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PERPENDICULAR TRANSPORT OF CHARGED PARTICLES IN COMPOSITE MODEL TURBULENCE: RECOVERY OF DIFFUSION

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ABSTRACT

The computation of charged particle orbits in model turbulent magnetic fields is used to investigate the properties of particle transport in the directions perpendicular to the large-scale magnetic field. Recent results by Qin, Matthaeus, & Bieber demonstrate that parallel scattering suppresses perpendicular diffusion to a subdiffusive level when the turbulence lacks transverse structure. Here numerical computations are used to show that in turbulence in which there is substantial transverse structure, a second regime of diffusive transport can be established. In both the subdiffusion regime and this "second diffusion" regime, perpendicular transport is intrinsically nonlinear. The regime of second diffusion persists for long times and may therefore be of interest in astrophysical transport problems such as the scattering and solar modulation of cosmic rays.

Subject headings: cosmic rays — diffusion — scattering — turbulence

1. INTRODUCTION

Recent numerical results (Qin, Matthaeus, & Bieber 2002) have demonstrated that the scattering of charged particles in the direction parallel to the mean magnetic field can suppress diffusive transport perpendicular to the magnetic field. In such cases, transport across the mean field is subdiffusive (Urch 1977; Kóta & Jokipii 2000). Here we show, again using numerical simulations, that for magnetic fluctuations having sufficient perpendicular structure, a regime of second diffusion can be recovered. Although the theoretical basis for this possibility has been discussed for some time (Skilling, McIvor, & Holmes 1974; Rechester & Rosenbluth 1978; Chandran & Cowley 1998), the present study appears to be the first direct evidence for the recovery of perpendicular diffusion in a collisionless plasma.

Diffusion of charged particles in directions perpendicular to the large-scale average magnetic field remains a perplexing topic (Giacalone & Jokipii 1999; Mace, Matthaeus, & Bieber 2000). Heliospheric observations relating to perpendicular transport have remained enigmatic; on the one hand, the persistence of sharp boundaries (Mazur et al. 2000) appears to suggest a diminished role for transverse diffusion, while other evidence indicates the enhanced access of charged particles to widely disparate latitudes (Maclennan, Lanzerotti, & Hawkins 2001), requiring robust transport across the mean Archimedean spiral magnetic field. Standard quasi-linear scattering theory (QLT; Jokipii 1966) provides a framework for understanding both parallel and perpendicular transport, but in this approximation the two effects are necessarily distinct from one another. From the QLT approach emerges the physically appealing picture that perpendicular diffusion is governed by the tendency of charged particles to follow magnetic field lines. Field lines follow a random walk, and therefore so do the particles. However, fundamental questions can be raised regarding the applicability of this field line random walk (FLRW) limit to particle transport in certain geometries, especially those with one or more ignorable coordinates (Jokipii, Kóta, & Giacalone 1993). Even more troubling perhaps is that numerical computations of low-energy particle transport have failed to confirm FLRW behavior or, for that matter, any articulated theory of perpendicular diffusive transport (Giacalone & Jokipii 1999; Mace et al. 2000).

One possible complication is that perpendicular transport might not be a diffusion process. There are at least two ways in which one can envision this occurring. First, charged particles can be trapped and therefore have bounded displacements. A variation on this is the idea that in some circumstances, the field lines themselves are nondiffusive (e.g., Isichenko 1991; Zimbardo & Veltri 1995). Particles trying to follow such trapped or bounded field lines would then be restricted to nondiffusive transport. A second major possibility is that parallel scattering causes charged particles to scatter back along the same or a similar field line—by "retracing their steps," the particles experience a reduction in the rate of increase of perpendicular displacements. In this "compound" transport scenario, particle transport is relegated to a subdiffusive rate even if the field lines are globally diffusive. The numerical results (Qin et al. 2002) that verified the compound subdiffusive phenomenon made use of model magnetic fluctuations that are only weakly dependent on the transverse coordinates. In this regard, we can examine the discussions of compound subdiffusion (Urch 1977; Kóta & Jokipii 2000; Qin et al. 2002) that in essence hold that the accumulation of mean square perpendicular displacements is suppressed when parallel scattering reverses particle guiding center trajectories relative to a field line. On the other hand, the particles actually sample a bundle of field lines, and these in general will not be identical. As simulations verify, subdiffusion ensues when the field lines sampled within this field line bundle are almost identical. However, under very similar circumstances, the particles might well become randomized if the field lines sampled by the particles' gyro-orbits have substantial dissimilarities. Thus, it is a reasonable hypothesis that perpendicular diffusion is recovered if the magnetic field lines have sufficient transverse structure. We now proceed to demonstrate this effect.

2. NUMERICAL SIMULATIONS

We use an accurate adaptive step fourth-order Runge-Kutta method to compute test particle trajectories in magnetostatic turbulence. The test particles, with mass m and velocity $\boldsymbol{v}$, obey

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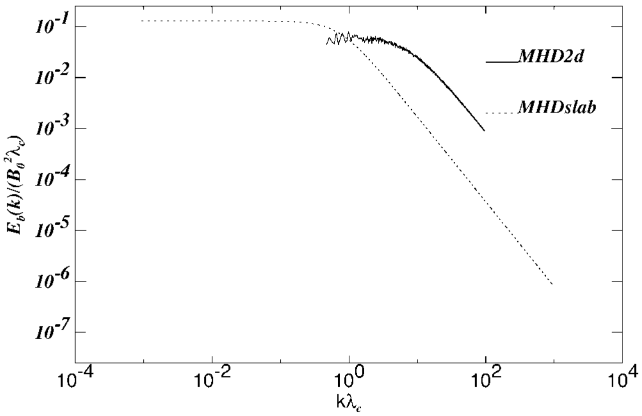

FIG. 1.—Spectrum of the slab and two-dimensional model components of magnetic turbulence with $E_{\rm slab} : E_{\rm 2D} = 20 : 80$, $b^2/B_0^2 = 1$, and $\lambda_x = 0.1\lambda$. (For more details, see the text.) The solid curve is the two-dimensional model component spectrum, and the dotted one is the slab model component spectrum.

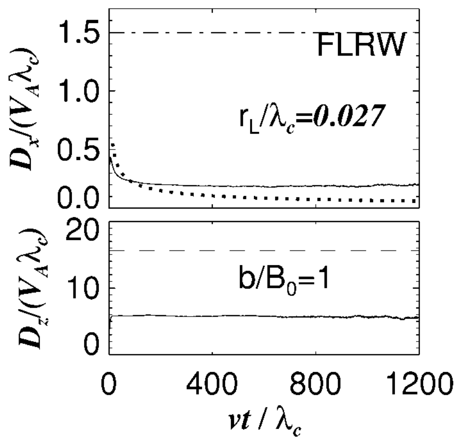

FIG. 2.—Second regime of perpendicular diffusion recovered in turbulence with strong transverse structure. Shown here is a test particle simulation in composite fluctuations ($E^{\rm slab} : E^{\rm 2D} = 20 : 80$), which is shown in Fig. 1 with $r_L/\lambda_c = 0.027$. In the lower panel, the solid line represents the running parallel diffusion coefficient vs. time. For reference, the dashed line is the approximate QLT result. In the upper panel, the solid line represents the running perpendicular diffusion coefficient. The dotted line corresponds to subdiffusion. The FLRW result is also shown. The running perpendicular diffusion coefficient is $\sim t^{-0.003}$ in the range vt/λ_c : [200, 1200].

the Newton-Lorentz equation

$$m\frac{d\boldsymbol{v}(t)}{dt} = \frac{q\boldsymbol{v}(t)}{c} \times \boldsymbol{B}(\boldsymbol{x}, t), \qquad (1)$$

where the laboratory frame electric field is neglected. The model magnetic field $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{b}$ consists of a uniform mean magnetic field B_0 (in the Cartesian z-direction) and a composite model of transverse magnetic fluctuations $\boldsymbol{b} = [b_x(x, y, z), b_y(x, y, z), 0]$ consisting of a two-dimensional part $\boldsymbol{b}^{\rm 2D}(x, y)$ and a one-dimensional "slab" part $\boldsymbol{b}^{\rm slab}(z)$. These composite model fluctuations are made strongly three-dimensional by setting the energies to be in the ratio $E^{\rm slab} : E^{\rm 2D} = 20 : 80$, while $(b_{\rm slab}^2 + b_{\rm 2D}^2)/B_0^2 = 1$.[2] The integration method is essentially identical to that employed by Mace et al. (2000) and Qin et al. (2002; see Qin 2002).

We calculate particle perpendicular and parallel diffusion coefficients for timescales up to 1000 vt/λ_c (particle speed × time/correlation length). The test particles are resonant in the mid-inertial range of both the slab spectrum and the two-dimensional spectrum. This ensures that the sampled fluctuations have strong transverse structure. Both slab and two-dimensional spectra have a functional form that becomes flat at wavenumbers k much less than the correlation scale, going over to a $k^{-5/3}$ form at high k. Figure 1 illustrates the spectra we employ. For the slab component, we use a box of size 10,000λ and $N_z = 2^{22} = 4{,}194{,}304$ grid points in the parallel direction. For the two-dimensional component, we use a box of size 100λ × 100λ and $N_x = 4096 \times N_y = 4096$ points.[3] The two-dimensional component is chosen to have a smaller correlation scale, $\lambda_x = 0.1\lambda$. In the numerical scheme, all lengths are scaled to λ. Because the slab wavenumber spacing is much smaller, its spectrum (*dotted line*) is smoother than the two-dimensional spectrum (*solid line*).[4]

To investigate diffusive behavior, we follow trajectories of typically 1000 test particles with a normalized per step accuracy of one part in 10^9. For trajectories calculated to a time of $vt/\lambda_c = 1200$, this corresponds to an average change of 0.0012% in a single particle energy. To investigate the approach to diffusive behavior, we compute running diffusion coefficients, $\tilde{\kappa}_{xx} = d\langle(\Delta x)^2\rangle/2\,dt$ and $\tilde{\kappa}_{zz} = d\langle(\Delta z)^2\rangle/2\,dt$, where the time derivative is computed using a first-order finite difference. When the mean square displacements are diffusive ($\propto t$), the running diffusion coefficient is identical to the usual one.

3. NUMERICAL RESULTS

By computing the parallel and perpendicular running diffusion coefficients simultaneously, we can investigate both types of diffusion as well as their interactions with one another. Figure 2 shows results for a set of 1000 particles immersed in the strong two-component magnetostatic fluctuations described above. The particles have a ratio of gyroradius to parallel correlation scale $r_L/\lambda_c = 0.027$ and are therefore nominally resonant in the mid-inertial range of the slab fluctuation spectrum. Note that the parallel running diffusion coefficient saturates at a nearly level value after about 10 crossing times of the correlation scale.[5]

As for perpendicular transport, the upper panel of Figure 2 shows that the running diffusion coefficient rapidly advances in the first ~10 time units. However, it falls far short of the FLRW limit. Instead, it reaches a local maximum at about the time the parallel running diffusion coefficient stabilizes in the lower panel. Thereafter, the perpendicular diffusion coefficient decreases qualitatively in the fashion expected in the subdiffusive regime. However, instead of behaving as $t^{-1/2}$, as would be expected for compound subdiffusion, here the behavior stabilizes at very nearly a constant value. A fit from $vt/\lambda_c = 200$ to 1200 indicates a variation as $t^{-0.003}$. This is very close to a

[2] Such a two-component/slab spectral model is a useful representation of anisotropic turbulence in the solar wind (Matthaeus, Goldstein, & Roberts 1990; Bieber, Wanner, & Matthaeus 1996).

[3] Here λ is the parameter that appears explicitly in the model slab spectrum, related to the slab correlation length λ_c as $\lambda = \pi^{-1/2}[\Gamma(\frac{1}{3})/\Gamma(\frac{5}{6})]\,\lambda_c$, where $\Gamma(\frac{1}{3})$ and $\Gamma(\frac{5}{6})$ are the gamma functions.

[4] Note that we have modeled the turbulent spectral anisotropy (Matthaeus et al. 1990; Bieber et al. 1996) with some care since it is known that this exerts considerable influence on scattering and transport (Bieber et al. 1994; Dröge 2000) and since extremely simple one-dimensional models (Jokipii 1966; Skilling et al. 1974) may obscure such effects.

[5] The measured parallel diffusion coefficient is about one-third the standard QLT (Jokipii 1966) result shown by the dashed line. The discrepancy, which is not central to the points we emphasize here, is presumably due to nonlinear effects associated with our use of strong $\delta B/B_0 = 1$ turbulence.

purely diffusive regime. However, it is at a level lower than either the FLRW or the peak value temporarily obtained near $vt/\lambda_c \approx 10$. This restoration of perpendicular diffusive behavior, far into the regime in which parallel scattering is important, may be thought of as a regime of "second diffusion."

This result has proved to be robust—a number of additional examples (not shown) verify the recovery of perpendicular diffusion, each following the pattern described in Figure 2. We note also that this effect occurs in the absence of either collisions (Rechester & Rosenbluth 1978) or trapping (Chandran & Cowley 1998).

4. DISCUSSION: SECOND PERPENDICULAR DIFFUSION

Inherent in the QLT approach is the assumption that various types of transport, such as parallel and perpendicular scattering, can be computed independently. This assumption enters in QLT when the random forces are integrated along unperturbed trajectories. The phenomenon of compound subdiffusion (sometimes misnamed "compound diffusion") underscores the possibility that one form of transport can "interfere" with the establishment of another. In compound subdiffusion, parallel scattering begins to interfere with perpendicular transport when a sufficient number of particles in the ensemble slow their progress along field lines. Some turn around. At this point, the assumption of constant parallel velocity in the ensemble (i.e., the QLT assumption of unperturbed trajectories) fails. As long as the particles are free-streaming along field lines, the question as to whether their perpendicular transport becomes diffusive falls back on the issue of the complexity of the magnetic field viewed along its field lines. Without parallel scattering, if the field lines themselves randomize diffusively, the particles will also experience perpendicular diffusion. To accentuate this point, we present an example in Figure 3 that shows that particle transport perpendicular to the mean magnetic field indeed does attain the FLRW limit when the spectrum is adjusted to entirely suppress parallel scattering. Results with the same $r_L/\lambda_c = 0.027$ but with an inertial range that supports parallel scattering were shown by Qin et al. (2002) to produce perpendicular subdiffusion.

In the strong parallel scattering limit, the above free-streaming scenario becomes inapplicable. Instead, the questions are as follows: What is the nature of the field lines encountered by the particles as they attempt to retrace their steps? Are the field lines so coherent that the perpendicular displacements are restricted and thus the mean square transverse displacement increases less than linearly in time? In the limiting case of an ignorable coordinate in the magnetic field model, there is no complexity whatsoever in the corresponding direction, and the Jokipii theorem (Jokipii et al. 1993) suggests that for this case, diffusion in that particular direction is suppressed. For cases lacking an ignorable coordinate, the situation is less clear. One may ask in that case the converse question: Is the transverse complexity great enough that a new kind of randomization occurs? In the latter case, diffusion relies on structure perpendicular to the field lines rather than the structure along them.

For a wide range of parameters of interest (Giacalone & Jokipii 1999), parallel scattering is faster than perpendicular transport, and consequently the onset of parallel scattering has an opportunity to influence the nature of perpendicular transport. For magnetic field models lacking strong transverse structure, particles are thwarted in their attempt to follow field lines, and after a brief period of diffusion at levels somewhat lower than the FLRW limit, a long period of subdiffusive transport emerges. An outstanding question is whether the subdiffusive

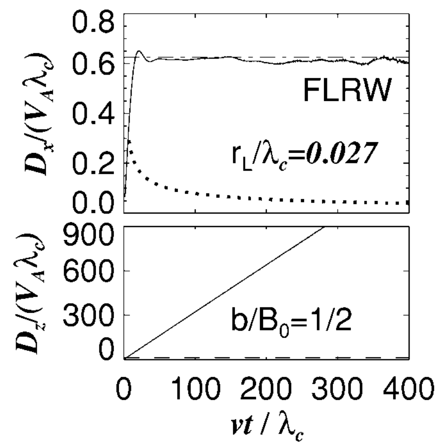

FIG. 3.—Here parallel diffusion has been suppressed, and the perpendicular transport quickly reaches the FLRW limit. This simulation uses nearly pure slab fluctuations ($E^{\rm slab} : E^{\rm 2D} = 9999 : 1$) similar to Fig. 1 of Qin et al. (2002) that showed subdiffusion, also using $r_L/\lambda_c = 0.027$, $b/B_0 = 0.5$, and $k_{\rm res\,min}\lambda_c = 37$. The difference is that here $k_{\rm max}\lambda_c = 0.24$, which suppresses parallel scattering by forming a wide resonance gap at a 90° pitch angle. In the lower panel, the solid line represents the running parallel diffusion coefficient vs. time. The dashed line is the approximate QLT result. In the upper panel, the solid line is the running perpendicular diffusion coefficient. The dotted line corresponds to subdiffusion.

regime is permanent or whether it gives way after some time to a second regime of diffusive scattering. While it is always difficult to rule out certain events for all time, particularly using a numerical approach, we believe that two partial answers have now been provided: First, for magnetic turbulence with very little transverse structure, subdiffusion is certainly a long-lived state (Qin et al. 2002) and mostly likely a permanent state. Second, for the present result, i.e., for magnetic turbulence with strong transverse structure, a regime of second diffusion is recovered.

The latter conclusion has a number of antecedents in the literature. Early on, Jokipii (1973) recognized that the structure of the magnetic field fluctuations perpendicular to the mean field might influence perpendicular transport, because the field lines sampled by a gyrating particle would tend to separate. In this way, Jokipii (1973) reasoned that FLRW perpendicular transport of particles would be accurate for low-energy particles, which have gyroradii too small to sample very much transverse structure.

Skilling et al. (1974) carried out a calculation of galactic cosmic-ray scattering, noting apparently for the first time that divergence of neighboring field line is crucial in situations in which escaping particles' parallel scattering brings them back into the galaxy. They noted that in order to scatter particles more than a gyroradius from their original field line, one needed to invoke field line separation. They used this idea to estimate, in effect, the distance a particle would scatter in the perpendicular direction for each unit of length transported along the magnetic field. The calculation of Skilling et al. (1974) is rather specialized in that it applies to a bounded inhomogeneous system (the galaxy) with externally located parallel scattering centers; however, it is noteworthy that this is perhaps the first calculation in which it transpires that collisionless perpendicular transport depends explicitly on parallel transport.

Somewhat later, it was recognized that the logical conclusion of the above reasoning is that perpendicular transport can be nondiffusive. Urch (1977) pointed out that particle motions are not free-streaming but rather represent a parallel random walk, so that when particles backscatter through 90°, the perpendicular displacement is decreased. Accordingly, the estimate $\Delta z/\Delta t \sim v$ is incorrect, and, instead, the parallel displacements are diffusive, $\Delta z/\Delta t \approx (2\kappa_{zz} t)^{1/2}$. Thus, one finds instead that[6] $\tilde{\kappa}_{xx} = D_\perp [\kappa_{xx}/(\pi t)]^{1/2}$. From this perspective, starting from weak transverse structure, what is needed is some effect to *restore* perpendicular diffusion.

The year following Urch (1977), the same issue was again discussed by Rechester & Rosenbluth (1978), but from a different perspective, namely, that when particles' gyrocenters follow magnetic field lines, then the phase-space density structure in directions perpendicular to the magnetic field evolves as an area-preserving map. Field line wandering can make the phase-space structure wrap up and fold, but without some additional effect, these surfaces cannot merge or break, so that in the presence of parallel scattering, the entire process can be undone. The assertion (given by Rechester & Rosenbluth (1978) essentially without proof) is that a small amount of scattering (in their case presumably due to collisions) can cause the complex transverse phase-space structure to blur slightly, so that parallel scattering can no longer cause the full restoration of the initial state. The conclusion is that this slight additional collisional effect restores perpendicular transport to the FLRW rate. These arguments are physically appealing but quantitatively inconsistent with our simulation results. There have been various calculations (e.g., Chandran & Cowley 1998) that have adopted this assertion as proven and, moreover, as applicable to the collisionless limit. However, to our knowledge, our current line of research is the first to examine the issue of the loss and restoration of perpendicular diffusion directly and quantitatively using accurate numerically determined particle orbits.

From our present simulation studies, we conclude that for magnetic turbulence with little transverse structure, subdiffusion is a long-lived state and mostly likely permanent state. But for magnetic turbulence with strong transverse structure, a regime of second diffusion is recovered. We view the "first" diffusion to be the evolution toward the FLRW limit. If there is no parallel scattering, this first diffusion limit is achieved (see Fig. 3). Parallel scattering suppresses this tendency, decreasing the mean square transverse separation relative to the FLRW expectation. Subsequently, either one gets subdiffusion or, if there is sufficient transverse structure, a new regime of secondary diffusions appears. This conclusion is qualitatively in accord with the physical reasoning of Jokipii (1973), Skilling et al. (1974), and Rechester & Rosenbluth (1978). However, none of these seem to have recognized that the collisionless limit can result in a stable diffusion regime at a rate lower than the FLRW limit, which is nevertheless manifestly nonlinear in a manner at least partially anticipated by these authors.

The recovery of diffusion has been seen to occur in the numerical examples relatively quickly (within ∼100 correlation length crossing times). Although we do not have a general theoretical perspective on the length of time for second diffusion to be realized, we note that the time observed in the simulations is short enough to be of interest in problems of heliospheric transport. In the solar wind at 1 AU, the correlation scale is about 0.02 AU. Diffusive transport theory is often applied over scales of many AU, up to the size of the heliosphere, for example, in the cosmic-ray modulation problem. Second perpendicular diffusion also persists for at least 1000 crossing times of the parallel correlation length and perhaps indefinitely. Thus, the observed regime of second diffusion may be relevant to certain heliospheric and astrophysical transport problems. It remains to quantify, either through simulations or through analytical approaches, the precise level of transverse complexity of the magnetic turbulence that is needed to attain this recovery of diffusion.

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[6] Kóta & Jokipii (2000) reached the same conclusion.

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