Exchanger Analytical Modeling ........................................................................ 15
   3.1. Straight Annular Heat Exchanger without and with Radial Fins .............................................. 15
   3.2. Helical Annular Heat Exchanger with Radial Fins .................................................................. 17
   3.3. Helical Annular Heat Exchanger with Radial Fins and Lean .................................................. 20
   3.4 Geometry Implications ............................................................................................................... 21
4 Results ........................................................................................................................................... 24
  4.1 Straight annular heat exchanger without and with radial fins .................................................. 25
  4.2 Helical annular heat exchanger with radial fins having no lean angle ..................................... 27
  4.3 Helical Annular Heat Exchanger with Radial Fins and Lean .................................................... 29
5 Parametric Study .......................................................................................................................... 33
  5.1 Heat Transfer and Compactness prioritized for optimization ................................................... 35
  5.2 Pressure Drop and Compactness for optimization ...................................................................... 40
  5.3 Heat Transfer and Pressure drop prioritized for optimization ................................................... 44
6 Conclusions and Future Work ..................................................................................................... 50
7 References ..................................................................................................................................... 52
8 Appendix A : Thermophysical properties of working fluids ......................................................... 54
9 Appendix B : Analytical modelling MATLAB code ......................................................................... 58
List of Figures

Figure 1-1-1: Parallel and Counterflow Heat Exchanger ................................................. 2
Figure 1-1-2: Single and multipass crossflow heat exchangers [23] .................................. 2
Figure 1-1-3: Relative heat transfer area to the difference in temperature to the inlet streams for different flow configurations [24] ............................................................... 3
Figure 1-1-4: Tube fin heat exchangers [24] .................................................................... 4
Figure 1-1-5: Printed circuit heat exchanger [25] .............................................................. 5
Figure 2-1: Cylindrical Annular Straight Counterflow Heat Exchanger ......................... 11
Figure 3-1: Cylindrical Helical Annular Counterflow Heat Exchanger with N = 0.5, \( \psi = 52.4^\circ \), \( \frac{L_{htx}}{L} = 1.26 \) for inner channels and N=1, \( \psi = 26.2^\circ \), \( \frac{L_{htx}}{L} = 2.26 \) for the outer channels ................................................................. 18
Figure 3-2: Cylindrical Helical Annular Counterflow Heat Exchanger with N = 0.5, \( \psi = 52.4^\circ \), \( \frac{L_{htx}}{L} = 1.26 \) for inner channels and N=1, \( \psi = 26.2^\circ \), \( \frac{L_{htx}}{L} = 2.26 \) for the outer channels with \( \theta = 45^\circ \) for both the channels ................................................................. 21
Figure 3-3: Helical Angle, \( \psi \) vs Heat exchanger length, L for fixed N = 1 ..................... 22
Figure 3-4: Number of helical turns, N vs Heat exchanger length, L for fixed \( \psi = 24.4^\circ \) .... 22
Figure 4-1: \( U_{ratio} \) and \( \Delta P \) vs N for water-water heat exchanger ......................... 29
Figure 4-2: \( U_{ratio} \) and \( \Delta P \) vs Heat exchanger length for water-water heat exchanger ..... 31
Figure 4-3: \( U_{ratio} \) and \( \Delta P \) vs Heat exchanger diameter for water-water heat exchanger .. 32
Figure 5-1: \( U_{ratio} \) vs \( \dot{m}_h \) vs \( \dot{m}_c \) for water – water heat exchanger when heat transfer and compactness are prioritized ................................................................. 36
Figure 5-2: \( U_{ratio} \) vs \( \dot{m}_h \) vs \( \dot{m}_c \) for nitrogen-water heat exchanger when heat transfer and compactness are prioritized ................................................................. 37
Figure 5-3: \( \Delta P \) vs \( \dot{m}_h \) for water - water heat exchanger when heat transfer and compactness are prioritized ................................................................. 38
Figure 5-4: \( \Delta P \) vs \( \dot{m}_c \) for water - water heat exchanger when heat transfer and compactness are prioritized ................................................................. 38
Figure 5-5: \( \Delta P \) vs \( \dot{m}_h \) for nitrogen - water heat exchanger when heat transfer and compactness are prioritized ................................................................. 39
Figure 5-6 : \( \Delta P \) vs \( \dot{m}_c \) for nitrogen - water heat exchanger when heat transfer and compactness are prioritized

Figure 5-7 : \( \Delta P \) vs \( \dot{m}_c \) for water - water heat exchanger when pressure drop and compactness are prioritized

Figure 5-8 : \( \Delta P \) vs \( \dot{m}_c \) for water - water heat exchanger when pressure drop and compactness are prioritized

Figure 5-9 : \( \Delta P \) vs \( \dot{m}_c \) for nitrogen - water heat exchanger when pressure drop and compactness are prioritized

Figure 5-10 : \( \Delta P \) vs \( \dot{m}_c \) for nitrogen - water heat exchanger when pressure drop and compactness are prioritized

Figure 5-11 : \( U_{ratio} \) vs \( \dot{m}_h \) vs \( \dot{m}_c \) for water – water heat exchanger when Pressure drop and compactness are prioritized

Figure 5-12 : \( U_{ratio} \) vs \( \dot{m}_h \) vs \( \dot{m}_c \) for nitrogen-water heat exchanger when Pressure drop and compactness are prioritized

Figure 5-13 : \( U_{ratio} \) vs \( \dot{m}_h \) vs \( \dot{m}_c \) for water – water heat exchanger when heat transfer and pressure drop are prioritized

Figure 5-14 : \( U_{ratio} \) vs \( \dot{m}_h \) vs \( \dot{m}_c \) for nitrogen – water heat exchanger when heat transfer and pressure drop are prioritized

Figure 5-15 :\( \Delta P \) vs \( \dot{m}_h \) for water - water heat exchanger when heat transfer and pressure drop are prioritized

Figure 5-16 : \( \Delta P \) vs \( \dot{m}_c \) for water - water heat exchanger when heat transfer and pressure drop are prioritized

Figure 5-17 : \( \Delta P \) vs \( \dot{m}_h \) for nitrogen - water heat exchanger when heat transfer and pressure drop are prioritized

Figure 5-18: \( \Delta P \) vs \( \dot{m}_c \) for nitrogen - water heat exchanger when heat transfer and pressure drop are prioritized
List of Tables

Table 3-1: Summary of important heat exchanger geometric parameters ........................................... 23
Table 4-1 : Heat exchanger design and performance parameters ......................................................... 24
Table 4-2 : Summary of analysis for straight heat exchanger without fins ........................................... 25
Table 4-3 : $U_{ratio}$ for straight annular heat exchanger with 8 radial fins in both the channels ............... 26
Table 4-4 : Frictional pressure drop in a straight annular heat exchanger ........................................... 27
Table 4-5 : $U_{ratio}$ for helical annular heat exchanger with radial fins having no lean ....................... 28
Table 4-6 : $U_{ratio}$ comparison for helical annular heat exchanger with $\theta = 0^\circ$ and $\theta = 45^\circ$, $N=1$ .................................................................................................................................................. 29
Table 4-7 : Frictional pressure drop in a helical annular heat exchanger for multiple $N$’s ............... 30
Table 5-1 : Design parameters for Optimized geometry ......................................................................... 33
Table 5-2 : Heat exchanger performance for optimized geometry .......................................................... 34
Table 5-3 : Optimized design parameters when heat transfer and compactness are prioritized ............ 35
Table 5-4 : $U_{ratio}$ for Optimized design parameters when heat transfer and Compactness are prioritized .................................................................................................................................................. 35
Table 5-5 : $\Delta P$ for Optimized design parameters when heat transfer and compactness are prioritized .................................................................................................................................................. 37
Table 5-6 : Optimized design parameters when Pressure drop and Compactness are prioritized ....... 40
Table 5-7 : $\Delta P$ for Optimized design parameters when Pressure drop and Compactness are prioritized .................................................................................................................................................. 41
Table 5-8 : $U_{ratio}$ for Optimized design parameters when heat transfer and Compactness are prioritized .................................................................................................................................................. 43
Table 5-9 : Optimized design parameters when heat transfer and pressure drop are prioritized ......... 44
Table 5-10 : $U_{ratio}$ for Optimized design parameters when heat transfer and pressure drop are prioritized .................................................................................................................................................. 45
Table 5-11 : $\Delta P$ for Optimized design parameters when heat transfer and pressure drop are prioritized .................................................................................................................................................. 47
List of Symbols

\begin{align*}
A &= \text{Area} \quad [\text{m}^2] \\
C &= \text{Specific heat} \quad [\text{J/kg-K}] \\
D &= \text{Diameter} \quad [\text{m}] \\
\text{De} &= \text{Dean number} \\
f &= \text{frictional factor} \\
h &= \text{Convective heat transfer coefficient or} \\
&\quad \text{Enthalpy of fluid} \quad [\text{W/m}^2\text{K}] \text{ or } [\text{J/kg}] \\
K &= \text{Thermal conductivity} \quad [\text{W/m-K}] \\
L &= \text{Heat exchanger length} \quad [\text{m}] \\
\dot{m} &= \text{Mass flow rate of fluid} \quad [\text{kg/s}] \\
n &= \text{Number of fins} \\
N &= \text{Number of turns} \\
\text{Nu} &= \text{Nusselt number} \\
P &= \text{Wetted perimeter} \quad [\text{m}] \\
p &= \text{Pressure} \quad [\text{kPa}] \\
\text{Pr} &= \text{Prandtl number} \\
Q &= \text{Energy transfer rate} \quad [\text{J/s}] \\
\text{Re} &= \text{Reynolds number} \\
t &= \text{Thickness} \quad [\text{m}] \\
T &= \text{Temperature} \quad [\text{K}] \\
U &= \text{Overall heat transfer coefficient} \quad [\text{W/m}^2\text{K}] \\
\Delta &= \text{Parameter difference} \\
\eta &= \text{Efficiency} \\
\mu &= \text{Dynamic Viscosity} \quad [\text{Pa-s}] \\
\rho &= \text{Density} \quad [\text{kg/m}^3] \\
\psi &= \text{Helical Angle} \quad [\text{deg}] \\
\theta &= \text{Lean angle} \quad [\text{deg}]
\end{align*}
Subscripts

ach = Achievable

crs = Cross-sectional area

c = Cold fluid

cric = Critical

cv = Curved pipe

f = Fins

h = Hot fluid or hydraulic

hlx = Helix

i or 1 = Inlet or inner

lm = Log mean

o or 2 = Outlet or outer

ovr = Overall

ratio = Ratio

req = Required

s = Straight or innermost
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1. Introduction

1.1. Background

Heat Exchangers are one of the most important components in many industrial processes and covers a wide range of industrial applications. Heat exchangers have been used in power plant, electronics, environmental engineering, manufacturing industry, air-conditioning, waste heat recovery, cryogenic processes, chemical processing steam power plants, transportation power systems, refrigeration units. Heat exchangers have come long way, from large ones transported in trucks, airplanes to small ones which can fit in the palm of our hands. Factors like cost of fabrication and installation, weight and size play important roles in choosing an appropriate design. Heat exchangers can be classified according to transfer process, construction, number of fluids, surface compactness, flow arrangement and heat transfer mechanisms.

1.1.1 Classification According to Flow Path

The four most common types based on flow configuration are parallel, counter flow and single pass, multiple crossflow as illustrated in Figure 1-1. In parallel or co-current flow, fluids enter at one end, flow in the same direction and leaves together at the same end. Fluids move in opposite directions in case of counter-flow or countercurrent heat exchangers and in case of single pass crossflow units, one fluid moves through the heat transfer matrix at right angles to the flow path of the other fluid. In multi-pass crossflow units, fluid pass each other more than once.

\[\text{Fluid 2 in} \quad \longrightarrow \quad \text{Fluid 2 out} \]
\[\text{Fluid 1 in} \quad \longrightarrow \quad \text{Fluid 1 out}\]

a) Parallel flow
The relative heat transfer surface area required to achieve the desired amount of heat transfer between the two fluids is the main criteria in choosing from the above-stated flow configurations. Figure 1-1-3 shows the relative heat transfer area to the difference in temperature to the inlet streams. Parallel flow heat exchangers are used when the fluid temperature change across the heat exchanger is a small percentage of the difference in temperature between the inlet fluid streams. In the case of counter flow heat exchangers, the temperature difference across the heat exchangers is very close to the difference in temperatures of the inlet fluid streams. Counterflow heat exchanger requires the least area compared to parallel and crossflow heat exchangers.
1.1.2 Classification According to Transfer Processes

Heat exchangers can be classified according to transfer process into direct and indirect contact types.

In an indirect contact heat exchanger, the fluids remain separated and heat transfers through a wall in a transient manner. It is further classified into direct-transfer type, storage type and fluid-bed exchangers. In Direct contact heat exchangers, heat transfers from hot to a cold fluid through a wall. There is no direct mixing between the fluids as they flow in separate passages and this type of heat exchanger is also called recuperative heat exchangers. Recuperators are further classified as prime surface and extended surface heat exchangers. Prime surfaces are those which do not employ fins and examples of prime surface heat exchangers are Plain tubular, shell and tube and plate heat exchangers. Fins are used to enhance heat transfer.

1.1.3 Classification According to Construction

Tubular heat exchangers are easy to manufacture and relatively cheap when compared to the rest of the variety and can accommodate a wide range of pressures and
temperatures. A common design called the shell and tube heat exchanger, consists of round tubes mounted on a cylindrical shell. The main parts of shell-tube heat exchangers are tube bundle, shell, front and rear end headers and baffles. Baffles are used as support structures and direct the flow perpendicular to the tubes. Various types of baffles and shell tube heat exchangers are available and are differentiated based on arrangement and flow configuration. The character of fluids used in heat exchangers are liquid-liquid, liquid-gas, gas-gas and liquid-liquid being the most common one in applications. Gas to gas heat exchangers are used in gas-turbine systems, cryogenic gas-liquefaction systems, and steel furnaces. Fins are employed in tubular heat exchangers and are called tube-fin exchangers. These are used when operating fluid pressures are less than 30 atm and operating temperature from low cryogenic applications to 870 °C. Figure 1-1-4 illustrates a tube fin heat exchanger.

**Plate heat exchangers** cannot accommodate high pressure or temperatures when compared to the tubular heat exchangers and is designed for moderate pressure and temperature differentials.

In case of lower pressure not exceeding 10 atm, temperatures not exceeding 800 °C, plate fin heat exchangers are preferred and are generally used in gas to gas applications.

![Figure 1-1-4: Tube fin heat exchangers](image)

a) Round tube and fin  

b) Flat tube and fin  

Figure 1-1-4: Tube fin heat exchangers [24]
Printed circuit heat exchanger is a compact version of the shell and tube heat exchanger. These are stacked and diffusion bonded, converting the plates into a solid metal block having precise flow passages. These types of heat exchangers as shown in figure 1-1-5 can withstand high pressures and temperature ranges when compares to shell tube heat exchangers. They are 4 to 6 times smaller than the conventional designs and have extremely high heat transfer coefficients with small flow passages. However, printed circuit heat exchangers are expensive when compared to the conventional ones. There is possibility for blockages within the passages due to channels being very fine in size. Materials used in manufacturing of printed circuit heat exchanger include stainless steel, nickel and super alloys Inconel 600 etc.

Boilers are one of the earliest applications of heat exchangers. Different types of boilers exist and are used from house heating applications to power stations. Condensers are also type of classification and major applications in steam power plants, chemical processing and nuclear power plants.

Figure 1-1-5 : Printed circuit heat exchanger [25]
1.1.4 Classification According to Compactness

Compactness of heat exchangers are measured based on the ratio of heat transfer surface area on one side of the heat exchanger to the volume. A heat exchanger having surface area density greater than 700 $m^2/m^3$ is classified under compact heat exchanger irrespective of the structural design. These types of heat exchangers are used in automobiles, aerospace vehicles, cryogenic systems and in refrigeration and air-conditioning where weight and size are important.

Helical coil heat exchangers are universally used in various industrial applications ranging from heat exchangers, power plant, electronics, environmental engineering, manufacturing industry, air-conditioning, waste heat recovery, cryogenic processes, to chemical processing, because of their compact size and high heat transfer performance. The flow and convective heat transfer in a helical coiled tube are complicated as compared with the straight tube, because strongly depend on the behavior of secondary flow. Enhancement in heat transfer due to helical coils has been reported by many researchers.

Several studies have investigated the flow and heat transfer characteristics for single-pipe and double-pipe helical heat exchangers, both experimentally [1–4], as well as numerically [5–8]. The secondary flow motion induced by the curvature effect and the resultant centrifugal force makes heat transfer coefficient greater than that in a straight pipe. Also, torsion of helically coiled tubes causes more complication in temperature and velocity fields. Rennie [1] and Rennie and Raghavan [2] experimentally reported the heat transfer in a coil-in-coil heat exchanger comprised of one loop. This configuration results in secondary flows in both the inner tube and in the annulus, as both sections. They also reported that increasing the tube Dean number or annulus Dean numbers resulted in an increase in the overall heat transfer coefficient. Kumar et al. [3,7] have investigated hydrodynamics and heat transfer characteristics of tube-in-tube helically coiled. In their analyses, they have concentrated on the turbulent flow regime and a new empirical correlation is developed for hydrodynamic and heat-transfer predictions in the outer tube of the tube-in-tube helically coiled.
Naphon [4] studied the thermal performance and pressure drop of the helical-coil heat exchanger with and without helical crimped fins. The results shown that with increasing hot water mass flow rate, the friction factor decreased. Rennie and Prabhanjan [5] numerically studied the heat transfer characteristics in a two-turn coil-in-coil helical coil heat exchanger. The results showed that the flow in the inner tube at the high tube-to-tube ratios was the limiting factor for the overall heat transfer coefficient. This dependency was reduced at the smaller tube-to-tube ratio, where the influence of the annulus flow was increased. Also, Rennie and Raghavan [6] numerically modeled of the heat exchanger for laminar fluid flow and heat transfer characteristics. Overall heat transfer coefficients for counter-current and parallel flows were calculated for inner Dean numbers in the range of 38–350 for the boundary conditions of constant wall temperature and constant heat flux.

1.2. Motivation

Thermal management requirements for aerospace applications continue to grow but the weight and volume allotment remains the same or is shrunk. Compact, high performance and lightweight heat exchangers are needed to meet the requirements. Several innovative heat transfer enhancement techniques are being considered for development of thermal management components that will meet these challenging demands. Thermal performance requirements for aircraft engine heat exchangers are becoming quite challenging, requiring development of novel heat transfer enhancement techniques and design concepts.

The current state-of-the-art, compact plate-fin designs, although quite efficient, cannot provide the high heat transfer rejection rate to meet future heat exchanger envelope and weight requirements. A number of enhanced heat transfer techniques are being developed to improve heat exchanger performance [9-11]. These include microchannels [12-14], high porosity, open-cell metallic and carbon and graphite foams [15-18], and novel heat transfer surfaces [19]. Early heat exchangers designers were adding these bulky units to existing engines with no cycle modifications to fully utilize the additional hardware. Compact heat exchangers have traditionally been sought in the aerospace industry due to the strong incentive to minimize exchanger weight and volume. Further, to achieve an overall system efficiency of greater than 70%, very low heat exchanger pressure drops are needed, initiating more challenges to
creating a compact design. Helical heat exchanger is one such design which helps in satisfying the heat transfer goals and by reducing the weight and volume.

The evolution of 3D printing and additive manufacturing technologies has changed design, engineering and manufacturing processes across industries and thus with the help of 3D printing technology it is possible to accurately print such compact helical models.. Although the helical passages increase the heat transfer and reduce the size of the device, 3D print build constraints mandate that the passages are constructed with a lean angle for structural support because there is no way to build-up the walls that define the helical passages due the being cantilevered perpendicular from the wall without support. To amend this issue, it is common practice in additive manufacturing to build cantilevered elements using a build-angle, here referred to as a lean angle. The lean angle allows a cantilevered wall to be built using 3D printing. Where traditional heat exchanging prototypes take months to develop, 3D printed heat exchanger can be completed in just a few weeks’ time, especially because alterations and experimental designs are produced much more quickly. 3D printing allows walls to be built as thin as 200 micrometers, making possible more heat exchanger applications than ever before. As thin as these walls are, they are still able to withstand high pressure and, as noted, are more leak-resistant than those on traditional models.

1.3. Objectives

The objectives of this thesis are to:

1) Develop an analytical model that can be used to optimize the design of a 3D printed, compact counter-flow heat exchanger.

2) Identify relevant performance metrics, including heat exchange, working fluid pressure drop, compactness, cost, manufacturability, etc.

3) Use the model to assess the performance of a new compact counter-flow heat exchanger design over a range of relevant fluids, flow conditions and targeted heat exchange.

4) Assess a variety of geometric configurations for important performance metrics, including heat exchanger, pressure loss, volume, mass, manufacturability, etc.

5) Optimize design for range of flow condition and based on a set of constraints.
1.4. Approach

An initial design and a set of specified parameters are used as a starting point and is used in analyzing a straight annular counterflow heat exchanger. To verify heat transfer goals, two important parameters, required and available heat transfer coefficient are compared and checked if values are equal. In other words if the ratio of achievable over required is equal to one, then heat transfer goals have been met. The same concept is applied to helical annular counterflow heat exchanger with and without lean. To check the efficiency of the design, frictional pressure drop is calculated across the heat exchanger for all the different configurations.

The model used to analyze the different heat exchanger types is run across a range of relevant fluids and flow conditions. Finally based on a set of geometric and performance constraints different heat exchanger models are designed and optimized over a range of mass flow rates.

1.5. Thesis Overview

Chapter 2 gives a detailed overview in the analytical modelling of an annular counterflow heat exchanger. Two important modeling goals, heat exchange and pressure drop are stressed upon.

Chapter 3 develops an analytical model for three different annular counterflow heat exchanger designs. Straight annular counterflow with and without fins, helical annular with fins having no lean and helical annular heat exchanger having leans are discussed in sections 3.1, 3.2 and 3.3 respectively. Nusselt number and frictional factor correlations have also been laid down carefully for all the types of heat exchangers.

Chapter 4 details the results for the three different heat exchangers discussed in Chapter 3 and are constantly checked if the design is able to match the heat transfer goals and pressure drop constraints.
Chapter 5 develops the parametric study which helps in optimizing the heat exchanger to achieve the heat transfer goals, improve efficiency and simultaneously minimize the weight and volume, i.e. a compact design.

Chapter 6 concludes this thesis. It summarizes the results and suggests areas for further research.
2. Counterflow Heat Exchanger Analysis Overview

Cylindrical, annular counterflow heat exchangers have been extensively investigated and are discussed in literature [4-6]. This section presents an overview of a special class of cylindrical, annular counterflow heat exchangers, which is used in many engineering applications where the central region of the heat exchanger is left open for several reasons, such as locating other internal components, and because locating the flow passages further radially outward increases surface area available for heat exchange. This type of heat exchanger is shown in Figure 2-1.

![Figure 2-1: Cylindrical Annular Straight Counterflow Heat Exchanger](image)

The heat exchanger shown in Figure 1 has two concentric annular channels. In this figure, the outer annular channel has 4 fins and thus 4 passages and the inner has 8 passages. The diameters shown in Figure 1 are centerline diameters, meaning that the diameters are those to the center of the walls. The outer diameter is $D_o$, the inner diameter is $D_i$ and the innermost is $D_s$. The wall thickness of the outer, $t_o$, inner, $t_i$, innermost, $t_s$ and fin thickness, $t_f$ are included in the geometric calculations however, the walls present no thermal resistance. Based on the thickness of the channel walls and the diameters the heights of the individual channels can be determined. For example, the outer channel height is $D_o - t_o - (D_i + t_i)$. The length of the heat exchanger is denoted by $L$. 

11
The important parameters for the heat exchanger are the inlet temperatures, pressures, and mass flow of the two working fluids, and the geometry of the device. For this device, and in the analyses, that follow, hot fluid flows in the inner channels and the cold fluid flows in the outer channel. The fluids always flow counter to each other.

The analyses assume that the heat exchanger operates in steady-state, is adiabatic, and that the flows enter the heat exchanger fully developed in both momentum and thermal profiles. Energy balance equations are used to find the required overall heat transfer coefficient. Equation (1) and (2) give the energy balance for the hot and cold fluid, respectively.

\[
q = \dot{m}_h (h_{h,o} - h_{h,i}) \quad (1)
\]
\[
q = \dot{m}_c (h_{c,i} - h_{c,o}) \quad (2)
\]

Where \( q \) is the heat transfer rate from either hot to cold fluid or from cold to hot fluid, \( \dot{m}_h \) is the mass flow rate of hot fluid, \( h_{h,i} \) is the inlet enthalpy of the hot fluid, \( h_{h,o} \) is the outlet enthalpy of the hot fluid, \( \dot{m}_c \) is the mass flow rate of cold fluid, \( h_{c,i} \) is the inlet enthalpy of the cold fluid, and \( h_{c,o} \) is the outlet enthalpy of the cold fluid. For example, with known mass flow rates, inlet temperatures, and the desired exit temperature of one of the other fluids, the heat transfer rate and exit temperature of the other fluid can be found.

Equation (3) gives the heat transfer rate from the hot fluid to the cold fluid or vice-versa in the heat exchanger

\[
q = U_{req} A_s \Delta T_{lm} \quad (3)
\]

\( A_s \) is the heat transfer surface area, \( U_{req} \) is the overall heat transfer coefficient that is required to achieve the desired heat exchange, and \( \Delta T_{lm} \) is the log mean temperature difference of the fluids, given by Equation (4).

\[
\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} \quad (4)
\]
\[ \Delta T_1 = T_{h,i} - T_{c,o} \quad (5) \]
\[ \Delta T_2 = T_{h,o} - T_{c,i} \quad (6) \]

Where \( \Delta T_1 \) and \( \Delta T_2 \) are the temperature differences of the fluid temperatures at the inlet and outlet of the heat exchanger channels, \( T_{h,i} \) and \( T_{h,o} \) are the temperatures of the hot fluid at inlet and exit respectively. Likewise, \( T_{c,i} \) and \( T_{c,o} \) are the temperatures of the cold fluid at inlet and exit respectively.

The achievable overall heat transfer coefficient is the inverse of the total thermal resistance between two fluids. Generally, the coefficient is determined by accounting for conduction and convection resistances between fluids separated by composite plane and cylindrical walls respectively. In this analysis, zero wall resistance is assumed and thus the achievable overall heat transfer coefficient is determined from the hot and cold fluid convection coefficients and from appropriate geometric parameters. Equation (7) gives the expression for achievable overall heat transfer coefficient.

\[ U_{ach} = \left( \frac{1}{h_h} + \frac{1}{h_c} \right)^{-1} \quad (7) \]

In this expression \( h_h \) and \( h_c \) are the hot and cold convective coefficients respectively. The convective heat transfer coefficient is found using equation (8):

\[ h = Nu \frac{k}{D_h} \quad (8) \]

In equation (6), \( Nu \) is the Nusselt number, \( k \) is the thermal conductivity of the fluid and \( D_h \) is the hydraulic diameter and is given by Equation (9):

\[ D_h = \frac{4A_{crs}}{p} \quad (9) \]
In the above equation $A_{crs}$ is the cross-sectional area and $p$ is the wetted perimeter. Nusselt number is a function of the two dimensionless quantities, Reynolds, $Re_D$ and Prandtl, $Pr$ number. Reynolds and Prandtl number is given by equation (10) and (12) respectively.

\[ Re_D = \frac{\rho v D_h}{\mu} \]  \hspace{1cm} (10)

\[ v = \frac{\dot{m}}{\rho A_{crs}} \]  \hspace{1cm} (11)

\[ Pr = \frac{C_p \mu}{k} \]  \hspace{1cm} (12)

In the above equations $\rho, C_p, \mu$ are the density, Specific heat at constant pressure and viscosity of the fluid respectively and $v$ is the fluid velocity. Therefore, convective heat transfer depends on the flow regime, fluid properties, geometry and convective heat transfer coefficients are analyzed for two different counterflow heat exchanger design/model, a straight and a helical annular heat exchanger.

The fluid pressure drop is an important parameter in heat exchanger analysis and minimizing is always favorable. The frictional pressure drop, $\Delta P$ along the length of the channel is given by Equation (13):

\[ \Delta P = \frac{fL\dot{m}}{2\rho D_h A_{crs}^2} \]  \hspace{1cm} (13)

Where $f$ is the frictional factor, $L$ is the length of the channel, $\dot{m}$ is the mass flow rate of the fluid, $\rho$ is the density of the fluid, $D_h$ is the hydraulic diameter of the pipe and $A_{crs}$ is the cross-sectional area of the channel. The frictional factor correlations for different types of heat exchangers are discussed in the next section.
3. Counterflow Heat Exchanger Analytical Modeling

This section develops analytical models for several types of counterflow heat exchangers in order to trade relevant device performance parameters such as overall heat transfer rates, resulting flow temperatures, pressure loss, as well as the physical characteristics of the device such as volume and mass. This work develops an analytical model which determines the heat transfer performance and pressure drop for a heat exchanger design and then the design is optimized to increase heat transfer performance and decrease pressure drop. Three geometric categories of counterflow heat exchanger are considered: subsection 3.1 examines cylindrical, annular geometries without and with radial fins, subsection 3.2 develops a model for a cylindrical, annular heat exchanger in which the flow passages are helically wrapped around the device, and subsection 3.3 extends the models in subsection 3.2 to include a lean angle of the radial fins that is required for the fabrication of a such a device using additive manufacturing.

3.1. Straight Annular Heat Exchanger without and with Radial Fins

This section describes the analytical modeling for a counterflow heat exchanger where both the cold flow and the hot flow passages are straight – meaning that the passages are parallel to the central axis of the heat exchanger and the flows move parallel to the central axis of the heat exchanger, as shown in Figure 1. Radial elements can be added which divides both the cold and the hot passages into individual channels. The radial elements act as fins to promote greater heat transfer and act as flow straighteners which keep the flows moving parallel to the axial direction of the heat exchanger. The penalty associated with adding these radial elements is that there is more flow-surface interaction, typically resulting in larger pressures losses of the working fluids. A schematic of a straight counter flow heat exchanger with 4 channels in the cold section and 8 channels in the hot section was shown in Figure 2-1. The cross-sectional area and the perimeter for passage as shown in figure 2-1 is calculated appropriately by taking diameter, wall and fin thickness into account. For example, the cross-sectional area and perimeter of a passage in the outer channel is given by equation (14) and (15).
\[ A_{crs} = \frac{1}{n} \left( \frac{\pi (D_o - t_o)^2}{4} - \frac{\pi (D_i + t_o)^2}{4} \right) - \left( \frac{t_i - t_o}{2} + D_o - D_i \right) t_f \]  
(14)

\[ p = \frac{1}{n} \left( \pi (D_o + D_i + \frac{t_i - t_o}{2}) \right) + 2 \left( D_o - D_i - \frac{t_o + t_i}{2} \right) - t_f \]  
(15)

Nusselt number is a function of Reynolds number and Prandtl number and equation (16) and (17) gives the Nusselt number correlations which are valid for straight channels.

\[ Nu_D = 4.36 \]  
(16)

\[ Nu_D = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{0.5}(Pr^{2/3} - 1)} \]  
(17)

\[ f = (0.790lnRe_D - 1.64)^{-2} \]  
(18)

The flow is assumed to be fully developed and is under a uniform heat flux. Equation (16) is used when the flow is laminar and Pr \( \geq 0.6 \). In the above expressions \( f \) is the Darcy frictional factor, \( Re_D \) is the Reynolds number (based on hydraulic diameter) and \( Pr \) is the Prandtl number. The correlation in equation (17) is valid for, \( 3,000 \leq Re_D \leq 5 \times 10^6 \), \( 0.5 \leq Pr \leq 2,000 \) and \( L \geq 10D_h \). Based on the flow regime and \( Pr \), an appropriate \( Nu \) correlation is chosen and the convective heat transfer coefficient is found for both the hot and cold fluids and ultimately the achievable overall heat transfer coefficient is found using equation (7).

The achievable overall heat transfer coefficient is given by equation (19) when fins are added. Fins increase the surface area exposed to heat transfer and they reduce the resistance to convective heat transfer and the overall fin efficiency is given by equation (20).

\[ U_{ach} = \left( \frac{1}{(\eta_0 h)_R} + \frac{1}{(\eta_0 h)_C} \right)^{-1} \]  
(19)

\[ \eta_0 = 1 - \frac{A_f}{A} (1 - \eta_f) \]  
(20)

In the above expressions \( \eta_0 \) is the overall fin efficiency, \( \eta_f \) is the efficiency of a single fin, \( A_f \) is the fin surface area and \( A \) is the total surface area. The efficiency of a fin is calculated using equation (21) and under the assumption that the tip of the fin is adiabatic.
\[ \eta_f = \frac{\tanh(mL)}{mL} \]  
\[ m = \frac{2h}{\sqrt{k_f t_f}} \]

In the above expressions \( t_f \) is the fin thickness, \( h \) is the convective heat transfer coefficient of the fluid and \( k_f \) is the thermal conductivity of the fin. In case of straight pipes, the frictional factor for laminar flow regime is given by equation (23) and Colebrook-white equation is used for turbulent regime as shown in equation (24)

\[ f_s = \frac{64}{Re_D} \]  
\[ \frac{1}{\sqrt{f_s}} = -2 \log_{10} \left[ \frac{\varepsilon/D_h}{3.7} - \frac{2.51}{Re_D \sqrt{f_s}} \right] \]

In the above equation \( f_s \) is the frictional factor for straight tubes and \( \varepsilon \) is the surface roughness of the pipe material.

### 3.2. Helical Annular Heat Exchanger with Radial Fins

This section presents a heat exchanger concept similar to that shown in Figure 2-1, however, the channels are now helical, rather than straight passages. A schematic of the helical annular counter flow heat exchanger concept with 8 channels in the cold (outer) section and 4 channels in the hot (inner) section is shown in Figure 3-1.
Figure 3-1 : Cylindrical Helical Annular Counterflow Heat Exchanger with N = 0.5, \( \psi = 37.6^\circ \), \( L_{hlx}/L = 1.26 \) for inner channels and N=1, \( \psi = 26.2^\circ \), \( L_{hlx}/L = 2.26 \) for the outer channels.

The helical passages are characterized by the number of turns, N, over the length of the heat exchanger, L, or the helical angle, \( \psi \). The length of the helical channel, \( L_{hlx} \), and helical angle, \( \psi \), are given by Equations (25) and (26).

\[
L_{hlx} = \sqrt{(2\pi N r)^2 + L^2} \\
\psi = \cos^{-1}(L/L_{hlx})
\]

In the above equations, N is the number of helical turns, r is the radiuses of helix i.e. distance from the center of the heat exchanger to the center of the channel and L is the length of the heat exchanger. In case of helical passages, the cross-sectional area and the wetted perimeter of a single passage is found by taking diameter, wall and fin thickness and helical angle, \( \psi \) into account. For example, cross-sectional area and perimeter of a passage in the outer channel is given by equation (27) and (28).
The secondary flow within the passages is an important characteristic of the helical heat exchanger. The dimensionless Dean number, De, is used in the analysis in addition to those used in straight round channels and is given by Equation (29). The critical Reynolds number, \( Re_{crit} \), is used to identify the transition from laminar to turbulent flow in curved or helical pipes, is calculated as shown in equation (30).

\[
De = Re_D (a/R)^{1/2}
\]

\[
Re_{crit} = 2100[1 + 12(R/a)^{-0.5}]
\]

In the above expressions \( a \) denotes the radius of the helical channel. For helical coils, no single \( Re_{crit} \) exists because of the varying curvature. For helical coils with constant heat flux, the Nusselt number has been developed by Manlapaz and Churchill [20] for laminar fully developed flow and is given by equation (31). Nusselt correlations for turbulent flow developed by Schmidt [20] is suggested for \( 2 \times 10^4 < Re < 1.5 \times 10^5 \) and \( 5 < R/a < 84 \) and is given by equation (34). For low Reynolds number Pratt’s correlation is recommended and is for \( 1.5 \times 10^3 < Re < 2 \times 10^4 \) and is given by equation (35).

\[
Nu_{cv} = \left[ \left( 4.364 + \frac{4.636}{x_3} \right)^3 + 1.816 \left( \frac{De}{x_4} \right)^{3/2} \right]^{1/3}
\]

\[
x_3 = \left( 1 + \frac{1342}{De^2 Pr} \right)^2
\]

\[
x_4 = 1 + \frac{1.15}{Pr}
\]

\[
Nu_{cv} = Nu_s \left[ 1 + 3.6 \left( \frac{a}{R} \right) \left( \frac{a}{R} \right)^{0.8} \right]
\]

\[
Nu_{cv} = Nu_s \left[ 1 + 3.4 \left( \frac{a}{R} \right) \right]
\]
In the above expressions, $N_u_{cv}$ is the Nusselt number for curved or helical pipes and $N_u_s$ is the Nusselt number for straight pipes. In helical coils, the flow generally becomes fully developed within the first half turn of the coil. The required and achievable convective heat transfer coefficient is calculated using equation (7) and (19). Frictional factor for a fully developed laminar flow in helical coil proposed by Manlapaz and Churchill [21] is given by equation (36)

$$\frac{f_{cv}}{f_s} = \left[1 - \frac{0.18}{\left[1 + (35/De)^2\right]^{0.5}}\right]^m + \left(1 + \frac{a/R}{3}\right)^2 \left(\frac{De}{88.33}\right)^{0.5}$$  \hspace{1cm} (36)

In the above equation $f_c$ is the frictional factor for curved pipes, $f_s$ is the frictional factor for straight pipes, $m = 2$ for $De < 20$; $m = 1$ for $20 < De < 40$; and $m = 0$ for $De > 40$. Appropriate $f_s$ can calculated based on $Re_D$ and from the correlations given by equation (23) and (24). Turbulent flow frictional factors as shown in equation (37) was developed by Srinivasan and can be used when $Re \left(\frac{R}{a}\right)^{-2} < 700$ and $7 < \frac{R}{a} < 104$.

$$f_{cv} \left(\frac{R}{a}\right)^{0.5} = 0.084 \left[Re \left(\frac{R}{a}\right)^{-2}\right]^{-0.2}$$  \hspace{1cm} (37)

3.3. Helical Annular Heat Exchanger with Radial Fins and Lean

The geometry shown in Figure 3-1 represents a highly compact and efficient device, however, the geometry cannot be fabricated using 3D printing because there is no way to build-up the helical passage walls due to them being cantilevered perpendicular from the wall without support. To amend this issue, a lean angle is used during the build. A schematic of the heat exchanger with 8 channels in the cold section and 4 channels in the hot section with fins having a lean angle is shown in Figure 3-2.
In case of radial fins with a lean angle, $\theta$ the area of the passage remains the same, but the wetted perimeter changes when compared to those of the model without lean and is shown by equation (38).

$$p = \left\{ \frac{1}{n_o} \left( \pi (D_o + D_i + \frac{t_i-t_o}{2}) \right) \right\} \cos \psi + 2 \left( D_o - D_i - \frac{t_o+t_i}{2} \right) \sec \theta - 2t_f$$

Thus, the hydraulic diameter changes and varies the Reynolds number and thus ultimately changing the achievable overall heat transfer coefficient. The frictional factor and the Nusselt number correlation are the same to that of the helical coils without lean.

### 3.4 Geometry Implications

This section shows change in helical angle, $\psi$ and number of turns, $N$ when heat exchanger length is varied. Figure 3-3 shows change in helical angle when heat exchanger length is varied for a fixed number of helical turns (in this case, $N = 1$).
Figure 3-3: Helical Angle, $\psi$ vs Heat exchanger length, $L$ for fixed $N = 1$

From the above figure, as heat exchanger length increases for a fixed $N$, the helical angle increases which in turn decrease the cross sectional area and perimeter as shown in equation (27) and (28). Figure 3-4 shows change in number of helical turns, $N$ when heat exchanger length is varied for a fixed helical angle (in this case, $\psi = 24.4^\circ$ (calculated for $N = 1$)).

Figure 3-4: Number of helical turns, $N$ vs Heat exchanger length, $L$ for fixed $\psi = 24.4^\circ$
In figure 3-4, as heat exchanger length increases for a fixed $\psi$, number of helical turns increases too, but there is no change in cross sectional are and perimeter. However, the helical length increases as shown by equation (25).

Table 3-1 summarizes the important geometric parameters for the heat exchangers described in this section

Table 3-1: Summary of important heat exchanger geometric parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\theta = 0^\circ$</th>
<th>$\theta = 0^\circ$</th>
<th>$\theta = 45^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N = 0$</td>
<td>$N = 0.5$</td>
<td>$N = 1$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>90°</td>
<td>42.2°</td>
<td>24.4°</td>
</tr>
<tr>
<td>$L_{h,x}$</td>
<td>1</td>
<td>1.49</td>
<td>2.42</td>
</tr>
<tr>
<td>$A_{crs}$</td>
<td>1</td>
<td>0.67</td>
<td>0.41</td>
</tr>
<tr>
<td>$P$</td>
<td>1</td>
<td>0.68</td>
<td>0.43</td>
</tr>
<tr>
<td>$D_h$</td>
<td>1</td>
<td>0.98</td>
<td>0.95</td>
</tr>
</tbody>
</table>

In Table 3-1, the straight channel case ($N = 0, \theta = 0^\circ$), the length, cross-sectional area, and wetted perimeter have been normalized to 1 as a baseline case. As the number of turns, $N$ is increased, helical angle $\psi$ decreases, the helical length of the channel increases, the cross-sectional area and wetted perimeter decreases. When a lean angle, $\theta$, is added to the helical cases, the length and cross-sectional area do not change, but the wetted perimeter increases, thus decreasing the hydraulic diameter. In helical case, there is an increase in length and thus the heat transfer area, which increases the heat transfer rate but also increases the pressure drop across the heat exchanger. Increasing the number of turns will result in higher heat transfer rate, but also a higher pressure drop.
4 Results

This section presents the results of a parametric study for the various heat exchanger geometries discussed above. The geometric constraints and flow conditions for the parametric study are summarized in Table 4-1.

Table 4-1: Heat exchanger design and performance parameters

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value or Range</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer diameter, $D_0$</td>
<td>$\leq 0.3 \text{ m}$</td>
<td>Constraint</td>
</tr>
<tr>
<td>Innermost diameter, $D_s$</td>
<td>$\geq 0.2 \text{ m}$</td>
<td>Constraint</td>
</tr>
<tr>
<td>Length, L</td>
<td>$\leq 0.4 \text{ m}$</td>
<td>Constraint</td>
</tr>
<tr>
<td>$U_{ratio}$</td>
<td>$= 1$</td>
<td>Constraint</td>
</tr>
<tr>
<td>Pressure drop, $\Delta P$</td>
<td>$\leq 10%$ of Inlet Pressure</td>
<td>Constraint</td>
</tr>
<tr>
<td>Hot fluid mass flow rate, $\dot{m}_h$</td>
<td>0.1 kg/s - 1 kg/s</td>
<td>Desired operating range</td>
</tr>
<tr>
<td>Cold fluid mass flow rate, $\dot{m}_c$</td>
<td>1 kg/s - 3 kg/s</td>
<td>Desired operating range</td>
</tr>
<tr>
<td>Hot fluid inlet temperature, $T_{h,i}$</td>
<td>368 K</td>
<td>Constraint</td>
</tr>
<tr>
<td>Hot fluid exit temperature, $T_{h,o}$</td>
<td>298 K</td>
<td>Constraint</td>
</tr>
<tr>
<td>Cold fluid inlet temperature, $T_{c,i}$</td>
<td>278 K</td>
<td>Constraint</td>
</tr>
<tr>
<td>Wall and fin thickness, $t_o, t_i, t_f$</td>
<td>1 mm</td>
<td>Constant</td>
</tr>
<tr>
<td>Number of turns</td>
<td>-</td>
<td>Variable</td>
</tr>
<tr>
<td>Inner diameter, $D_i$</td>
<td>-</td>
<td>Variable</td>
</tr>
<tr>
<td>Fluids</td>
<td>Water, Nitrogen</td>
<td>Constant</td>
</tr>
</tbody>
</table>

The constraints are set by the heat exchanger necessitated performance, variable parameters can be adjusted to achieve required performance.

The working fluids are water/water and water/nitrogen. The objective is to cool the incoming hot fluid from 368 K to 298 K using cold fluid which enters the heat exchanger at 278 K. Both fluids enter the heat exchanger with static pressure of 202 kPa. For the analysis
the initial geometry are $D_0 = 0.3$ m, $D_i = 0.287$ m, $D_s = 0.275$ m, $L = 0.4$ m, and the fin and wall thickness are all 1 mm. Diameters and wall thickness set the inner and outer channel heights.

The energy balance (Equation 1 and 2) and log mean temperature difference (Equation 4) are used to find the heat transfer rate or the power required to lower the temperature of the hot fluid and find the exit temperature of the cold fluid. As discussed in section II Equations (3) and (7) are used to find $U_{req}$ and $U_{ach}$ for different fluid mass flow rates. The next subsections present the performance results of the various heat exchanger geometries.

4.1 Straight annular heat exchanger without and with radial fins

Table 4-2 shows the $U_{ratio}$ variation for different mass flow rates for straight heat exchanger without radial fins for sets of working fluid combination.

<table>
<thead>
<tr>
<th>$\dot{m}_h$ (kg/s)</th>
<th>$\dot{m}_c$ (kg/s)</th>
<th>q (kW)</th>
<th>$T_{h,o}$ (K)</th>
<th>$U_{req}$ (kW/ $m^2.K$)</th>
<th>$U_{ach}$ (kW/ $m^2.K$)</th>
<th>$U_{ratio}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water – Water Heat exchanger</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>303</td>
<td>351</td>
<td>45.71</td>
<td>0.537</td>
<td>0.01</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>303</td>
<td>304</td>
<td>22.28</td>
<td>0.991</td>
<td>0.04</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>30</td>
<td>286</td>
<td>1.925</td>
<td>0.132</td>
<td>0.07</td>
</tr>
<tr>
<td>0.1</td>
<td>3</td>
<td>30</td>
<td>281</td>
<td>1.850</td>
<td>0.246</td>
<td>0.13</td>
</tr>
<tr>
<td>Nitrogen – Water Heat exchanger</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>73</td>
<td>297</td>
<td>5.081</td>
<td>0.191</td>
<td>0.04</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>73</td>
<td>285</td>
<td>4.603</td>
<td>0.455</td>
<td>0.10</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>7</td>
<td>280</td>
<td>0.445</td>
<td>0.069</td>
<td>0.15</td>
</tr>
<tr>
<td>0.1</td>
<td>3</td>
<td>7</td>
<td>279</td>
<td>0.441</td>
<td>0.091</td>
<td>0.20</td>
</tr>
</tbody>
</table>

In case of a straight heat exchanger without fins, $U_{ratio}$ is less than 1 for different mass flow rate cases. This means the hot fluid is not cooled to the desired temperature for this design. To
improve the $U_{ratio}$ and to achieve the required drop in temperature for the hot fluid, fins are employed, which in turn increases the heat transfer area and thus the heat transfer rate. Table 4-3 summarizes changes in $U_{ratio}$ when 8 fins are employed in both inner and outer channel. Energy transfer rate, $q$ and the exit temperature, $T_{h,o}$ of the cold fluid remains the same for different mass flow rate cases.

**Table 4-3: $U_{ratio}$ for straight annular heat exchanger with 8 radial fins in both the channels**

<table>
<thead>
<tr>
<th>$m_h$ (kg/s)</th>
<th>$m_c$ (kg/s)</th>
<th>$U_{ratio}$</th>
<th>$L_{req}$ (m) to achieve $U_{ratio} = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0.04</td>
<td>0.10</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>0.07</td>
<td>0.16</td>
</tr>
<tr>
<td>0.1</td>
<td>3</td>
<td>0.14</td>
<td>0.21</td>
</tr>
</tbody>
</table>

In case of straight heat exchanger with fins, $U_{ratio}$ nearly is the same when compared to the one without fins. There is a very small improvement in $U_{ratio}$, but not significant enough to cool down the hot fluid to the desired temperature. $L_{req}$ is the heat exchanger length required to achieve $U_{ratio} = 1$. The frictional pressure loss for both sets of working fluids in a straight heat exchanger with and without radial fins is summarized in Table 4-4.
Table 4-4: Frictional pressure drop in a straight annular heat exchanger

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Mass flow rate (kg/s)</th>
<th>ΔP for heat exchanger without fins (kPa)</th>
<th>ΔP for heat exchanger with fins (kPa)</th>
<th>ΔP for L_{req} (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water – Water Heat exchanger</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hot Water</td>
<td>1</td>
<td>0.04</td>
<td>0.04</td>
<td>3.64</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.001</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>Cold Water</td>
<td>1</td>
<td>0.04</td>
<td>0.04</td>
<td>3.67</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.33</td>
<td>0.36</td>
<td>2.61</td>
</tr>
<tr>
<td>Nitrogen – Water Heat exchanger</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrogen</td>
<td>1</td>
<td>10.36</td>
<td>11.53</td>
<td>298</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.15</td>
<td>0.16</td>
<td>0.78</td>
</tr>
<tr>
<td>Water</td>
<td>1</td>
<td>0.03</td>
<td>0.03</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.34</td>
<td>0.36</td>
<td>1.74</td>
</tr>
</tbody>
</table>

Tables above summarizes the $U_{ratio}$ and $\Delta P$ for a straight heat exchanger with and without fins, $U_{ratio} < 1$ in all the cases. There is increase in $U_{ratio}$ for the design with fins when compared to that of design without fins, however the pressure loss increases too. A long heat exchanger might satisfy $U_{ratio}$ and pressure drop constraints, however the design is not suitable if weight and compactness are considered. An improved design is needed to bring $U_{ratio}$ to 1 and thus helically coiled heat exchanger is the next design tested.

### 4.2 Helical annular heat exchanger with radial fins having no lean angle

Helically coiled heat exchangers coiled offers advantages over conventional shell and straight tube heat exchangers in terms of heat transfer rates. It accommodates a large heat transfer area in a small space, with high heat transfer coefficients. Tubes are wrapped around cylinder in a helical shape and number of turns or helical angle are varied which changes the length of the heat exchanger and ultimately the heat transfer area. Due to helical shape, a secondary flow (centrifugal force) is created within the channel and allows for better mixing.
and there is also an increase in the heat exchanger length leading to an increase in heat transfer area and thus a higher $U_{ratio}$. Increasing the number of turns or decreasing the helical angle increases the $U_{ratio}$. Table 4-5 showcases how $U_{ratio}$ changes with increasing coil turns in a helical annular heat exchanger having radial fins with no lean.

Table 4-5: $U_{ratio}$ for helical annular heat exchanger with radial fins having no lean

<table>
<thead>
<tr>
<th>$m_h$ (kg/s)</th>
<th>$m_c$ (kg/s)</th>
<th>$U_{ratio}$</th>
<th>Water – Water</th>
<th>Nitrogen-Water</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$N = 0.5$</td>
<td>$N = 1$</td>
<td>$N = 1.25$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.03</td>
<td>0.10</td>
<td>0.15</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0.11</td>
<td>0.29</td>
<td>0.42</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>0.30</td>
<td>0.65</td>
<td>0.85</td>
</tr>
<tr>
<td>0.1</td>
<td>3</td>
<td>0.42</td>
<td>0.79</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 4-5 shows $U_{ratio}$ is high for helical heat exchanger when compared to a straight heat exchanger. Increase in turns leads to higher $U_{ratio}$. In a few cases $U_{ratio}$ exceeds 1 and it means that the hot fluid is getting overcooled, i.e. beyond the desired temperature. When hot water-cold water is used as the working fluids, $U_{ratio}=1$ is never attained for half turn or for a complete turn. In case of 1.25 turns, $U_{ratio}=1$ is achieved, thus cooling the hot water to the desired temperature. When nitrogen-water is used as the working fluid, 0.5, 1 and 1.25 turns does not meet the heat transfer goals. $U_{ratio}$ is either less than or greater than 1 for all different mass flow rate combinations. $U_{ratio}=1$ can be achieved by varying the mass flow rates between the given range for hot and cold fluid. For example, with mass flow rates $m_h = 0.385$ kg/s and $m_c = 1$ kg/s and heat exchanger with 1.25 turns gives $U_{ratio}=1$ when nitrogen-water is used as working fluid.

As shown in table 4-3 the length required to bring in $U_{ratio}=1$ in case of $m_h = 0.1$ kg/s and $m_c = 3$ kg/s for straight counterflow water-water heat exchanger is 2.9 m. For the same mass flow rate combination in helical counterflow heat exchanger, for $N$ corresponding to 1.25 turns, $U_{ratio}=1$ is achieved. The helical length corresponding to 1.25 turns is 1.17 m. The heat transfer goal has been met in a relatively shorter length which is 1.17 m, than the one
calculated before which is 2.9 m. The reasoning for this interesting observation is, in the helical heat exchanger the cross sectional area decreases too in the process of increasing the number of turns. In decreasing the cross sectional area there is an increase in velocity and thus Reynolds number goes up, i.e. it becomes more turbulent. With the flow being more turbulent it helps in better mixing and with secondary flow formed, the heat exchange is quicker.

Figure 4-1 illustrates how $U_{ratio}$ and $\Delta P$ changes with increase in number of helical turns for water-water heat exchanger for a fixed mass flow rate, $m_h = 1$ kg/s and $m_c = 1$ kg/s.

![Figure 4-1: $U_{ratio}$ and $\Delta P$ vs N for water-water heat exchanger](image)

In figure 4-1 the dotted blue line represents the heat transfer goal and the red dotted line represents the pressure drop threshold.

### 4.3 Helical Annular Heat Exchanger with Radial Fins and Lean

Due to build constraints, the fins in the heat exchanger are at a lean angle and the table 4-6 summarizes change in $U_{ratio}$ with and without lean for $N = 1$. 

29
Table 4-6: $U_{ratio}$ comparison for helical annular heat exchanger with $\theta = 0^\circ$ and $\theta = 45^\circ$, N=1

<table>
<thead>
<tr>
<th>$\dot{m}_h$ (kg/s)</th>
<th>$\dot{m}_c$ (kg/s)</th>
<th>$U_{ratio}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Water – Water</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta = 0^\circ$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.10</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0.29</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>0.65</td>
</tr>
<tr>
<td>0.1</td>
<td>3</td>
<td>0.79</td>
</tr>
</tbody>
</table>

From the above table having a lean on the fins increases $U_{ratio}$ marginally. The frictional pressure loss in a helical heat exchanger is summarized in table 4-7 for distinctive design cases.

Table 4-7: Frictional pressure drop in a helical annular heat exchanger for multiple N’s

<table>
<thead>
<tr>
<th>Fluids</th>
<th>Mass flow rate (kg/s)</th>
<th>$\Delta P$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$N = 0.5, \theta = 0^\circ$</td>
</tr>
<tr>
<td>Water – Water Heat exchanger</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hot-Water</td>
<td>1</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.006</td>
</tr>
<tr>
<td>Cold-Water</td>
<td>1</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4.47</td>
</tr>
<tr>
<td>Nitrogen – Water Heat exchanger</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrogen</td>
<td>1</td>
<td>185</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>2.91</td>
</tr>
<tr>
<td>Water</td>
<td>1</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4.43</td>
</tr>
</tbody>
</table>
Increase in number of turn’s leads to an increase in heat exchanger helical length, decrease in cross sectional area and ultimately an increase in pressure loss. Pressure loss is directly proportional to the length and inversely proportional to the square of the cross sectional area as shown in equation (13).

Figure 4-2: $U_{ratio}$ and $\Delta P$ vs Heat exchanger length for water-water heat exchanger

Figure 4-2 shows changes in $U_{ratio}$ and $\Delta P$ for increase in heat exchanger length for fixed number of helical turns $N = 1$, fixed heat exchanger diameter and fixed mass flow rate, $m_h = 0.1 \text{ kg/s}$ and $m_c = 3 \text{ kg/s}$. There is a decrease in $U_{ratio}$ till $L = 0.5 \text{ m}$ and then there is an increase after $0.5 \text{ m}$. The change in trend is due to change in flow regime, turbulent to laminar. Figure 4-3 shows changes in $U_{ratio}$ and $\Delta P$ for increase in heat exchanger diameter ($D_o$) for fixed number of helical turns $N = 1$, fixed heat exchanger length and fixed mass flow rate, $m_h = 0.1 \text{ kg/s}$ and $m_c = 3 \text{ kg/s}$. 
Figure 4-3: $U_{ratio}$ and $\Delta P$ vs Heat exchanger diameter for water-water heat exchanger

As heat exchanger diameter increases $U_{ratio}$ increases too. There is a decrease in pressure drop till diameter is 0.2 m and then pressure drop starts to increase after 0.2 m. Again the reason for change in trend is the flow regime change, i.e. turbulent to laminar.
5 Parametric Study

In the previous section, an analytical model for various heat exchanger types were discussed and analyzed. Even though helical heat exchangers are a compact design when compared to a standard straight tube in tube straight heat exchanger, this section investigates the possibility of designing a heat exchanger which is compact and lower in weight, but also achieves the required goal of a conventional design. Table 5-1 and 5-2 summarizes the optimized geometry and the resulting performance respectively for fixed mass flow rate of $\dot{m}_h = 0.1 \text{ kg/s}$ and $\dot{m}_c = 3 \text{ kg/s}$.

Table 5-1 : Design parameters for Optimized geometry

<table>
<thead>
<tr>
<th>Design</th>
<th>Diameters (m)</th>
<th>Number of fins</th>
<th>Number of helical turns</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_0$</td>
<td>$D_i$</td>
<td>$D_s$</td>
<td>Inner</td>
</tr>
<tr>
<td>Water – Water Heat exchanger</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
<td>0.269</td>
<td>0.247</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
<td>0.269</td>
<td>0.247</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>0.26</td>
<td>0.221</td>
<td>0.209</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>0.26</td>
<td>0.221</td>
<td>0.209</td>
<td>4</td>
</tr>
<tr>
<td>Nitrogen-Water Heat exchanger</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
<td>0.285</td>
<td>0.273</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
<td>0.275</td>
<td>0.255</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>0.26</td>
<td>0.243</td>
<td>0.229</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>0.27</td>
<td>0.239</td>
<td>0.217</td>
<td>8</td>
</tr>
</tbody>
</table>

The design parameters summarized above have been based on the constraints and variables summarized in table 4-1. All wall and fin thickness are 1 mm and fins are leaned at 45° in the above design models.
Table 5-2: Heat exchanger performance for optimized geometry

<table>
<thead>
<tr>
<th>Design</th>
<th>$U_{ratio}$</th>
<th>$\Delta P$ (kPa)</th>
<th>Volume ($m^3$)</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Hot fluid</td>
<td>Cold fluid</td>
<td></td>
</tr>
<tr>
<td>Water – Water Heat exchanger</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.019</td>
<td>13.42</td>
<td>0.0196</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.027</td>
<td>18.65</td>
<td>0.0071</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.101</td>
<td>7.59</td>
<td>0.0078</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.322</td>
<td>15.37</td>
<td>0.0141</td>
</tr>
<tr>
<td>Nitrogen-Water Heat exchanger</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>8.33</td>
<td>11.60</td>
<td>0.0212</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>20.13</td>
<td>7.53</td>
<td>0.0071</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>9.39</td>
<td>14.55</td>
<td>0.0078</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>15.13</td>
<td>20.76</td>
<td>0.0103</td>
</tr>
</tbody>
</table>

In case of water-water heat exchanger, design 1 is preferred if pressure loss is to be minimized or in other words, higher efficiency is desired. Design 4 is picked if compactness, i.e., weight and volume is important. Similarly, in case of Nitrogen-water design 1 is preferred if minimum pressure drop is wanted and design 4 if compactness is prioritized. There is a tradeoff between pressure loss and compactness in all the above designs. For mass calculations, aluminum having a density of 2700 kg/m$^3$ is used.

The different geometries shown in Table 5-1 work only for $\dot{m}_h = 0.1$ kg/s and $\dot{m}_c = 3$ kg/s. When the same geometries are run at different flow rates within the range, it does not meet the heat transfer goals, and the pressure drop is not within the constraints too. Therefore, a better optimized design is needed which works for the entire mass flow rate range.

Factors like thermal performance, pressure drop, heat exchanger weight, and volume (compactness) are important in designing and optimizing a heat exchanger. Based on the vendor demands, one of these factors can be prioritized in designing.
5.1 Heat Transfer and Compactness prioritized for optimization

This section presents design and performance when heat transfer and compactness are prioritized. The design parameters for water-water and nitrogen-water heat exchanger are shown in table 5-3.

Table 5-3 : Optimized design parameters when heat transfer and compactness are prioritized

<table>
<thead>
<tr>
<th></th>
<th>Diameters (m)</th>
<th>Number of fins</th>
<th>Number of helical turns</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D₀ Dᵢ Dₛ</td>
<td>Inner</td>
<td>Outer</td>
<td></td>
</tr>
<tr>
<td>Water – Water Heat exchanger</td>
<td>0.3 0.277 0.255</td>
<td>8</td>
<td>8</td>
<td>4.5</td>
</tr>
<tr>
<td>Nitrogen-Water Heat exchanger</td>
<td>0.3 0.287 0.251</td>
<td>8</td>
<td>8</td>
<td>2.25</td>
</tr>
</tbody>
</table>

Table 5-4 summarizes changes in $U_{ratio}$ for the design parameters shown in table 5-3 for different mass flow rate combinations. The volume for both the heat exchangers is 0.0212 m³ and the mass for water-water is 2.24 kg and that of nitrogen-water is 2.27 kg.

Table 5-4 : $U_{ratio}$ for Optimized design parameters when heat transfer and Compactness are prioritized

<table>
<thead>
<tr>
<th>$\dot{m}_h$ (kg/s)</th>
<th>$\dot{m}_c$ (kg/s)</th>
<th>$U_{ratio}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Water – Water</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>2.96</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>3.95</td>
</tr>
<tr>
<td>0.1</td>
<td>3</td>
<td>4.47</td>
</tr>
</tbody>
</table>

As shown in the above table $U_{ratio}$ is greater than or equal to 1 for the entire mass flow rate range. For cases where $U_{ratio}$ is greater than 1, mass flow rate of the hot fluid should be
increased or that of the cold fluid must be decreased, in order to bring $U_{ratio}$ to 1. However increasing or decreasing mass flow rates to satisfy heat transfer goals means going outside the mass flow rate range. For example in case of water-water heat exchanger, the mass flow rate of the cold fluid must be decreased to 0.1 kg/s if the hot fluid flows at 0.1 kg/s to achieve $U_{ratio} = 1$. Figure 5-1 and 5-2 illustrates change in $U_{ratio}$ for different mass flow rate combinations within the range for both water-water and nitrogen-water heat exchanger respectively.

Figure 5-1 : $U_{ratio}$ vs $\dot{m}_h$ vs $\dot{m}_c$ for water – water heat exchanger when heat transfer and compactness are prioritized
Figure 5-2: $\frac{U_{ratio}}{U_{req}}$ vs $m_h$ vs $m_c$ for nitrogen-water heat exchanger when heat transfer and compactness are prioritized

The biggest drawback with this design is the high pressure drop which accompanies with meeting heat transfer goals and compactness as shown in table 5-5. Pressure drop is beyond threshold for majority of the flow rate range.

Table 5-5: $\Delta P$ for Optimized design parameters when heat transfer and compactness are prioritized

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Mass flow rate (kg/s)</th>
<th>$\Delta P$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Water – Water Heat exchanger</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hot Water</td>
<td>1</td>
<td>87.20</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.398</td>
</tr>
<tr>
<td>Cold Water</td>
<td>1</td>
<td>98.98</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>834.43</td>
</tr>
<tr>
<td><strong>Nitrogen – Water Heat exchanger</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrogen</td>
<td>1</td>
<td>640</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>10.08</td>
</tr>
<tr>
<td>Water</td>
<td>1</td>
<td>60.22</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>465.6</td>
</tr>
</tbody>
</table>
Figure 5-3 and 5-4 shows pressure drop variation with change in mass flow rates for the hot and cold fluid channel respectively for the water-water heat exchanger.

**Figure 5-3**: $\Delta P$ vs $\dot{m}_h$ for water-water heat exchanger when heat transfer and compactness are prioritized.

**Figure 5-4**: $\Delta P$ vs $\dot{m}_c$ for water-water heat exchanger when heat transfer and compactness are prioritized.
Figure 5-5 and 5-6 shows pressure drop vs mass flow rates in hot and cold fluid channels respectively for the nitrogen-water heat exchanger.

Figure 5-5: $\Delta P$ vs $\dot{m}_h$ for nitrogen-water heat exchanger when heat transfer and compactness are prioritized

Figure 5-6: $\Delta P$ vs $\dot{m}_c$ for nitrogen-water heat exchanger when heat transfer and compactness are prioritized
5.2 Pressure Drop and Compactness for optimization

This section presents a design and its performance when pressure drop and compactness are prioritized. The design parameters for water-water and nitrogen-water heat exchanger are shown in table 5-6.

<table>
<thead>
<tr>
<th>Diameters (m)</th>
<th>Number of fins</th>
<th>Number of helical turns</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_0$</td>
<td>$D_i$</td>
<td>$D_s$</td>
<td>Inner</td>
</tr>
<tr>
<td>Water – Water Heat exchanger</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.26</td>
<td>0.229</td>
<td>0.215</td>
<td>8</td>
</tr>
<tr>
<td>Nitrogen-Water Heat exchanger</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.26</td>
<td>0.245</td>
<td>0.203</td>
<td>8</td>
</tr>
</tbody>
</table>

Figure 5-7 and 5-8 shows variation in pressure drop with change in mass flow rates for the hot and cold fluid channel respectively for the water-water heat exchanger.

Figure 5-7 : $\Delta P$ vs $m_h$ for water - water heat exchanger when pressure drop and compactness are prioritized
Figure 5-8: $\Delta P$ vs $n_c$ for water - water heat exchanger when pressure drop and compactness are prioritized

Table 5-7 summarizes the pressure drop for water-water and nitrogen-water heat exchangers and they are within the threshold for the given mass flow rate range.

Table 5-7: $\Delta P$ for Optimized design parameters when Pressure drop and Compactness are prioritized

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Mass flow rate (kg/s)</th>
<th>$\Delta P$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Water-Water Heat exchanger</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hot Water</td>
<td>1</td>
<td>16.36</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.083</td>
</tr>
<tr>
<td>Cold Water</td>
<td>1</td>
<td>2.38</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td><strong>Nitrogen – Water Heat exchanger</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrogen</td>
<td>1</td>
<td>18.26</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.288</td>
</tr>
<tr>
<td>Water</td>
<td>1</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>15.28</td>
</tr>
</tbody>
</table>
Figure 5-9 and 5-10 shows pressure drop variation with change in mass flow rates for the hot and cold fluid channels respectively for the nitrogen-water heat exchanger.

**Figure 5-9**: $\Delta P$ vs $\dot{m}_h$ for nitrogen - water heat exchanger when pressure drop and compactness are prioritized

**Figure 5-10**: $\Delta P$ vs $\dot{m}_c$ for nitrogen - water heat exchanger when pressure drop and compactness are prioritized
Table 5-8 summarizes changes in $U_{ratio}$ for the design parameters in table 5-6 for different mass flow rate combinations.

**Table 5-8 : $U_{ratio}$ for Optimized design parameters when heat transfer and Compactness are prioritized**

<table>
<thead>
<tr>
<th>$m_h$ (kg/s)</th>
<th>$m_c$ (kg/s)</th>
<th>$U_{ratio}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Water-Water</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.12</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0.40</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>0.78</td>
</tr>
<tr>
<td>0.1</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

In table 5-8 $U_{ratio}$ is less than or equal to 1 for different mass flow rate combinations and is the main disadvantage when pressure drop and compactness are prioritized. $U_{ratio}$ can be increased to 1 by either increasing the flow rate of cold fluid or by decreasing the hot fluid mass flow rate. The better option would be decreasing the mass flow rate of hot fluid as it keeps the pressure drop within the constraints. Figure 5-11 and 5-12 illustrates change in $U_{ratio}$ for different mass flow rate combinations within the mass flow rate range for both water-water and nitrogen-water heat exchanger respectively.

![Figure 5-11](image-url)

**Figure 5-11 : $U_{ratio}$ vs $m_h$ vs $m_c$ for water – water heat exchanger when Pressure drop and compactness are prioritized**
The volume for both the heat exchangers is 0.016 m³ and the mass for water-water is 1.66 kg and that of nitrogen-water is 1.70 kg.

5.3 Heat Transfer and Pressure drop prioritized for optimization

This section presents design and performance when heat transfer and pressure drop are prioritized. The design parameters for water-water and nitrogen-water heat exchanger are shown in table 5-9.

Table 5-9: Optimized design parameters when heat transfer and pressure drop are prioritized

<table>
<thead>
<tr>
<th>Diameters (m)</th>
<th>Number of fins</th>
<th>Number of helical turns</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_0$</td>
<td>$D_i$</td>
<td>$D_s$</td>
<td>Inner</td>
</tr>
<tr>
<td>Water-Water Heat exchanger</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>0.723</td>
<td>0.709</td>
<td>8</td>
</tr>
<tr>
<td>Nitrogen-Water Heat exchanger</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>0.729</td>
<td>0.665</td>
<td>8</td>
</tr>
</tbody>
</table>
Table 5-10 summarizes changes in $U_{ratio}$ for the design parameters shown in table 5-9 for different mass flow rate combinations.

Table 5-10: $U_{ratio}$ for Optimized design parameters when heat transfer and pressure drop are prioritized

<table>
<thead>
<tr>
<th>$m_h$ (kg/s)</th>
<th>$m_c$ (kg/s)</th>
<th>$U_{ratio}$</th>
<th>Water-Water</th>
<th>Nitrogen-Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>3.25</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>7.08</td>
<td>1.98</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>3</td>
<td>9.50</td>
<td>2.05</td>
<td></td>
</tr>
</tbody>
</table>

In table 5-10 too $U_{ratio}$ is greater than or equal to 1 for the entire mass flow rate change. For cases where $U_{ratio}$ is greater than 1, mass flow rate of the hot fluid should be increased or that of the cold fluid must be decreases, in order to bring $U_{ratio}$ to 1. However increasing or decreasing mass flow rates to satisfy heat transfer goals means going outside the mass flow rate range. Figure 5-13 and 5-14 illustrates change in $U_{ratio}$ for different mass flow rate combinations within the range for both water-water and nitrogen-water heat exchanger respectively.

Figure 5-13: $U_{ratio}$ vs $m_h$ vs $m_c$ for water – water heat exchanger when heat transfer and pressure drop are prioritized
Figure 5-14: $U_{ratio}$ vs $m_h$ vs $m_c$ for nitrogen – water heat exchanger when heat transfer and pressure drop are prioritized

Figure 5-15 and 5-16 shows pressure drop vs mass flow rates for the hot and cold fluid channel respectively for the water-water heat exchanger.

Figure 5-15: $\Delta P$ vs $m_h$ for water - water heat exchanger when heat transfer and pressure drop are prioritized
Figure 5-16: $\Delta P$ vs $m_c$ for water - water heat exchanger when heat transfer and pressure drop are prioritized

Table 5-11 summarizes the pressure drop for water-water and nitrogen-water heat exchangers and values are within the threshold for the given mass flow rate range

**Table 5-11: $\Delta P$ for Optimized design parameters when heat transfer and pressure drop are prioritized**

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Mass flow rate (kg/s)</th>
<th>$\Delta P$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Water-Water Heat exchanger</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hot Water</td>
<td>1</td>
<td>14.91</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.09</td>
</tr>
<tr>
<td>Cold Water</td>
<td>1</td>
<td>2.63</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td><strong>Nitrogen – Water Heat exchanger</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrogen</td>
<td>1</td>
<td>18.5</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.291</td>
</tr>
<tr>
<td>Water</td>
<td>1</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>16.40</td>
</tr>
</tbody>
</table>
Figure 5-17 and 5-18 shows pressure drop vs mass flow rates for the hot and cold fluid channel respectively for the nitrogen-water heat exchanger.

![Figure 5-17: $\Delta P$ vs $m_h$ for nitrogen-water heat exchanger when heat transfer and pressure drop are prioritized](image1)

![Figure 5-18: $\Delta P$ vs $m_c$ for nitrogen-water heat exchanger when heat transfer and pressure drop are prioritized](image2)
In this case the volume for both the heat exchangers is 0.331 m³ and the mass for water-water is 14.2 kg and that of nitrogen-water is 14.3 kg.

To summarize a tradeoff between heat exchange, pressure loss and compactness is observed while designing an optimized model for given set of geometry constraints.
6 Conclusions and Future Work

This work explored the design and development of a novel high-performance, compact, counter flow heat exchanger design to utilize cold water to reduce the operating temperature of cryogenic nitrogen and hot water. New additive manufacturing approaches allow for 3D-printing of intricate design features that are not available using traditional approaches, however, additional constraints, such as a cantilever build angle to support the deposited metal material during the build.

Using a counter-flow and counter-helical design, an elevated level of heat exchange can be achieved in a compact volume, and the structure is robust enough to withstand the required operating pressures of the two fluids. Several designs were examined, based on a series of given design constraints, and several candidate options were identified. Specific findings from this work are:

- Helical heat exchangers offer significant advantage in heat exchange over straight tubular heat exchangers due to better mixing caused by the secondary flow in the helical coils. In case of helical heat exchangers there is an increase in heat transfer surface area for the same heat exchanger length and diameter.

- Increase in number of helical turns increases the heat transfer area and thus the heat exchange. There is also decrease in cross sectional area which makes the flow more turbulent and with secondary flows being involved, heat exchange is better and improved. However, an increase in pressure drop is also observed in such cases which affects the efficiency of the heat exchanger.

- Based on set of geometric constraints:
  - If heat transfer and compactness are prioritized for optimization, pressure loss increases and goes beyond threshold
  - If pressure drop and compactness are prioritized for optimization, heat transfer goals are not met
  - If heat transfer and pressure drop are prioritized for optimization, then compactness in design are to be sacrificed
In all above optimized designs, to bring $U_{ratio} = 1$, it is required to go outside the mass flow rate range.

The same analysis and concept can be applied in designing heat exchangers for space applications with different fluids. Future work will include numerical and experimental investigations of the proposed highly compact and highly efficient heat exchanger design and an uncertainty/error analysis before experimenting the optimized design.
7 References


8. Appendix A: Thermophysical properties of working fluids

1) Water

Density vs Temperature

Specific heat at constant pressure vs Temperature
Thermal conductivity vs Temperature

Dynamic viscosity vs Temperature
2) Nitrogen

Density vs Temperature

Specific heat at constant pressure vs Temperature
Thermal conductivity vs Temperature

Dynamic viscosity vs Temperature
9. Appendix B : Analytical modelling MATLAB code

clear all;
clear workspace;
clc;
format short g
load Water.mat;
load Nitrogen.mat;

PMpah = [0.01 0.1 2];
Mpah = 0.2;  % Hot fluid Inlet static pressure
Pah = Mpah*1e6;
PMpac = [0.1 1 10];
Mpac = 0.2;  % Cold fluid Inlet static pressure
Pac = Mpac*1e6;
ThiK = 368;  % inlet hot temp in K
Thif = (ThiK - 273.15)*1.8 + 32; % inlet hot temp(F)
TciK = 278;  % inlet cold temp in K
ThoK = 298;  % outlet cold temp in
mdoth = 0.1; % mdot of hot(in kg/s)
mdotc = 3;  % mdot of cold(in kg/s)

% Calling Fluid thermal properties function
[densityh2i,Cphi,mvh2i,kh2i,Cvh2i,Prh2i,gammai] = fluidpropsh2(Tk1,PMpah,Densitya,Cpa,mva,ka,Cva,Mpah,ThiK);
[densityh2o,Cpho,mvh2o,kh2o,Cvh2o,Prh2o,gammao] = fluidpropsh2(Tk1,PMpah,Densitya,Cpa,mva,ka,Cva,Mpah,ThoK);
Cpci = (interp2(Tk1c',PMpac,Cpca,TciK,Mpac))*1000;
Q = mdoth*((Cphi*ThiK)-(Cpho*ThoK));  % in W
tempT = (Q/(mdotc*Cpci)) + (TciK);
ico = (Q/mdotc) + (Cpci*TciK);
Toutc = TciK:0.1:372;  % temperary tempature variable
error = 1;
x = 1;
while (error > 0.01)
    Cpcoc = (interp2(Tk1c',PMpac,Cpca,Toutc(x),Mpac))*1000;  % The loop is to iterate a solution for
    n = (Toutc(x)*Cpcoc);
    error = ((ico-n));  % (Temp. dependent Cp useed)
    x = x+1;
end
TcoK = Toutc(x-1);  % Temp. out for cold fluid (K)
Tcof = (TcoK - 273.15)*1.8 + 32;
\[ C_{pco} = (\text{interp2}(Tk1c',PMpac,Cpca,TcoK,Mpac))*1000; \]

% Finding the Mean temperature in the HEX for hot and cold fluids (Ref: HANDBOOK OF HEAT TRANSFER, pg: 17.48, 17.49)

% Difference in temperature inlet to HEX (hot - cold)
\[ dt1 = (ThiK-TcoK); \]
% Difference in temperature outlet to HEX (hot - cold)
\[ dt2 = (ThoK-TciK); \]
% Mean temperature (geometric mean of dt1 and dt2)
\[ dtms = (dt1*dt2)^0.5; \]
\[ ihi = C_{phi}*ThiK; \]
\[ ici = C_{pci}*TciK; \]
\[ iho = C_{pho}*ThoK; \]
\[ ihm = iho + ((ihi-iho)*((dtms-dt2)/(dt1-dt2))); \]
\[ icm = ico + ((ici-ico)*((dtms-dt2)/(dt1-dt2))); \]
\[ Thtemp = ThoK:0.1:ThiK; \]
\[ Tctemp = TciK:0.1:TcoK; \]
\[ error = 1; \]
\[ x = 1; \]
\[ \text{while (error > 0.01)} \]
\[ C_{phm} = (\text{interp2}(Tk1',PMpah,Cpa,Thtemp(x),Mpa)h)*1000; \]
\[ \text{The loop is to iterate a solution for} \]
\[ n = (Thtemp(x)*C_{phm}); \]
\[ error = (ihm-n); \]
\[ x = x+1; \]
\[ \text{end} \]
\[ ThmK = Thtemp(x-1); \]
\[ error = 1; \]
\[ x = 1; \]
\[ \text{while (error > 0.01)} \]
\[ C_{pcm} = (\text{interp2}(Tk1c',PMpac,Cpca,Tctemp(x),Mpac))*1000; \]
\[ \text{The loop is to iterate a solution for} \]
\[ n = (Tctemp(x)*C_{pcm}); \]
\[ \text{mean temperature for hot gas.} \]
\[ error = ((icm-n)); \]
\[ x = x+1; \]
\[ \text{end} \]
\[ TcmK = Tctemp(x-1); \]
% Cold fluid mean temperature (K)
% Mean temperature difference of hot and cold fluid
\[ dtm = ThmK - TcmK; \]
% Log mean temperature:
\[ \text{LogdelT} = (((ThiK-TcoK)-(ThoK-TciK))/(log((ThiK-TcoK)/(ThoK-TciK)))); \]
% HEX Geometry

L = 0. % Length of straight HEX

to = 0.001; % Outer wall thickness
ti = 0.001; % Inner wall thickness
ts = 0.001; % Inner solid wall thickness
ho = 0.005; % Outer channel height
hi = 0.005; % Inner channel height
Dmax = 0.3; % OD of HEX

% Centerline Outer Diameter
Do = Dmax - (2*(to/2));

% Centerline Inner Diameter
Di = Dmax - (2*to) - (2*ho) - (2*(ti/2));

% Centerline Innermost Diameter
Ds = Dmax - (2*to) - (2*(ho+hi)) - (2*ti) - (2*(ts/2));

tf = 0.001; % Fin thickness

nfinsh = 8; % number of fins (hot channel)
nfinsc = 8; % number of fins (cold channel)

% ID of HEX
ID = Dmax - (2*(to+ti+ts)) - (2*ho) - (2*hi);

% Straight HEX No fin

Ah = (pi/4)*(((Di - ti)^2) - ((Ds + ts)^2));

Ac = (pi/4)*(((Do - to)^2) - ((Di + ti)^2));

Ph = (pi)*((Di - ti) + (Ds + ts));

Pc = (pi)*((Do - to) + (Di + ti));

DhH = 4*Ah/Ph;

Dhc = 4*Ac/Pc;

As = pi*(Di-(di))*L;

[densityh2i,Cphi,mvh2i,Prh2i,gammai] = fluidpropsh2(Tk1,PMpah,Densitya,Cpa,mva,ka,Cva,Mpah,ThiK);
[densityh2o,Cpho,mvh2o,Prh2o,gammao] = fluidpropsh2(Tk1,PMpah,Densitya,Cpa,mva,ka,Cva,Mpah,ThoK);
[densityh2m,Cpcm,mvh2m,Prh2m,gammam] = fluidpropsh2(Tk1,PMpah,Densitya,Cpa,mva,ka,Cva,Mpah,ThmK);

[densityci,Cpci,mvci,kci,Cvci,Prci] = fluidpropsc(Tk1c,PMpac,Densityca,Cpca,mvca,kca,Cvca,Mpac,TciK);
[densityco,Cpco,mvco,kco,Cvco,Prco] = fluidpropsc(Tk1c,PMpac,Densityca,Cpca,mvca,kca,Cvca,Mpac,TcoK);
[densitycm,Cpcm,mvcm,kcm,Cvcm,Prcm] = fluidpropsc(Tk1c,PMpac,Densityca,Cpca,mvca,kca,Cvca,Mpac,TcmK);
[Uh₂ᵢ, Reh₂ᵢ] = AchU₁(mvh₂ᵢ, mdoth, Ph, Prh₂ᵢ, kh₂ᵢ, DhH);
[Uh₂ₒ, Reh₂ₒ] = AchU₁(mvh₂ₒ, mdoth, Ph, Prh₂ₒ, kh₂ₒ, DhH);
[Uh₂ₘ, Reh₂ₘ] = AchU₁(mvh₂ₘ, mdoth, Ph, Prh₂ₘ, kh₂ₘ, DhH);
[Ucᵢ, Recᵢ] = AchU₁(mvcᵢ, mdotc, Pc, Prcᵢ, kᵢ, Dhᵢ);
[Ucₒ, Recₒ] = AchU₁(mvcₒ, mdotc, Pc, Prcₒ, kₒ, Dhₒ);
[Ucₘ, Recₘ] = AchU₁(mvcₘ, mdotc, Pc, Prcₘ, kₘ, Dhₘ);

Rw = log((Di+(ti/2))/(Di-(ti/2)))/(2*pi*SS316*L);

% Wall resistance (Conduction)
U₁ = ((1/Uh₂ᵢ)+Rw+(1/Ucᵢ))^-1;
Um = ((1/Uh₂ₘ)+Rw+(1/Ucₘ))^-1;
U₂ = ((1/Uh₂ₒ)+Rw+(1/Ucₒ))^-1;

Ustar = Um*(dtm/dtms);
U = ((1/(6*U₁))+(2/(3*Um))+(1/(6*U₂)))^-1

% Simpson's Rule
Ureq = Q/(As*LogdelT)
% in W/m^2.K
ratio = U/Ureq

% Pressure Drop across gas pipe
e = 8e^-6;

Surface roughness of aluminium alloy 10 mg powder

if (Reh₂ₘ >= 2300)
    fh = 1;
    error₁ = 1;
    while (error₁ > 1e-6)
        fLHS = 1/((fh)^0.5);
        fRHS = -2*log10((e/(3.7*DhH))+(2.51/(Reh₂ₘ*((fh)^0.5))));
        error₁ = (fRHS - fLHS);
        fh = fh - 0.00001;
    end
else
    fh = 64/Reh₂ₘ;
end

if (Recₘ >= 2300)
    fc = 1;
    error₂ = 1;
    while (error₂ > 1e-6)
        fLHS = 1/((fc)^0.5);
        fRHS = -2*log10((e/(3.7*Dhc))+(2.51/(Recₘ*((fc)^0.5))));
        error₂ = (fRHS - fLHS);
        fc = fc - 0.00001;
    end
else
    fc = 64/Recₘ;
end
\[
\text{DeltaP}2h = \frac{(fh \times L \times (mdoth \times mdoth))}{(2 \times \text{densityh2m} \times DhH \times Ah \times Ah)};
\]
\[
\text{DeltaP}2c = \frac{(fc \times L \times (mdotc \times mdotc))}{(2 \times \text{densitycm} \times Dhc \times Ac \times Ac)};
\]

% % Straight HEX with 'n' fins
%--------------------------------------------------------

\[
\text{Areafi} = \frac{(1}{nfinsh} \times \frac{(\pi}{4}) \times (((Di - ti)^{2}) - ((D_s + ts)^{2})) \times (tf \times hi);
\]
\[
\text{Perimeterfi} = \frac{(1}{nfinsh} \times \frac{(\pi}{2}) \times (((Di - ti) + (Ds + ts))) \times (2\times hi) \times (2\times tf);
\]
\[
\text{Areafo} = \frac{(1}{nfinsc} \times \frac{(\pi}{4}) \times (((Do - to)^{2}) - ((Di + ti)^{2})) \times (tf \times ho);
\]
\[
\text{Perimeterfo} = \frac{(1}{nfinsc} \times \frac{(\pi}{2}) \times (((Do - ti) + (Di + ts))) \times (2\times ho) \times (2\times tf);
\]
\[
\text{DhH} = 4 \times \text{Areafi}/\text{Perimeterfi};
\]
\[
\text{Dhc} = 4 \times \text{Areafo}/\text{Perimeterfo};
\]
\[
\text{As} = \pi \times (Di-(ti)) \times L;
\]

[densityh2i, Cphi, mvh2i, kh2i, Cvhi, Prh2i, gamma1] = fluidpropsh2(Tk1, PMpah, Densitya, Cpa, mva, ka, Cva, Mpah, Thik);
[densityh2o, Cpho, mvh2o, kh2o, Cvhi, Prh2o, gammao] = fluidpropsh2(Tk1, PMpah, Densitya, Cpa, mva, ka, Cva, Mpah, Thok);
[densityh2m, Cph2m, mvh2m, kh2m, Cvhi, Prh2m, gamman] = fluidpropsh2(Tk1, PMpah, Densitya, Cpa, mva, ka, Cva, Mpah, Thmk);

[densityci, Cpci, mvci, kci, Cvc, Prci] = fluidpropsc(Tk1c, PMpac, Densitya, Cpa, mva, kca, Cva, Mpac, Tck);
[densityco, Cpc, mvco, kco, Cvc, Prco] = fluidpropsc(Tk1c, PMpac, Densitya, Cpa, mva, kca, Cva, Mpac, Tco);
[densitycm, Cpcm, mvcm, kcm, Cvc, Prcm] = fluidpropsc(Tk1c, PMpac, Densitya, Cpa, mva, kca, Cva, Mpac, Tcm);

[Uh2i, Reh2i] = AchU2(mvh2i, mdoth, Perimeterfi, Prh2i, kh2i, DhH, nfinsh);
[Uh2o, Reh2o] = AchU2(mvh2o, mdoth, Perimeterfi, Prh2o, kh2o, DhH, nfinsh);
[Uh2m, Reh2m] = AchU2(mvh2m, mdoth, Perimeterfi, Prh2m, kh2m, DhH, nfinsh);
[Uci, Reci] = AchU2(mvci, mdotc, Perimeterfo, Prci, kci, Dhc, nfinsc);
[Uco, Reco] = AchU2(mvco, mdotc, Perimeterfo, Prco, kco, Dhc, nfinsc);
[Ucm, Recm] = AchU2(mvcm, mdotc, Perimeterfo, Prcm, kcm, Dhc, nfinsc);
% Calculating Fin efficiency
SS316 = 21.4;
Afi = nfinsh*2*(hi)*L;
% Area of inner channel fins
Afo = nfinsc*2*(ho)*L;
% Area of outer channel fins
Ai = Afi + (((Di-ti)*pi)-(nfinsh*tf))*L;
% Total Area of heat transfer (inner channel)
Ao = Afo + (((Di+ti)*pi)-(nfinsc*tf))*L;

% Total Area of heat transfer (outer channel)

mh = ((2*Uh2m)/(SS316*tf))^0.5;
mc = ((2*Ucm)/(SS316*tf))^0.5;

nfh = (tanh(mh*L))/(mh*L);

nfc = (tanh(mc*L))/(mc*L);

noh = 1 - ((Afi/Ai)*(1-nfh));

noc = 1 - ((Afo/Ao)*(1-nfc));

Rw = log((Di+(ti/2))/(Di-(ti/2)))/(2*pi*SS316*L);
% Wall resistance (Conduction)

U1 = ((1/(noh*Uh2i))+Rw+(1/(noc*Uci)))^-1;
Um = ((1/(noh*Uh2m))+Rw+(1/(noc*Ucm)))^-1;
U2 = ((1/(noh*Uh2o))+Rw+(1/(noc*Uco)))^-1;

Ustar = Um*(dtm/dtms);

U = ((1/(6*U1))+(2/(3*Um))+(1/(6*U2)))^-1;

% Simpson's Rule

Ureq = Q/(Ai*LogdelT);
% in W/m^2.K
ratio = U/Ureq
Lreq = Q/(U*LogdelT*(((nfinsh*2*(hi))+(((Di-ti)*pi)-(nfinsh*tf))))

% Pressure Drop across gas pipe

e = 10e-6;
% Surface roughness of aluminium alloy 10 mg powder

if (Reh2m >= 2300)
    fh = 1;
    error1 = 1;
    while (error1 > 1e-6)
        fLHS = 1/((fh)^0.5);
        fRHS = -
        2*log10((e/(3.7*DhH))+(2.51/(Reh2m*((fh)^0.5))));
        error1 = (fRHS - fLHS);
        fh = fh - 0.00001;
    end
else
    fh = 64/Reh2m;
end
if (Recm >= 2300)
    fc = 1;
    error2 = 1;
    while (error2 > 1e-6)
        fLHS = 1/((fc)^0.5);
        fRHS = -
        2*log10((e/(3.7*Dhc))+(2.51/(Recm*((fc)^0.5))));
        error2 = (fRHS - fLHS);
        fc = fc - 0.00001;
    end
else
    fc = 64/Recm;
end
DeltaP2h = (fh*L*(mdoth*mdoth)/(nfinsh*nfinsh))/(2*densityh2m*DhH*Areafi*Areafi);
DeltaP2c = (fc*L*(mdotc*mdotc)/(nfinsc*nfinsc))/(2*densitycm*Dhc*Areafo*Areafo);
DeltaPLreqi = (fh*Lreq*(mdoth*mdoth)/(nfinsh*nfinsh))/(2*densityh2m*DhH*Areafi*Areafi);
DeltaPLreqo = (fc*Lreq*(mdotc*mdotc)/(nfinsc*nfinsc))/(2*densitycm*Dhc*Areafo*Areafo);

%--------------------------------------------------------------------------%
% Helical with 'n' fins
%--------------------------------------------------------------------------%
degl = 360;
% Angle of Revolution (inner channel)
deg2 = 360;
% Angle of Revolution (outer channel)
leani = 45;
leano = 45;
tperch1 = deg1/360;
% no. of turns per inner channel
tperch2 = deg2/360;
% no. of turns per outer channel
radtoceni = (Ds/2) + (ts/2) + (hi/2);
% radius of inner channel (from center of HEX)
radtoceno = (Di/2) + (ti/2) + (ho/2);
% radius of outer channel (from center of HEX)
Lchi = (((2*pi*radtoceni*tperch1)^2) + ((L)^2))^(0.5);
% helical inner channel length
Lcho = (((2*pi*radtoceno*tperch2)^2) + ((L)^2))^(0.5);
% helical outer channel length
phii = acosd(L/Lchi);
% helical inner channel angle
phio = acosd(L/Lcho);
% helical outer channel angle
Areafi = (((1/nfinsh)*(pi/4)*(((Di-ti)^2) - ((Ds+ts)^2)) - (tf*hi))*cosd(phii);
Perimeterfi = (((1/nfinsh)*((pi)*((Di-ti) + (Ds+ts)))*cosd(phii)) + (2*hi*secd(leani)) - (2*tf);
Areafo = (((1/nfinsc)*(pi/4)*(((Do-to)^2) - ((Di+ti)^2))) - (tf*ho))*(cosd(phio));
Perimeterfo = (((1/nfinsc)*((pi)*((Do-to) + (Di+ts)))*cosd(phio)) + (2*ho*secd(leano)) - (2*tf);
DhhelH = 4*Areafi/Perimeterfi;
Dhhelc = 4*Areafo/Perimeterfo;
ai = DhhelH/2;
ao = Dhhelc/2;
% radius of inner channel
Ro = radtoceni;
% radius of outer channel
[densityh2i,Cphi,mvh2i,kh2i,Cvh2i,Prh2i,gammai] = fluidpropsh2(Tk1,PMpah,Densitya,Cpa,mva,ka,Cva,Mpah,ThiK);
[densityh2o,Cpho,mvh2o,kh2o,Cvh2o,Prh2o,gammao] = fluidpropsh2(Tk1,PMpah,Densitya,Cpa,mva,ka,Cva,Mpah,ThoK);
[densityh2m,Cph2m,mvh2m,kh2m,Cvh2m,Prh2m,gammam] = fluidpropsh2(Tk1,PMpah,Densitya,Cpa,mva,ka,Cva,Mpah,ThmK);
[densityci,Cpci,mvci,kci,Cvci,Prci] = fluidpropsc(Tk1c,PMpac,Densityca,Cpca,mvca,kca,Cvca,Mpac,TciK);
[densityco,Cpco,mvco,kco,Cvco,Prco] = fluidpropsc(Tk1c,PMpac,Densityca,Cpca,mvca,kca,Cvca,Mpac,TcoK);
[densitycm,Cpcm,mvcm,kcm,Cvcm,Prcm] = fluidpropsc(Tk1c,PMpac,Densityca,Cpca,mvca,kca,Cvca,Mpac,TcmK);

[Uh2i,Reh2i] = AchU3(mvh2i,mdoth,Perimeterfi,Prh2i,kh2i,DhhelH,nfinsh,ai,Ri);
[Uh2o,Reh2o] = AchU3(mvh2o,mdoth,Perimeterfi,Prh2o,kh2o,DhhelH,nfinsh,ai,Ri);
[Uh2m,Reh2m] = AchU3(mvh2m,mdoth,Perimeterfi,Prh2m,kh2m,DhhelH,nfinsh,ai,Ri);
\[ [\text{Uci, Reci}] = \text{AchU3(mvci, mdotc, Perimeterfo, Prci, kci, Dhhelc, nfinsc, ao, Ro)}; \]
\[ [\text{Uco, Reco}] = \text{AchU3(mvco, mdotc, Perimeterfo, Prco, kco, Dhhelc, nfinsc, ao, Ro)}; \]
\[ [\text{Ucm, Recm}] = \text{AchU3(mvcm, mdotc, Perimeterfo, Prcm, kcm, Dhhelc, nfinsc, ao, Ro)}; \]

\[ \text{SS316} = 21.4; \]
\[ \text{Afh} = \text{nfinsh} * 2 * (hi) * Lchi; \]
\[ \text{Aih} = \text{Afh} + (((Di-ti) * pi) - (nfinsh * tf)) * Lchi; \]
\[ \text{mh} = ((2*Uh2m)/(SS316*tf))^{0.5}; \]
\[ \text{nfh} = (\text{tanh}(\text{mh} * Lchi)) / (\text{mh} * Lchi); \]
\[ \text{noh} = 1 - (\text{Afh} / \text{Aih}) * (1 - \text{nfh}); \]
\[ \text{Afc} = \text{nfinsc} * 2 * (ho) * Lcho; \]
\[ \text{Aic} = \text{Afc} + (((Di+ti) * pi) - (nfinsc * tf)) * Lcho; \]
\[ \text{mc} = ((2*Ucm)/(SS316*tf))^{0.5}; \]
\[ \text{nfc} = (\text{tanh}(\text{mc} * Lcho)) / (\text{mc} * Lcho); \]
\[ \text{noc} = 1 - (\text{Afc} / \text{Aic}) * (1 - \text{nfc}); \]

\[ \text{Rw} = \text{log}((\text{Di} + (\text{ti}/2))/((\text{Di} - (\text{ti}/2)))/(2*pi*SS316*Lchi)); \quad \% \text{Wall resistance} \]
\[ (\text{Conduction}) \]
\[ \text{U1} = ((1/(\text{noh} * Uh2i)) + \text{Rw} + (1/(\text{noc} * Uci)))^{-1}; \]
\[ \text{Um} = ((1/(\text{noh} * Uh2m)) + \text{Rw} + (1/(\text{noc} * Ucm)))^{-1}; \]
\[ \text{U2} = ((1/(\text{noh} * Uh2o)) + \text{Rw} + (1/(\text{noc} * Uco)))^{-1}; \]
\[ \text{Ustar} = \text{Um} * (\text{dtm}/\text{dtms}); \]
\[ \text{U} = ((1/(6*U1)) + (2/(3*Um)) + (1/(6*U2)))^{-1}; \]

\% Simpson's Rule
\[ \text{Ureq} = Q / (\text{Aih} * \text{LogdelT}); \]
\% in W/m^2.K
\[ \text{ratio} = \text{U} / \text{Ureq} \]

\% %Pressure Drop across gas pipe
\% Reference: (https://neutrium.net/fluid_flow/friction-factor-for-flow-in-coils-and-curved-pipe/)\n\[ e = 10^{-6}; \quad \% \]

\% Surface roughness of aluminium alloy 10 mg powder
\[ \text{resepi} = 2100 * (1 + (12 * ((ai/Ri)^0.5))); \]
\[ \text{resepo} = 2100 * (1 + (12 * ((ao/Ro)^0.5))); \]
\[ \text{Dehm} = \text{Reh2m} * ((ai/Ri)^0.5); \]
\[ \text{Decm} = \text{Recm} * ((ao/Ro)^0.5); \]
\[ \text{if} (\text{Reh2m} <= \text{resepi}) \]
\[ \quad \text{if} (\text{Dehm} < 20) \]
\[ \quad \text{if} (\text{Reh2m} >= 2300) \]
\[ \quad \text{fh} = 1; \]
\[ \quad \text{error1} = 1; \]
while (error1 > 1e-6)
    fLHS = 1/((fh)^0.5);
    fRHS = -
    2*log10((e/(3.7*DhHelH))+(2.51/(Reh*2*fh^0.5)));
    error1 = (fRHS - fLHS);
    fh = fh - 0.00001;
end
else
    fh = 64/Reh2m;
end

%fhelixh = fh*(((1-(0.18/(1+((35/Dehm)^2))^0.5)))^2)+(((1+(ai/(Ri*3)))^2)*(Dehm/88.33)))^0.5);
fhelixh = fh*(((1-(0.18/(1+((35/Dehm)^2))^0.5)))^2)+(((1+(ai/(Ri*3)))^2)*(Dehm/88.33)))^0.5);
elseif ((Dehm>20)&&(Dehm<40))
    if (Reh2m >= 2300)
        fh = 1;
        error1 = 1;
        while (error1 > 1e-6)
            fLHS = 1/((fh)^0.5);
            fRHS = -
            2*log10((e/(3.7*DhHelH))+(2.51/(Reh*2*fh^0.5)));
            error1 = (fRHS - fLHS);
            fh = fh - 0.00001;
        end
    end
    else
        fh = 64/Reh2m;
    end
    %fhelixh = fh*(((1-(0.18/(1+((35/Dehm)^2))^0.5)))^1)+(((1+(ai/(Ri*3)))^2)*(Dehm/88.33)))^0.5);
else
    if (Reh2m >= 2300)
        fh = 1;
        error1 = 1;
        while (error1 > 1e-6)
            fLHS = 1/((fh)^0.5);
            fRHS = -
            2*log10((e/(3.7*DhHelH))+(2.51/(Reh*2*fh^0.5)));
            error1 = (fRHS - fLHS);
            fh = fh - 0.00001;
        end
    end
    else
        fh = 64/Reh2m;
    end
end
fhelixh = fh*(((1-(0.18/(1+((35/Dehm)^2))^0.5)))^0)+((1+(ai/(Ri*3))^2)*(Dehm/88.33))^0.5);
end
else
fhelixh = ((0.084*Reh2m*((Ri/ai)^-2))^0.2)/((Ri/ai)^0.5);
end

if (Recm<=resepo)
  if (Decm < 20)
    if (Recm >= 2300)
      fc = 1 ;
      error2 = 1;
      while (error2 > 1e-6)
        fLHS = 1/((fc)^0.5);
        fRHS = -2*log10((e/(3.7*Dhhelc))+(2.51/(Recm*((fc)^0.5)))));
        error2 = (fRHS - fLHS);
        fc = fc - 0.00001;
      end
    else
      fc = 64/Recm;
    end
    fhelixc = fc*(((1-(0.18/(1+((35/Decm)^2))^0.5)))^2)+((1+(ao/(Ro*3))^2)*(Decm/88.33))^0.5);
  elseif ((Decm>20)&&(Decm<40))
    if (Recm >= 2300)
      fc = 1 ;
      error2 = 1;
      while (error2 > 1e-6)
        fLHS = 1/((fc)^0.5);
        fRHS = -2*log10((e/(3.7*Dhhelc))+(2.51/(Recm*((fc)^0.5)))));
        error2 = (fRHS - fLHS);
        fc = fc - 0.00001;
      end
    else
      fc = 64/Recm;
    end
    fhelixc = fc*(((1-(0.18/(1+((35/Decm)^2))^0.5)))^1)+((1+(ao/(Ro*3))^2)*(Decm/88.33))^0.5);
  else
    if (Recm >= 2300)
      fc = 1 ;
      error2 = 1;
    end
  end
end
while (error2 > 1e-6)
    fLHS   = 1/((fc)^0.5);
    fRHS   = -
    2*log10((e/((3.7*Dhelc))+(2.51/(Recm*((fc)^0.5)))));
    error2  = (fRHS - fLHS);
    fc      = fc - 0.00001;
end
else
    fc         =  64/Recm;
end

fhelixc = fc*(((1-
(0.18/((1+((35/Decm)^2))^0.5)))^0)+((1+(ao/(Ro*3)))^2)*(Decm/
88.33))^0.5);
end
else
    fhelixc = ((0.084*Recm*((Ro/ao)^-2))^0.2)/((Ro/ao)^0.5);
end

DeltaP2h        =
(fhelixh*Lchi*(mdoth*mdoth/(nfinsh*nfinsh)))/(2*densityh2m*Dhhel
h*Areafi*Areafi)
DeltaP2c        =
(fhelixc*Lcho*(mdotc*mdotc/(nfinsc*nfinsc)))/(2*densitycm*Dhhel
c*Areafo*Areafo)

Volume          = (pi*(Dmax*Dmax/4)*L)-
(pi*(ID,ID/4)*L)-
(nfinsh*Areafi*Lchi)-(nfinsc*Areafo*Lcho);
Mass            = 2700*Volume

%---------------------------------------------------------
% Sub Functions
%---------------------------------------------------------

%Fluid properties Function
function [density2,Cp2,mv2,k2,Cv2,Pr2,gamma] = fluidpropsh2(Tk1,Ph,Density,Cp,mv,k,Cv,z,y)
    density2   = interp2(Tk1',Ph,Density,y,z);
% in kg/m^3
    Cp2        = (interp2(Tk1',Ph,Cp,y,z))1000;
% in J/kg.K
    mv2        = interp2(Tk1',Ph,mv,y,z);
% in kg/m.s
    k2         = interp2(Tk1',Ph,k,y,z);
% in J/m.s.K
    Cv2        = (interp2(Tk1',Ph,Cv,y,z))1000;
% in J/kg.K
    Pr2        = ((Cp2*mv2)/k2);
gamma = Cp2/ Cv2;

function [density2c,Cp2c,mv2c,k2c,Cv2c,Pr2c] = fluidpropsc(Tk1ct,Pc,Densityct,Cpct,mvct,kct,Cvct,zt,yt)
density2c = interp2(Tk1ct',Pc,Densityct,yt,zt);
% in kg/m^3
Cp2c = (interp2(Tk1ct',Pc,Cpct,yt,zt))*1000;
% in J/kg.K
mv2c = interp2(Tk1ct',Pc,mvct,yt,zt);
% in kg/m.s
k2c = interp2(Tk1ct',Pc,kct,yt,zt);
% in J/m.s.K
Cv2c = (interp2(Tk1ct',Pc,Cvct,yt,zt))*1000;
% in J/kg.K
Pr2c = ((Cp2c*mv2c)/k2c);
gammac = Cp2c/ Cv2c;
end

function [Hh,Reh] = AchU1(visc,mf,peri,Pr,k,hydd)
    Reh = (4*mf)/(peri*visc);
    if (Reh <= 2300)
        Nuh = 4.36;
    elseif (Reh >= 3000)
        fsh = ((0.790*log(Reh))-1.64)^(-2);
        Nuh = ((fsh/8)*(Reh-1000)*Pr)/(1+(12.7*((fsh/8)^0.5))*(((Pr)^(2/3))-1));
    else
        warning('Transition Regime !!! (Turbulent Nusslet number correlation used)');
        fsh = ((0.790*log(Reh))-1.64)^(-2);
        Nuh = ((fsh/8)*(Reh-1000)*Pr)/(1+(12.7*((fsh/8)^0.5))*(((Pr)^(2/3))-1));
    end
    Hh = (Nuh*k)/(hydd);
end

function [Hh,Reh] = AchU2(visc,mf,peri,Pr,k,hydd,n)
    Reh = (4*(mf/n))/(peri*visc);
    if (Reh <= 2300)
        Nuh = 4.36;
    elseif (Reh >= 3000)
        fsh = ((0.790*log(Reh))-1.64)^(-2);
        Nuh = ((fsh/8)*(Reh-1000)*Pr)/(1+(12.7*((fsh/8)^0.5))*(((Pr)^(2/3))-1));
    else
        warning('Transition Regime !!! (Turbulent Nusslet number correlation used)');
        fsh = ((0.790*log(Reh))-1.64)^(-2);
        Nuh = ((fsh/8)*(Reh-1000)*Pr)/(1+(12.7*((fsh/8)^0.5))*(((Pr)^(2/3))-1));
    end
    Hh = (Nuh*k)/(hydd);
end
warning('Transition Regime !!! (Turbulent Nusslet number correlation used)');
fsh = ((0.790*log(Reh))-1.64)^(-2);
Nuh = ((fsh/8)^1.64/(Reh-1000)*Pr)/(1+(12.7*(((fsh/8)^0.5)*(((Pr)^2/3)-1)));
end
Hh = (Nuh*k)/(hydd);
end

function [Hh,Reh] = AchU3(visc,mf,peri,Pr,k,hydd,n,a,R)
Reh = (4*(mf/n))/(peri*visc);
if (Reh <= 2300)
x1 = (1 + (1342/((Reh*Reh*(a/R))*Pr)))^2;
x2 = 1 + 1.15/Pr;
Nuhcoil = (((4.364+(4.636/x1))^3)+(1.816*(((Reh*a)/(R*x2))^1.5)))^(1/3);
elseif (Reh >= 2300 && Reh<=20000)
fsh = ((0.790*log(Reh))-1.64)^(-2);
Nuh = ((fsh/8)^1.64/(Reh-1000)*Pr)/(1+(12.7*(((fsh/8)^0.5)*(((Pr)^2/3)-1)));
Nuhcoil = Nuh*(1 + (3.4*a/R));
else
fsh = ((0.790*log(Reh))-1.64)^(-2);
Nuh = ((fsh/8)^1.64/(Reh-1000)*Pr)/(1+(12.7*(((fsh/8)^0.5)*(((Pr)^2/3)-1)));
Nuhcoil = Nuh*(1+(3.6*(1-(a/R))*((a/R)^0.8)));
end
Hh = (Nuhcoil*k)/(hydd);
end